AC field measurements and wave analysis tools

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Outline:

- **Introduction**
  - Waves in plasmas
  - Cold plasma theory – CMA diagram
  - Examples of waves in the Earth's magnetosphere
- **Analysis methods for multi-component measurements.**
  - STAFF-SA and WBD instruments onboard Cluster
  - Plane wave methods
  - Wave distribution function
  - Backward ray tracing
- **Examples of measurements of different types of waves in space plasmas**
  - Auroral hiss
  - Auroral kilometric radiation
  - Electron and proton whistlers
  - Equatorial noise
  - Whistler mode chorus
  - Magnetosheath "lion roar" emissions

Waves in plasmas

Plane wave

$$\vec{B}(t,\vec{x}) = \vec{B}_0 + \Re\left\{ \vec{\mathcal{B}} \exp\left[i(\omega t - \vec{k}\cdot\vec{x})\right] \right\}$$

wave vector

wave frequency

complex amplitudes

Maxwell's equations

$$\vec{k}\times\vec{\mathcal{E}} = \omega\vec{\mathcal{B}}$$

$$\vec{k}\times\vec{\mathcal{B}} = -\frac{\omega}{c^2}\left(\vec{\mathcal{E}} - \frac{i}{\omega\varepsilon_0}\vec{\mathcal{J}}\right) = -\frac{\omega}{c^2}\,\boldsymbol{\epsilon}\cdot\vec{\mathcal{E}}$$

plasma dielectric tensor

Wave equation

$$N^2\left(\vec{\kappa}\times\vec{\kappa}\times\vec{\mathcal{E}}\right) + \boldsymbol{\epsilon}\cdot\vec{\mathcal{E}} = 0$$

refractive index $N = \frac{\|\vec{k}\|\,c}{\omega}$,

$\vec{\kappa} = \frac{\vec{k}}{\|\vec{k}\|}$ wave normal direction

Equation of motion: **COLD PLASMA** approximation

$$\vec{\mathcal{J}} = \sum_s n_s q_s \vec{\mathcal{V}}_s \qquad m_s \vec{\mathcal{V}}_s \, i\omega = q_s \left(\vec{\mathcal{E}} + \vec{\mathcal{V}}_s \times \vec{B}_0 \right)$$

Dielectric tensor

Cartesian coordinates

$$\boldsymbol{\epsilon} \cdot \vec{\mathcal{E}} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \cdot \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ \mathcal{E}_z \end{pmatrix} \qquad \vec{B}_0 = \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix}$$

$$S = \frac{1}{2}(R+L) \qquad R = 1 - \sum_s \frac{\Pi_s^2}{\omega(\omega+\Omega_s)}$$

$$D = \frac{1}{2}(R-L) \qquad L = 1 - \sum_s \frac{\Pi_s^2}{\omega(\omega-\Omega_s)}$$

$$P = 1 - \sum_s \frac{\Pi_s^2}{\omega^2}$$

cyclotron frequency

$$\Omega_s = \frac{q_s B_0}{m_s}$$

plasma frequency

$$\Pi_s^2 = \frac{n_s q_s^2}{m_s \varepsilon_0}$$

Wave equation

$\angle(\vec{k}, \vec{B}_0)$

$$\begin{pmatrix} S - N^2\sin^2\theta & -iD & N^2\cos\theta\sin\theta \\ iD & S - N^2 & 0 \\ N^2\cos\theta\sin\theta & 0 & P - N^2\sin^2\theta \end{pmatrix} \cdot \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ \mathcal{E}_z \end{pmatrix} = 0$$

