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Basic Analysis Techniques & Multi-Spacecraft Data — Computer Session —

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4 Gradient estimation in measured magnetic fields

In this exercise the reciprocal vector method is applied to estimate electrical currents using measurements of the Cluster-II FGM instruments. This method for spatial derivative estimation is briefly described in the appendix (section 4.3). Here we look at events that were previously analysed by Dunlop et al. [2002] (note that these authors used a slightly different but equivalent method, namely, the so-called curlometer technique). The reciprocal vector method is coded in a bundle of IDL programs called `cdat`.

4.1 How to use the IDL program `cdat`

All IDL programs and data files needed for this exercise can be found in the subdirectory `ComputerSessions/BasicAnalysisTechniques_Vogt/ex4/` of the workshop web page[1]. For convenience, you may download the zip archives `ex4pro.zip` and `ex4dat.zip` which contain all the files. You may want to create a subdirectory for this exercise on your local PC

```
Linux> mkdir ex4
Linux> cd ex4
```

put the zip archives into the subdirectory, and then unzip them

```
Linux> unzip ex4pro.zip
Linux> unzip ex4dat.zip
```

You should now find two subdirectories `pro/` and `dat/` in `ex4/` containing the program and data files.

The driver routine is called `cdat`. In order to use it, go to the subdirectory `pro/` where `cdat` resides,

```
Linux> cd pro
```

then simply edit the parameter section (either in an editor like `xemacs`, or using the `idlde` editor), and type

```
IDL> .r cdat
```

at the IDL prompt.

[1] http://www.faculty.iu-bremen.de/jvogt/cospar/cbw6/

The resulting graphics are not displayed on the screen but are directed to a number of postscript files:

`curlB.ps` Shown are the three components of the estimated $\nabla \times \mathbf{B}$, $\nabla \cdot \mathbf{B}$, the length scale parameters $R/\sqrt{3}$ and $\sqrt{3}/K$ (see appendix 4.3.2) to assess the geometric quality of the spacecraft configuration (the curves should coincide for a regular tetrahedron).

`curlB_mu0.ps` Displayed are the three components of the estimated $\nabla \times \mathbf{B}$ and $\nabla \cdot \mathbf{B}$, both divided by μ_0. Electrical current density is $\mathbf{j} = \nabla \times \mathbf{B}/\mu_0$.

`gradB.ps` The graphics shows the elements of the magnetic gradient matrix $\nabla\mathbf{B}$.

`fgm.ps,reprovec.ps` Here the magnetic field data at the four cluster spacecraft and the reciprocal vectors are shown (mainly for debugging purposes).

4.1.1 Pre-selected events

The `cdat` program offers a number of parameters which control the file selection, the analysis, and the output. In order to facilitate the use of `cdat` in this exercise, a few events have been pre-selected for the analysis. The pre-selection is controlled by the parameter `EVENTDATE` which is the first one to appear in the parameter section. If `EVENTDATE` is assigned one of the predefined values, then all other parameters are chosen automatically. The data files for the pre-selected events are contained in the zip archive `ex4dat.zip`.

The pre-selected events were analysed in a paper by Dunlop et al. (M. W. Dunlop, A. Balogh, and K.-H. Glassmeier Four-point Cluster application of magnetic field analysis tools: The Curlometer, *J. Geophys. Res., 107*(A11), 1384, doi:10.1029/2001JA005088, 2002[2]). You are encouraged to look at the article for a check of your results and a detailed discussion of the events.

4.1.2 Program parameters

If you choose `EVENTDATE='none'`, then you have to set a number of parameters which control the execution of `cdat`. Here are the most important ones.

`DATA_READ` Controls whether data in the buffer are used or new data are read from the specified files.

`TIME_FORMAT` Choose always `TIME_FORMAT='IGM_STRING'`.

`FILENAMES` Array of strings containing the filenames. Usually, only the substring which contains the date has to be edited. The files as a whole are read sequentially into the buffer, selection of time intervals is done afterwards.

