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Joachim Vogt

Basic Analysis Techniques & Multi-Spacecraft Data — Computer Session —

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5 Magnetospheric boundary analysis

Fundamental (single-spacecraft) boundary analysis techniques based on the minimum variance principle are addressed in a small programming exercise. The AMPTE/IRM data set used in chapter 8 (by B. Sonnerup and M. Scheible) of the ISSI Cluster data analysis book should serve as a reference case. Since IDL provides a number of modules like `EIGENQL`, `PCOMP`, or the family of `SVD` routines that can be used for an eigenvector analysis of the (co)variance matrix, the actual programming effort can be kept small.

A genuine multi-spacecraft approach to boundary analysis takes the crossing times t_α and locations $\mathbf{r}_\alpha$ to estimate boundary parameters such as the boundary normal vector $\hat{\mathbf{n}}$ and the boundary velocity V (in the spacecraft frame of reference). This method will be addressed in a separate subsection 5.4.

5.1 Theory: minimum variance analysis

Because of $\nabla \cdot \mathbf{B} = 0$, at a planar (one-dimensional) magnetospheric boundary the normal component of the magnetic field should be constant. Although this ideal situation is never perfectly met in space, it motivates the principle of *minimum variance analysis on magnetic field (MVAB)*: the magnetic field variation should be minimum in the direction of the boundary normal. This statement can be made mathematically precise in the least square sense which leads to an eigenvector analysis of the *(co)variance matrix*

$$\mathsf{M} \;=\; \left\langle (\mathbf{B} - \langle \mathbf{B} \rangle)(\mathbf{B} - \langle \mathbf{B} \rangle)^\dagger \right\rangle \;=\; \begin{pmatrix} \mathrm{cov}(B_x, B_x) & \mathrm{cov}(B_x, B_y) & \mathrm{cov}(B_x, B_z) \\ \mathrm{cov}(B_y, B_x) & \mathrm{cov}(B_y, B_y) & \mathrm{cov}(B_y, B_z) \\ \mathrm{cov}(B_z, B_x) & \mathrm{cov}(B_z, B_y) & \mathrm{cov}(B_z, B_z) \end{pmatrix}$$

where $\langle \cdots \rangle$ denotes averaging, and $(\cdots)^\dagger$ the transpose (turns column vectors into row vectors). The matrix M has only positive eigenvalues, and the eigenvector with the smallest eigenvalues is taken as the boundary normal.

A derivation and a detailed discussion of the minimum variance technique can be found in chapter 8 (by B. Sonnerup and M. Scheible) of the book *Analysis Methods for Multi-Spacecraft*

Data, G. Paschmann and P. Daly (Eds.), ESA Publication Division (Noordwijk, Netherlands), 1998. The book is available as free pdf from the ISSI server (http://www.issi.unibe.ch/PDF-Files/analysis_methods_1_1a.pdf). For the 3rd COSPAR Capacity Building Workshop a local copy can be accessed from the workshop web page workshop web page[1]. For brevity, we refer to the book chapter by Sonnerup and Scheible as ISSI-C8.

5.2 The data set

The data set used in ISSI-C8 can be found in the subdirectory `ComputerSessions/BasicAnalysisTechniques_Vogt/ex5/` of the workshop web page directory[2]. Two formats are available: `ampte.dat` is an ascii file and can thus be read with an editor, and `ampte.sav` is an IDL binary file. Inside IDL, the ascii file can be read using the routine `read-ampte.pro` that resides in the same directory as the data files.

Download the data files `ampte.dat` and `ampte.sav` as well as the program `read-ampte.pro`. Create a subdirectory for this exercise on your local PC

```
Linux> mkdir ex5
Linux> cd ex5
```

where all files and your programs should be stored. The data set becomes available in IDL either by the command

```
IDL> restore, 'ampte.sav'
```

or by typing

```
IDL> .r read-ampte
```

5.3 Programming

You are now supposed to write a short IDL program which performs a minimum variance analysis on a given data set, and to test this program by means of the sample data set discussed in ISSI-C8. After you have read that book chapter and you have made yourself familiar with the underlying principles of minimum variance analysis, you should think about structuring your program. Here are some hints on how you may proceed.

1. First write a module that constructs the variance matrix of the data.
2. The next step should be an eigenvector analysis of the variance matrix. Have a look at the IDL function `EIGENQL` and its keyword `EIGENVECTORS`. Write all three eigenvalues and, as an estimate for the boundary normal, the eigenvector of the smallest eigenvalue to a log file.
3. Compute the projections B_1, B_2, B_3 of the data set onto the eigenvectors of the variance matrix.
4. Produce hodogram plots of (B_1 vs.B_2) and (B_1 vs.B_3). Print the eigenvalue ratios λ_1/λ_2 and λ_1/λ_3 underneath.

[1] http://www.faculty.iu-bremen.de/jvogt/cospar/cbw6/Etc/issibook.pdf

[2] http://www.faculty.iu-bremen.de/jvogt/cospar/cbw6/

Note that there are some built-in IDL procedures which already do part of the job, e.g., the procedure `PCOMP`, or the family of `SVD` routines. Consult the IDL help system for a description.

5.4 Multi-spacecraft data: crossing times approach

As described in the associated lecture, boundary crossing times t_α of the four Cluster-II spacecraft can be used to estimate the 'slowness' vector $\mathbf{m}$ as follows

$$\mathbf{m} = \sum_\alpha t_\alpha \mathbf{k}_\alpha \ .$$

Here the $\mathbf{k}_\alpha$ are the reciprocal vectors of the Cluster-II tetrahedron, the speed $V = 1/|\mathbf{m}|$, and the boundary normal vector $\hat{\mathbf{n}} = \mathbf{m}/V$.

On the basis of the `cdat` procedures (see sheet 4) you may write an interactive program which

- first displays the magnetic field data on the screen so that the crossing times can be determined visually,
- then reads the crossing times from keyboard input (you may also use the `cursor` command and determine the crossing times by mouse clicks), and,
- finally, computes the (average) reciprocal vectors, the slowness vector $\mathbf{m}$ as well as the boundary normal vector $\hat{\mathbf{n}}$ and the speed V.