Dispersion relation

$$AN^4 + BN^2 + C = 0$$

Examples of solutions

$$\theta = 0 \quad : \quad N^2 = R$$

R-mode

$$N^2 = L$$

L-mode

$$\theta = \frac{\pi}{2} \quad : \quad N^2 = \frac{RL}{S}$$

X-mode

$$N^2 = P$$

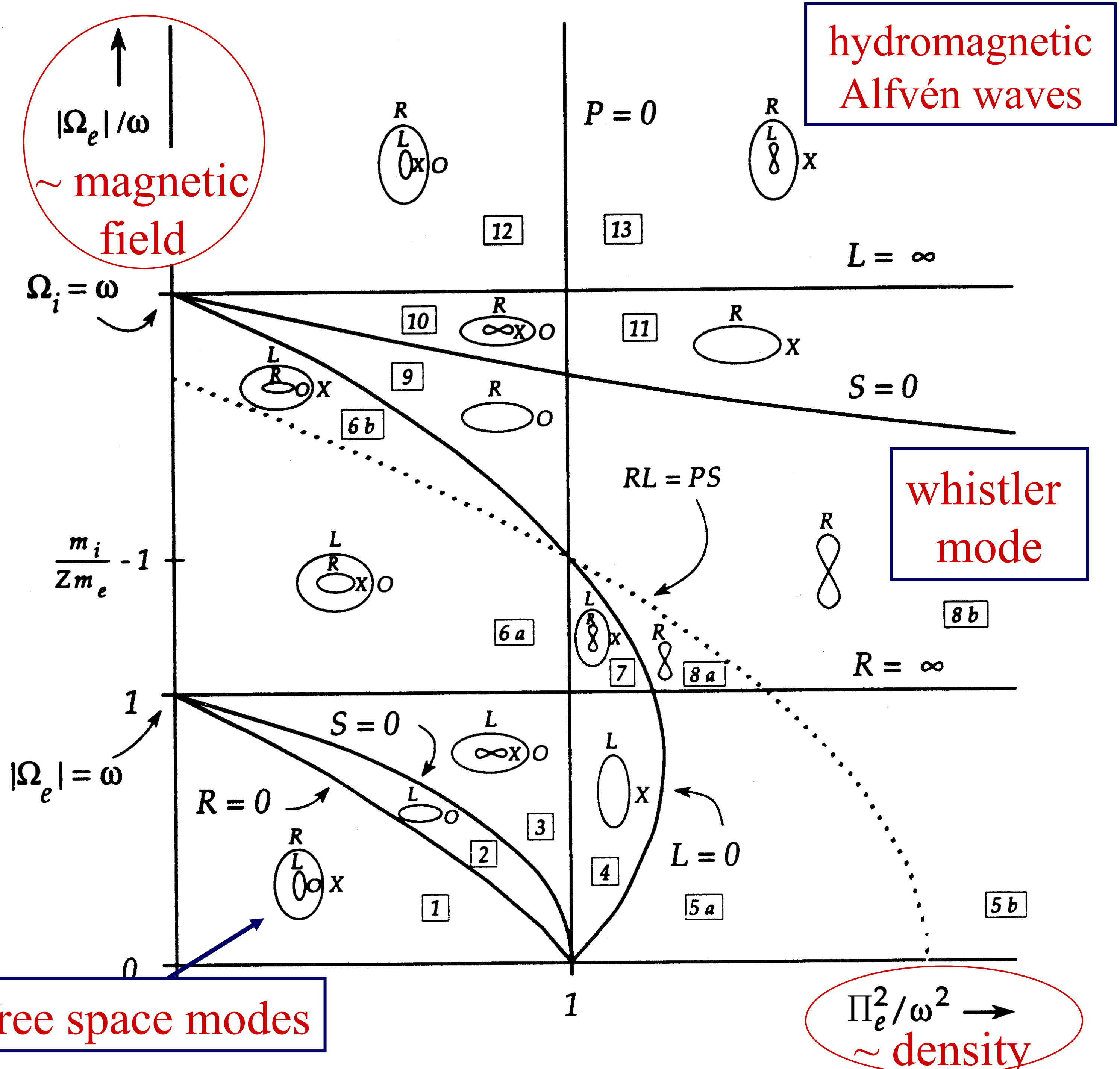
O-mode

CMA diagram

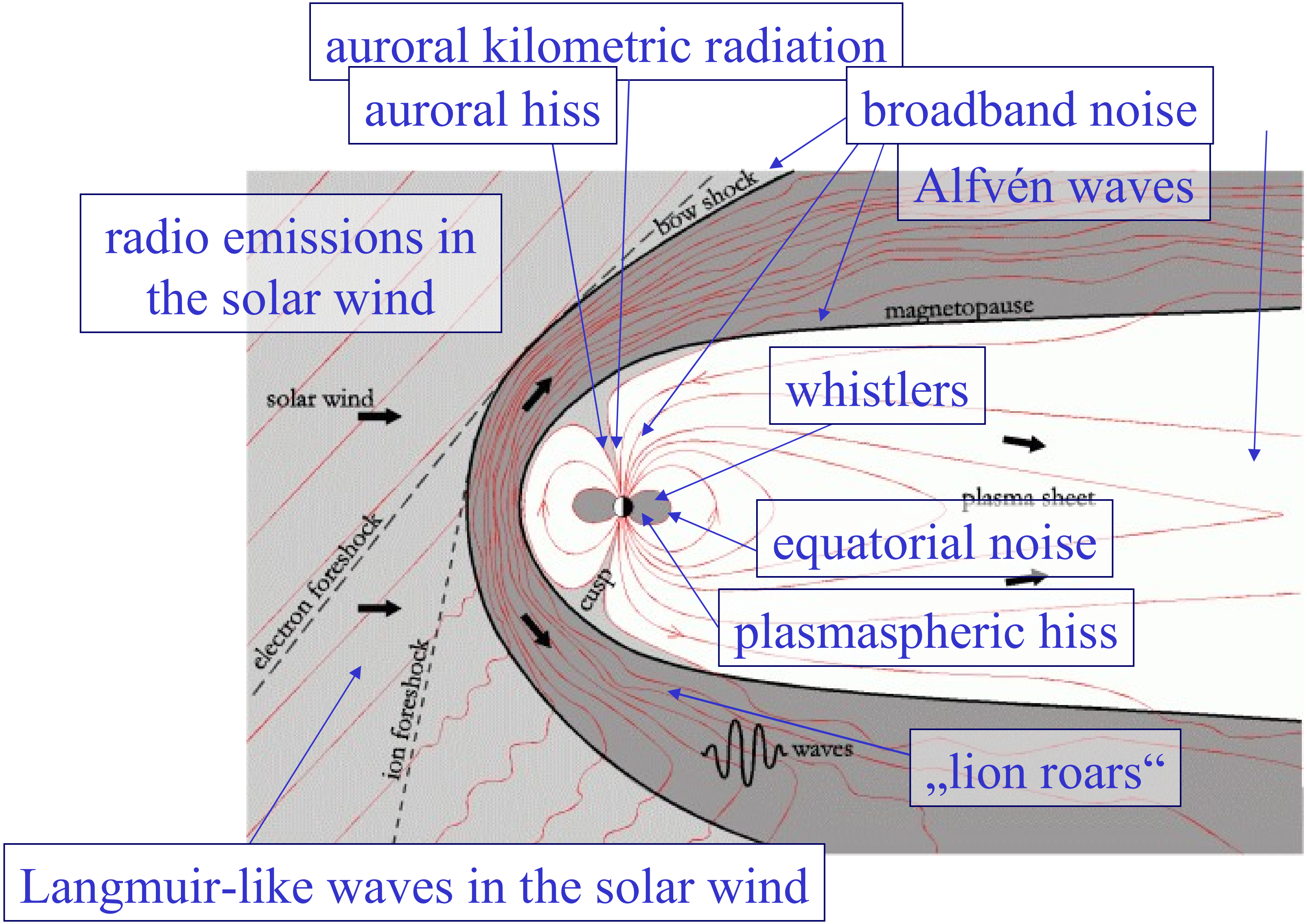
(Clemmow, Mullaly, and Allis)

Ingredients:

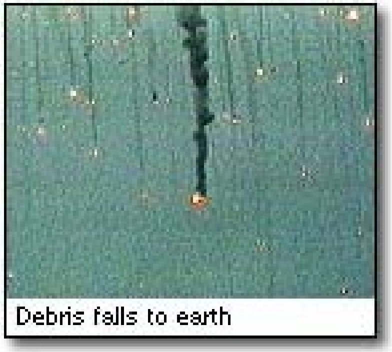
- Maxwell's equations
- Equation of motion of a collisionless, cold, two-component plasma
- Linear theory
- Homogeneous medium