`TBEG,TEND` Start and end time in UT.

`DT` Sampling interval in seconds.

[2]http://www.agu.org/pubs/crossref/2002/2001JA005088.shtml

`DATE,INFOSTRING` Information printed into the lower right corner of the plots.

`OFFSETS_CORR,OFFSETS_CALC` Based on a minimization criterion, spin axes offsets are calculated and stored in the files `offsets.sav` and `offsets.log` if `OFFSETS_CALC=1` is chosen. Spin axis offset correction is applied to the data set if `OFFSETS_CORR=1` is set: in this case the offsets calculated in a previous run are read from the file `offsets.sav` and subtracted. This implies that you have to run `cdat` twice if offsets are supposed to be corrected, the first time with `OFFSETS_CALC=1` and `OFFSETS_CORR=0`, and the second time with `OFFSETS_CORR=1` and `OFFSETS_CALC=0`.

`SPINAXESONLY,OFFSETS_CRIT,OFFSETS_EXPL` Parameters related with offset correction. In this exercise there is no need to change the default values.

`GRADIENTS_CALC,GRADIENTS_SAVE,GRADIENTS_PLOT` These parameters control whether spatial derivatives should be calculated, saved, and/or plotted.

`WSMO_SECONDS` Width of the smoothing window in seconds.

4.1.3 Suggestions

After you have made yourself familiar with the parameter section of `cdat`, you may want to play around with the program as follows.

- Choose `EVENTDATE='none'`, and set manually all the parameters needed to analyse one of the events in the Dunlop et al. [2002] paper. Compare your results with those from a run where `EVENTDATE` was set explicitly to the respective date.
- Select a different portion of the orbit or zoom into a specific part of the plot by changing the time interval parameters `TBEG` and `TEND`.
- Select `OFFSETS_CALC=1` and look at the offset correction suggested by the program. Then set `OFFSETS_CORR=1` and run the program again to see how much the divergence is reduced.
- Look at other files in the subdirectory `dat/` of `ex4/` and analyse them using the different options of the `cdat` program.
- Plot the orbits of the Cluster satellites for the events chosen here using, e.g., the link to NASA's satellite situation center[3] .

4.2 Links and references

In 1996 and 1997 the International Space Science Institute (ISSI)[4] hosted a team of space scientists who compiled analysis methods for the Cluster mission in a book entitled *Analysis Methods for Multi-Spacecraft Data*. It is available as free pdf from the server of the *International Space Science Institute (ISSI)* in Bern, Switzerland. For the 3rd COSPAR Capacity Building Workshop a local copy can be accessed from the workshop web page[5].

[3]http://sscweb.gsfc.nasa.gov/
[4]http://www.issi.unibe.ch/
[5]http://www.faculty.iu-bremen.de/jvogt/cospar/cbw6/Etc/issibook.pdf

Here is (once again, for completeness) the reference where the pre-selected events have been chosen from: M. W. Dunlop, A. Balogh, and K.-H. Glassmeier Four-point Cluster application of magnetic field analysis tools: The Curlometer, *J. Geophys. Res., 107*(A11), 1384, doi:10.1029/2001JA005088, 2002[6] .

Information about the Cluster-II mission (science, orbit information, display and retrieval of public data) can be found here:

- Cluster Science Data System[7]
- German Cluster Data Centre[8]

Software for analysis and display of spacecraft data: QSAS (Queen Mary)[9] .