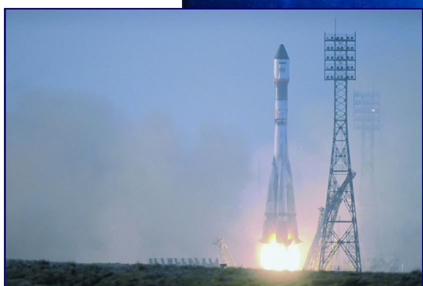
Examples of waves in the Earth's magnetosphere and its nearest environment



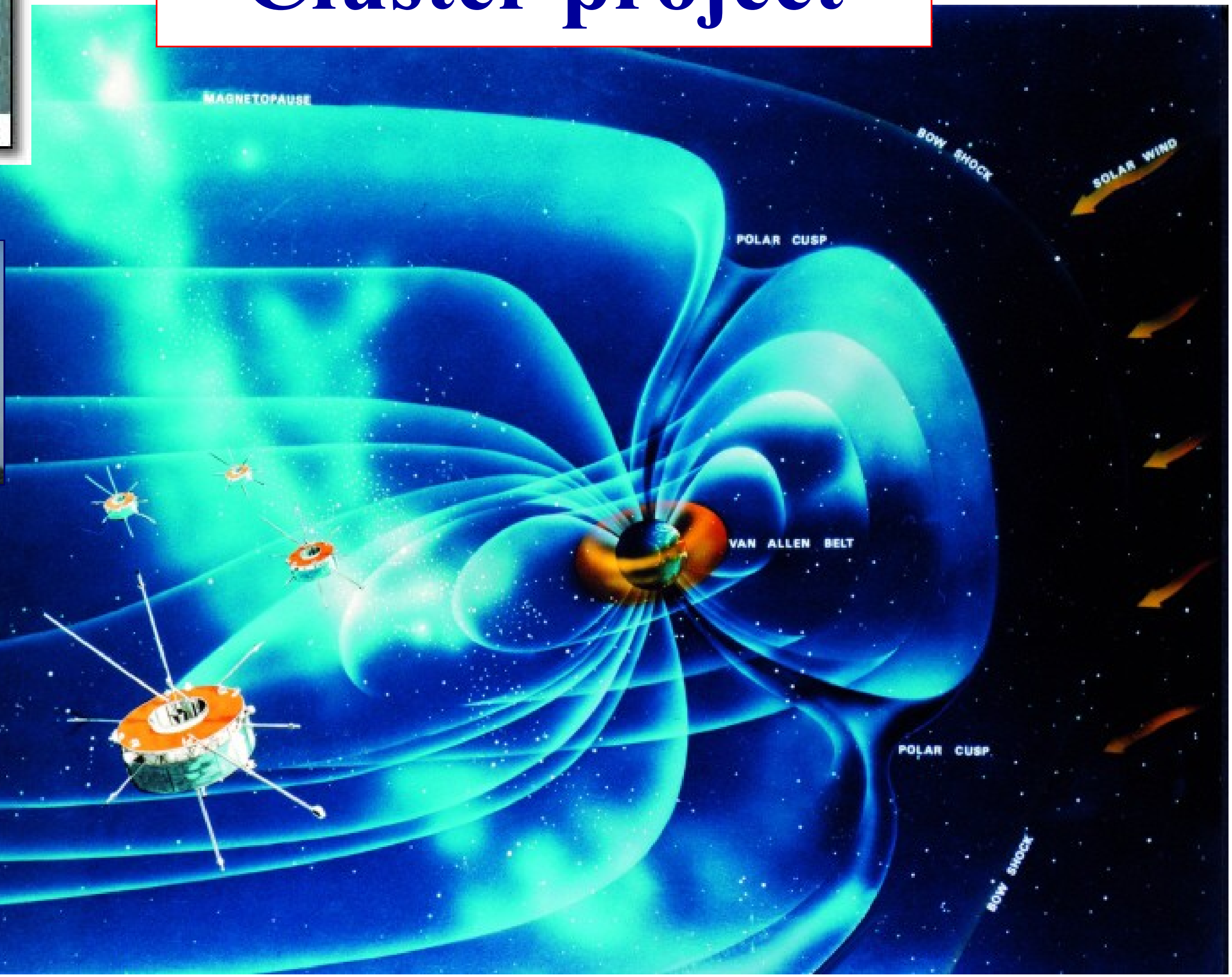
Cluster project



1996



2000



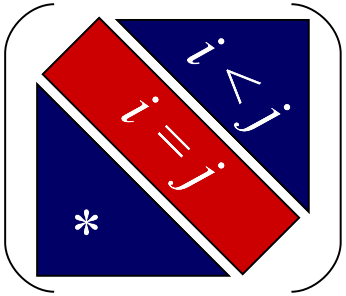
STAFF Instrument

(Spatio Temporal Analysis of Field Fluctuations)

- search coil sensors (tri-axial, 0.1 Hz- 4 kHz)
- waveform analyzer (0.1 -180 Hz)
- spectrum analyzer

On-board analysis

- 3 magnetic (STAFF) and 2 electric signals (EFW)
- 27 frequency channels 8-4000 Hz, 26% bandwidth
- complex amplitudes: $\mathcal{B}_x, \mathcal{B}_y, \mathcal{B}_z, \mathcal{E}_x, \mathcal{E}_y$
- spectral matrices 5x5: $\hat{S}_{ij} = \langle \, \xi_i \, \xi_j^* \, \rangle$
  - power-spectral densities (time resolution 0.125s-2s)
  - relative phase shifts, mutual coherency (1s-4s)