General spacecraft orbit information: NASA's satellite situation center[10]

Public data centers (geomagnetic indices etc):

- WDC-C2 KYOTO AE index service[11]
- Auroral Electrojet (AE) Index (NOAA)[12]
- NASA's geophysical data base[13]
- OMNIWeb Data System[14]
- ISTP/IACG CDAW Data Set[15]
- NOAA Data Center[16]
- International Service of Geomagnetic Indices (ISGI)[17]
- National Geophysical Data Center (NGDC)[18]

[6] http://www.agu.org/pubs/crossref/2002/2001JA005088.shtml
[7] http://sci2.estec.esa.nl/cluster/csds/csds.html
[8] http://cl1.plasma.mpe-garching.mpg.de/cdms/
[9] http://www.space-plasma.qmw.ac.uk/
[10] http://sscweb.gsfc.nasa.gov/
[11] http://swdcdb.kugi.kyoto-u.ac.jp/aedir/
[12] http://www.ngdc.noaa.gov/stp/GEOMAG/ae.html
[13] http://nssdc.gsfc.nasa.gov/space
[14] http://nssdc.gsfc.nasa.gov/omniweb/
[15] http://cdaw.gsfc.nasa.gov/
[16] http://www.ngdc.noaa.gov/seg//potfld/geomag.html
[17] http://tango.cetp.ipsl.fr/ isgi/homepag1.htm
[18] http://www.ngdc.noaa.gov/seg/potfld/geomag.shtml

4.3 Appendix: Theory of spatial gradient estimation

So far (at least) two methods to estimate spatial derivatives from Cluster data have been introduced by various authors, namely, the so-called curlometer technique and the reciprocal vector (or barycentric coordinates) method. Both methods are equivalent in the sense that linear field variations are evaluated (note that the linear approximation of a field is unique if the values at four distinct points in space are known). Detailed discussions can be found in various chapters of *Analysis Methods for Multi-Spacecraft Data*, G. Paschmann and P. Daly (Eds.), ESA Publication Division (Noordwijk, Netherlands), 1998.

In this exercise we only address the reciprocal vector method because the structure of the estimation formulas appears to be more transparent.

4.3.1 The reciprocal vector method

Briefly, barycentric coordinates provide a convenient means to linearly interpolate a physical quantity g inside a satellite cluster tetrahedron by using the measured values g_α at the four spacecraft positions $\mathbf{r}_\alpha$:

$$\tilde{g}(\mathbf{r}) \;=\; \sum_{\alpha=0}^{3} \mu_\alpha(\mathbf{r}) g_\alpha$$

where:

$$\mu_\alpha(\mathbf{r}) \;=\; 1 + \mathbf{k}_\alpha \cdot (\mathbf{r} - \mathbf{r}_\alpha)$$

$\tilde{g}$ denotes the linear function that interpolates between the measurements. The vectors $\mathbf{k}_\alpha$ are given by the formula:

$$\mathbf{k}_\alpha \;=\; \frac{\mathbf{r}_{\beta\gamma} \times \mathbf{r}_{\beta\lambda}}{\mathbf{r}_{\beta\alpha} \cdot (\mathbf{r}_{\beta\gamma} \times \mathbf{r}_{\beta\lambda})}$$

$(\alpha, \beta, \gamma, \lambda)$ must be a permutation of $(0, 1, 2, 3)$. Relative position vectors are denoted by $\mathbf{r}_{\alpha\beta} = \mathbf{r}_\beta - \mathbf{r}_\alpha$. The set $\{\mathbf{k}_\alpha\}$ is called the reciprocal base of the tetrahedron.

Vector functions $\mathbf{B}$ can be handled in a similar way by applying the above formulas to the cartesian components. Since $\tilde{g}$ and $\tilde{\mathbf{B}}$ are linear functions, the calculation of the derivatives can be done quite easily. The results are:

$$\begin{aligned}
\nabla g \;\simeq\; \nabla \tilde{g} \;&=\; \sum_{\alpha=0}^{3} \mathbf{k}_\alpha g_\alpha \\
\hat{\mathbf{e}} \cdot \nabla g \;\simeq\; \hat{\mathbf{e}} \cdot \nabla \tilde{g} \;&=\; \sum_{\alpha=0}^{3} (\hat{\mathbf{e}} \cdot \mathbf{k}_\alpha) g_\alpha \\
\nabla \cdot \mathbf{B} \;\simeq\; \nabla \cdot \tilde{\mathbf{B}} \;&=\; \sum_{\alpha=0}^{3} \mathbf{k}_\alpha \cdot \mathbf{B}_\alpha \\
\nabla \times \mathbf{B} \;\simeq\; \nabla \times \tilde{\mathbf{B}} \;&=\; \sum_{\alpha=0}^{3} \mathbf{k}_\alpha \times \mathbf{B}_\alpha
\end{aligned}$$