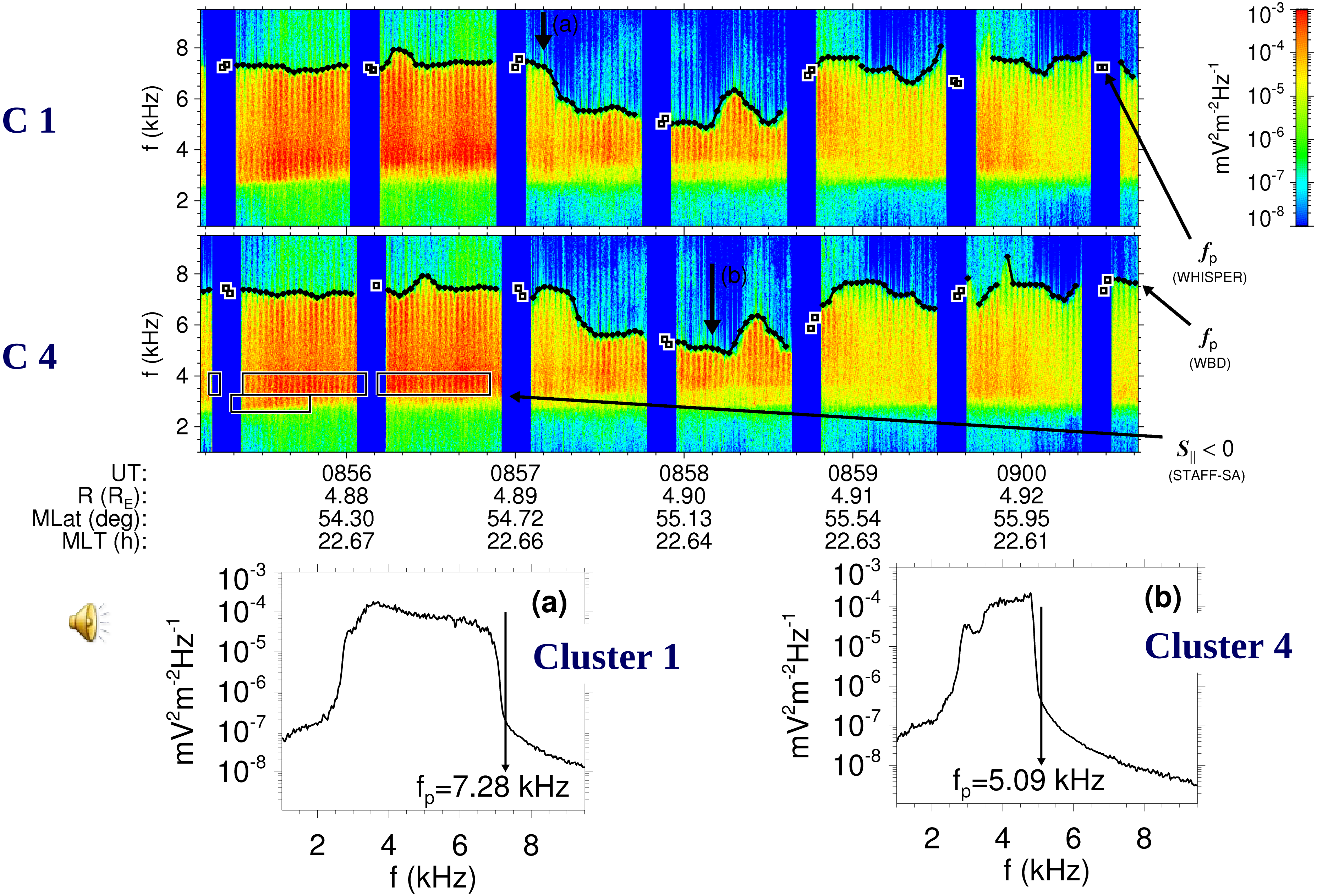


Wideband (WBD) Instrument

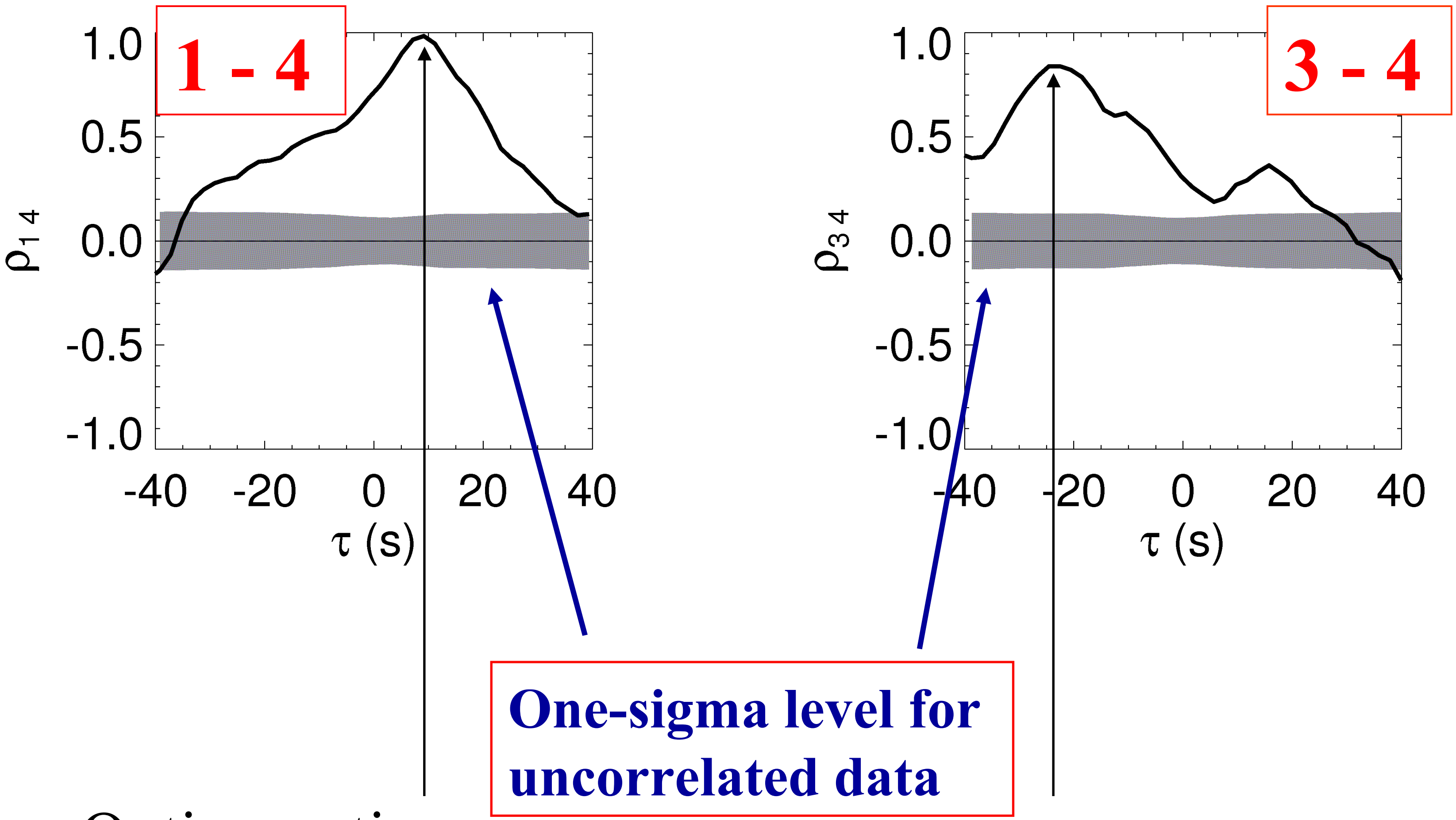
- high-resolution measurements of electric or magnetic fields
- digitized waveforms
- selected frequency bands from 25 Hz to 577 kHz.

Instrument mode used for common WBD-STAFF studies :

- continuous waveforms
- single electric field component
- pass-band filtered between 50 Hz and 9.5 kHz
- sampled at 27.44 kHz

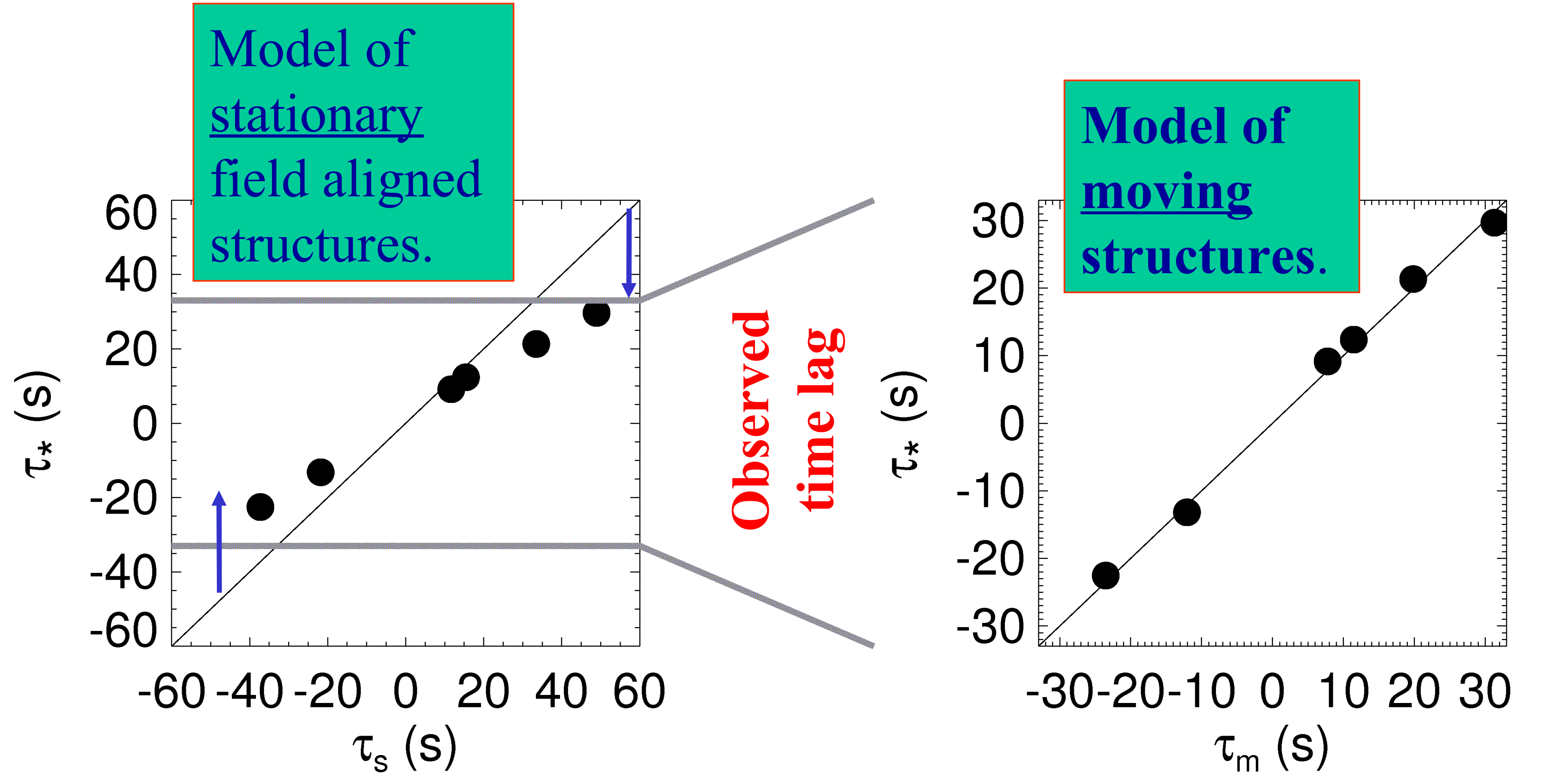

C 1
C 4
f (kHz)
(a)
(b)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$mV^2m^{-2}Hz^{-1}$
f_p (WHISPER)
f_p (WBD)
$S_{||} < 0$ (STAFF-SA)
UT: 0856 0857 0858 0859 0900
R (R_E): 4.88 4.89 4.90 4.91 4.92
MLat (deg): 54.30 54.72 55.13 55.54 55.95
MLT (h): 22.67 22.66 22.64 22.63 22.61
(a)
Cluster 1
f_p=7.28 kHz
(b)
Cluster 4
f_p=5.09 kHz
f (kHz)