The element (i, j) of the matrix $\nabla\mathbf{B}$ is given by:

$$\frac{\partial V_j}{\partial x_i} \;\equiv\; (\nabla\mathbf{B})_{ij} \;\simeq\; \sum_{\alpha=0}^{3} (\mathbf{k}_\alpha \mathbf{B}_\alpha)_{ij} \;\equiv\; \sum_{\alpha=0}^{3} k_{\alpha i} V_{\alpha j}$$

With regard to error estimation it is important to notice that $\nabla \times \mathbf{B}$ and $\nabla \cdot \mathbf{B}$ are just linear combinations of various $(\nabla\mathbf{B})_{ij}$'s and thus of terms like $k_{\alpha i} V_{\alpha j}$, with $i = j$ or $i \neq j$.

In short,

$$\nabla\mathbf{B} \;\simeq\; \nabla \cdot \tilde{\mathbf{B}} \;=\; \sum_{\alpha=0}^{3} \mathbf{k}_\alpha \mathbf{B}_\alpha^\dagger \;.$$

Note that the superscript † denotes the transposition (of a matrix, or, if applied to a vector, to turn column vectors into row vectors and vice versa).

4.3.2 Inter-spacecraft distance and geometric error parameters

In this exercise we assume (without loss of generality) that we analyse the data in a frame moving with the barycenter of the tetrahedron

$$\mathbf{r}^{\mathrm{bc}} \;=\; \frac{1}{4}\sum_{\alpha=0}^{3} \mathbf{r}_\alpha \;,$$

thus $\mathbf{r}^{\mathrm{bc}} = 0$. Then the parameter

$$R \;=\; \sqrt{\frac{1}{4}\sum_{\alpha=0}^{3} |\mathbf{r}_\alpha|^2}$$

is a measure of the distance between the four Cluster spacecraft that can be used for scaling purposes. Note that R^2 is 1/4 the trace of the spacecraft position tensor

$$\mathsf{R} \;=\; \sum_{\alpha=0}^{3} \mathbf{r}_\alpha \mathbf{r}_\alpha^\dagger \;.$$

(This expression differs by a factor of 1/4 from the definition of the volumetric tensor given by Harvey in the ISSI Cluster data analysis book.)

In a similar manner we can define the reciprocal tensor as

$$\mathsf{K} \;=\; \sum_{\alpha=0}^{3} \mathbf{k}_\alpha \mathbf{k}_\alpha^\dagger$$

which can be shown to be the inverse of the spacecraft position tensor. The square root of the trace of K, i.e.,

$$K \;=\; \sqrt{\sum_{\alpha=0}^{3} |\mathbf{k}_\alpha|^2}$$

constitutes an inverse length scale which turns out to be important in the analysis of geometric errors in gradient estimation. First-order (isotropic) error estimation yields for the geometric error of a spatial derivative $D\mathbf{B}$:

$$\delta|D\mathbf{B}| \;=\; \sqrt{\frac{f}{3}}\,K\,\delta B$$

where δB denotes a typical error of the field measurement. The parameter f can be understood as the number of degrees of freedom of the differential operator D; use $f = 3$ if $D\mathbf{B} = \nabla \cdot \mathbf{B}$, $f = 2$ if $D\mathbf{B} = \nabla \times \mathbf{B}$, and $f = 1$ if $D\mathbf{B} = \hat{\mathbf{e}} \cdot \nabla\mathbf{B}$ (directional derivative or partial derivative). For details, see the ISSI Cluster data analysis book.