Spearman’s rank correlation coefficients



Optimum time lag: $\tau_* = +9$ s

Optimum time lag : $\tau_* = -22$ s



$$\tau_s = -\mathbf{q}\cdot\hat{\mathbf{n}}/\mathbf{v_s}\cdot\hat{\mathbf{n}}$$

Separation vector

Orbital velocity

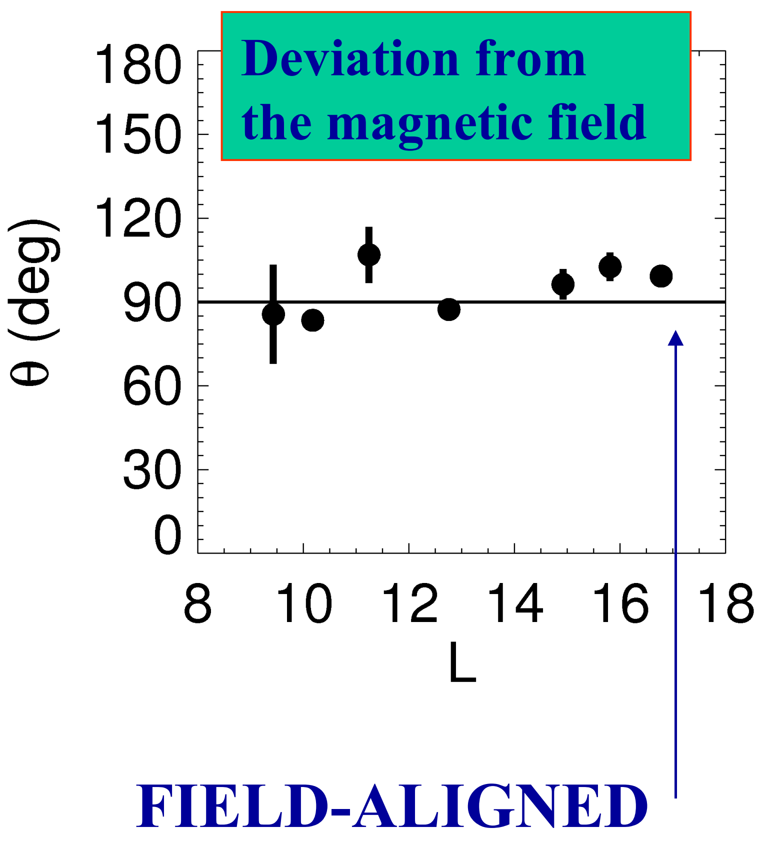
$$\tau_m = \mathbf{q}\cdot\hat{\mathbf{n}}/(u-\mathbf{v_s}\cdot\hat{\mathbf{n}})$$

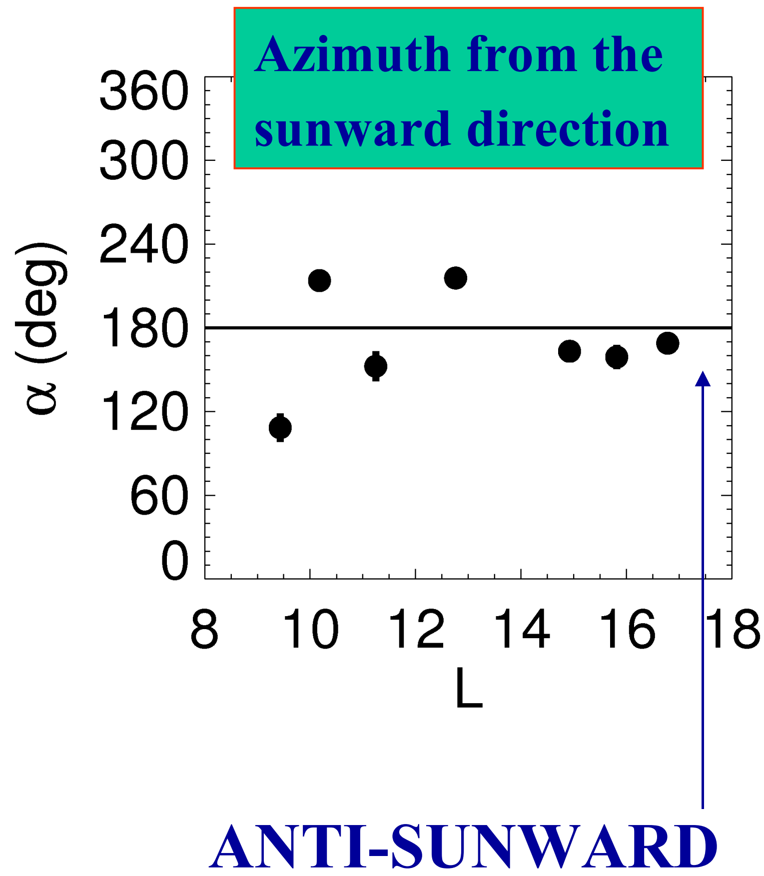
Separation vector

Speed of motion

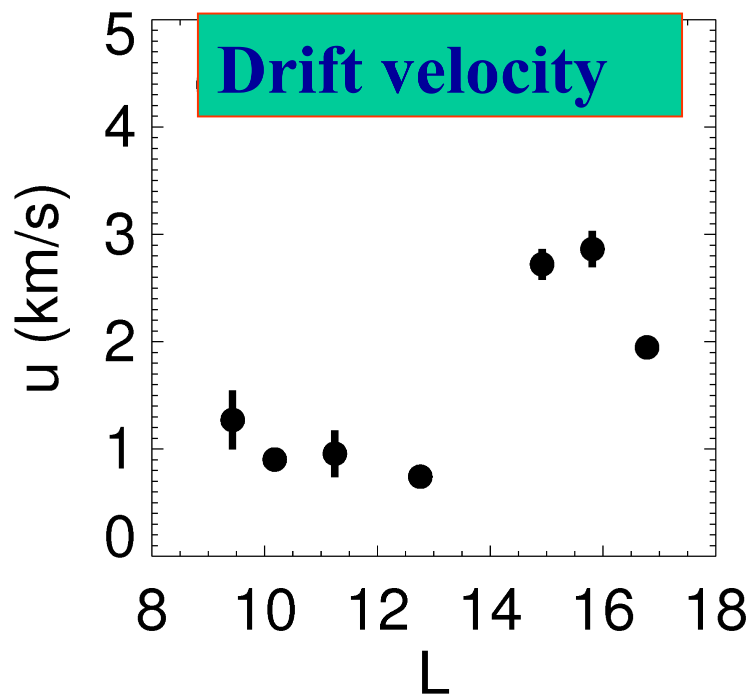
Orbital velocity

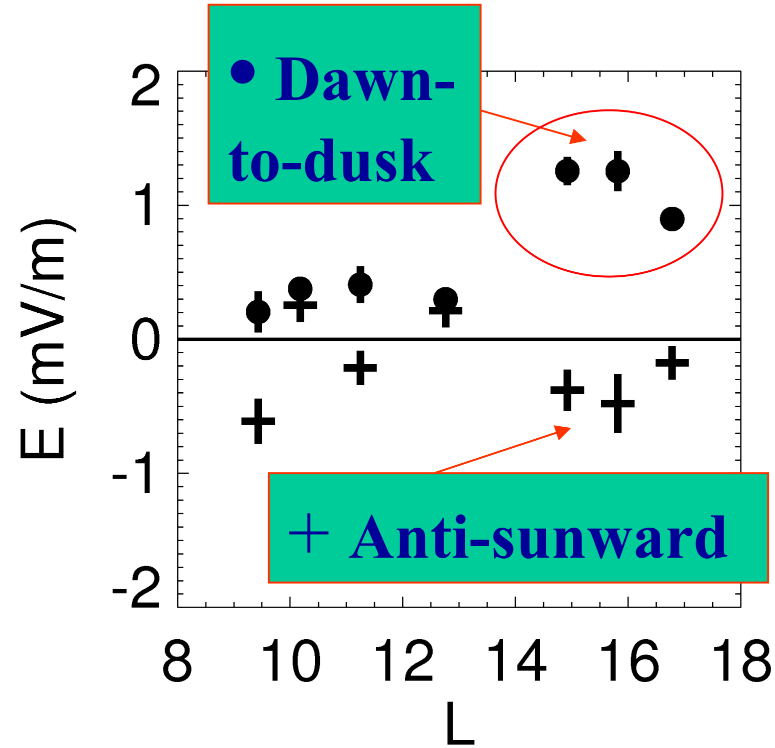
Parameters of the model of moving structures : Normal direction





Parameters of the model of moving structures : Drift velocity and convection electric field





Duration: 30-100 s
Diameter of field aligned depletions: 100-500 km
scales down to <u>10-50 km</u> in the ionosphere

1.3 mV/m duskward convection field in the polar cap
<u>50 kV</u> potential across the polar cap

Plane wave methods

Faraday's law

$$\vec{k} \times \vec{\mathcal{E}} = \omega \vec{\mathcal{B}} \qquad \Longrightarrow \qquad \begin{array}{ll} \vec{\mathcal{E}} \cdot \vec{\mathcal{B}} = 0 & (\vec{\mathcal{E}} \perp \vec{\mathcal{B}}) \\ \vec{k} \cdot \vec{\mathcal{B}} = 0 & (\vec{k} \perp \vec{\mathcal{B}}) \end{array}$$

-

Spectral matrix of analytic magnetic components

$$S_{ij} = \langle \mathcal{B}_i \, \mathcal{B}_j^* \rangle$$

Overdetermined set of equations

6 equations

2 unknowns

$$\mathsf{A} \cdot \vec{k} = \begin{pmatrix} \Re S_{11} & \Re S_{12} & \Re S_{13} \\ \Re S_{12} & \Re S_{22} & \Re S_{23} \\ \Re S_{13} & \Re S_{23} & \Re S_{33} \\ 0 & -\Im S_{12} & -\Im S_{13} \\ \Im S_{12} & 0 & -\Im S_{23} \\ \Im S_{13} & \Im S_{23} & 0 \end{pmatrix} \cdot \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = 0$$

Experimental spectral matrix

$$\hat{S}_{ij} = \langle\, \hat{\mathcal{B}}_i\, \hat{\mathcal{B}}_j^* \,\rangle \quad \longrightarrow \quad \hat{\mathsf{A}} \cdot \hat{\vec{\kappa}} = 0$$

Matrix 6x3
with orthonormal
columns

Singular value decomposition

Matrix
6x3

$$\left(\hat{\mathsf{A}}\right) = \left(\mathsf{U}\right) \cdot (\mathsf{W}) \cdot (\mathsf{V}^{\mathrm{T}})$$

Matrix 3x3
with orthonormal
rows

Diagonal matrix 3x3
of singular values

$$(\mathsf{W}) = \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \qquad w_1 \le w_2 \le w_3$$

Wave vector direction

$$\hat{\vec{\kappa}} = (V_{11},\ V_{21},\ V_{31})$$

Row of **V**$^{\mathrm{T}}$
corresponding to the minimum
singular value

Axes of the polarization ellipse

$$\hat{\vec{a}} = (w_3 V_{13},\ w_3 V_{23},\ w_3 V_{33})$$
$$\hat{\vec{b}} = (w_2 V_{12},\ w_2 V_{22},\ w_2 V_{32})$$

Ellipticity

$$L_p = w_2 / w_3$$

Ratio of
the axes of the polarization ellipse:
1 for the circular polarization
0 for the linear polarization

Planarity

$$F = 1 - \sqrt{w_1 / w_3}$$

Ratio of
standard deviations of
a Gaussian noise:
1 for a planar polarization
0 for unpolarized signals

Wave distribution function (WDF) methods

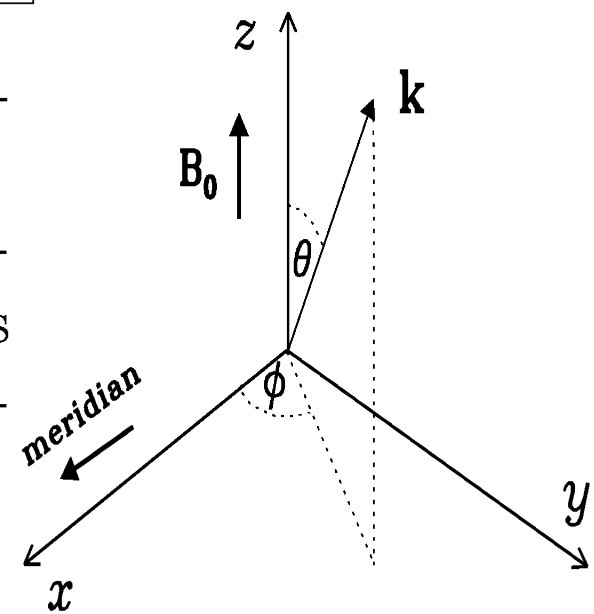
<u>WDF</u>: continuous distribution ***G*** of wave energy with respect to the wave-vector directions.

$$S_{ij} = \sum_m \int a_{mij}(\theta,\, \phi)\, G_m(\theta,\, \phi)\; d^2(\theta,\, \phi)$$

S_{ij} …theoretical prediction of the spectral matrix;

$a_{mij}(\theta,\, \phi)$ …matrix of integration kernels: theoretical predictions of normalized spectral matrices for plane waves with different wave-normal directions;

m …wave mode



- Dispersion relation - theoretical polarization - integration kernels
- Least squares methods to find the WDF which is most consistent with the experimental spectral matrix.

Inverse ray tracing

•Following the ray backward from the point of observation.

•Initialization by a wave normal direction found experimentally.

Basic equations: Given the dispersion relation

$$\omega = \omega(\vec{r}, \vec{k}, t),$$

we have

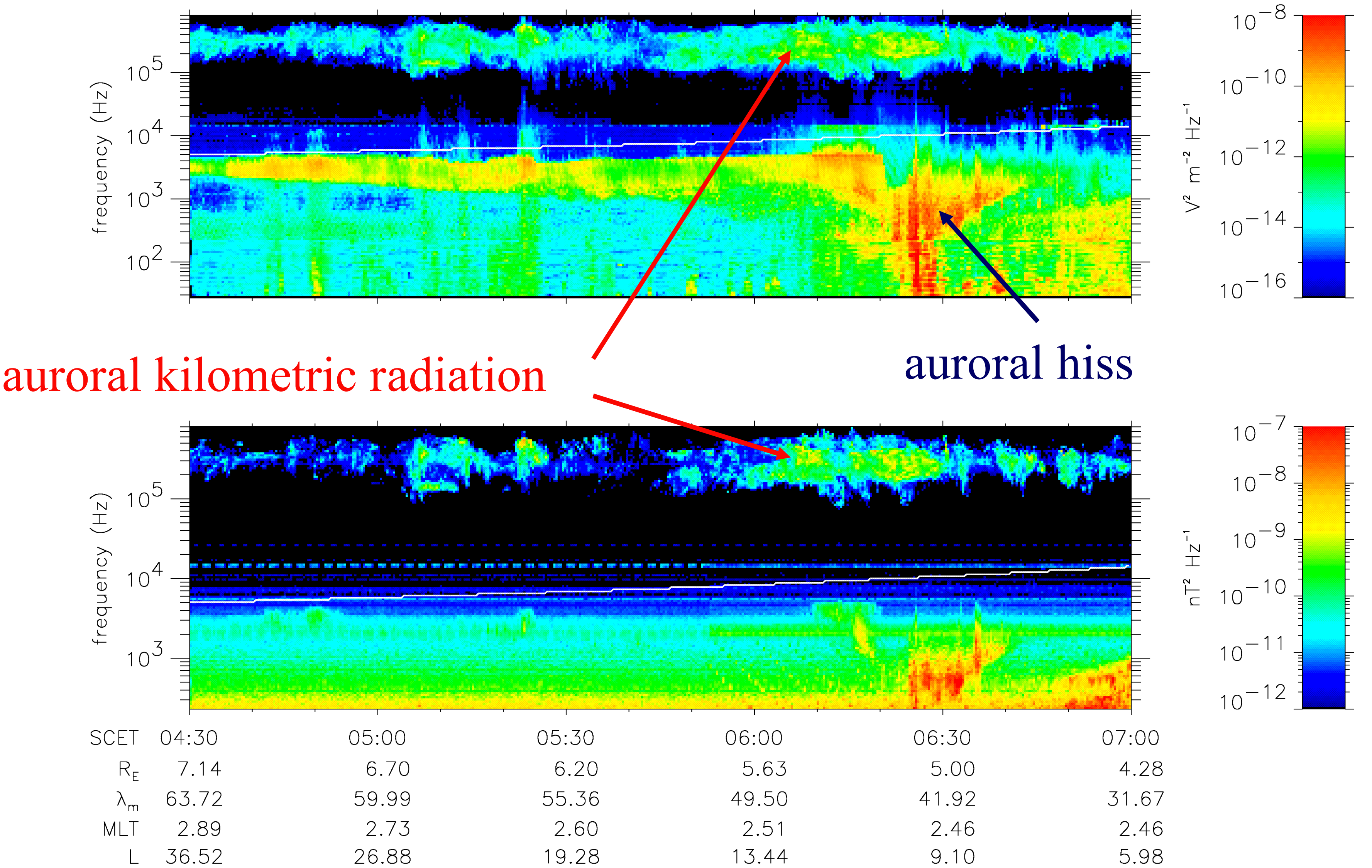
$$\frac{d\vec{r}}{dt} = \frac{\partial \omega}{\partial \vec{k}}$$

$$\frac{d\vec{k}}{dt} = -\frac{\partial \omega}{\partial \vec{r}}$$

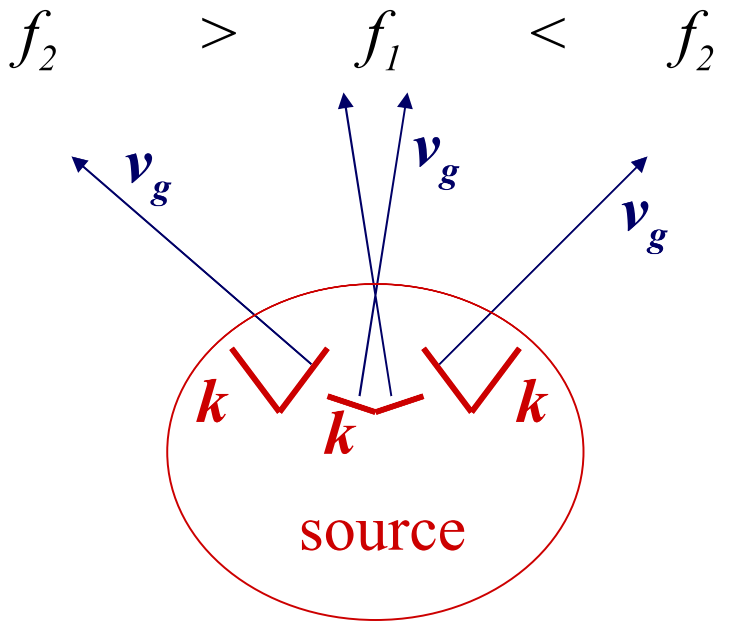
Basic limitations:

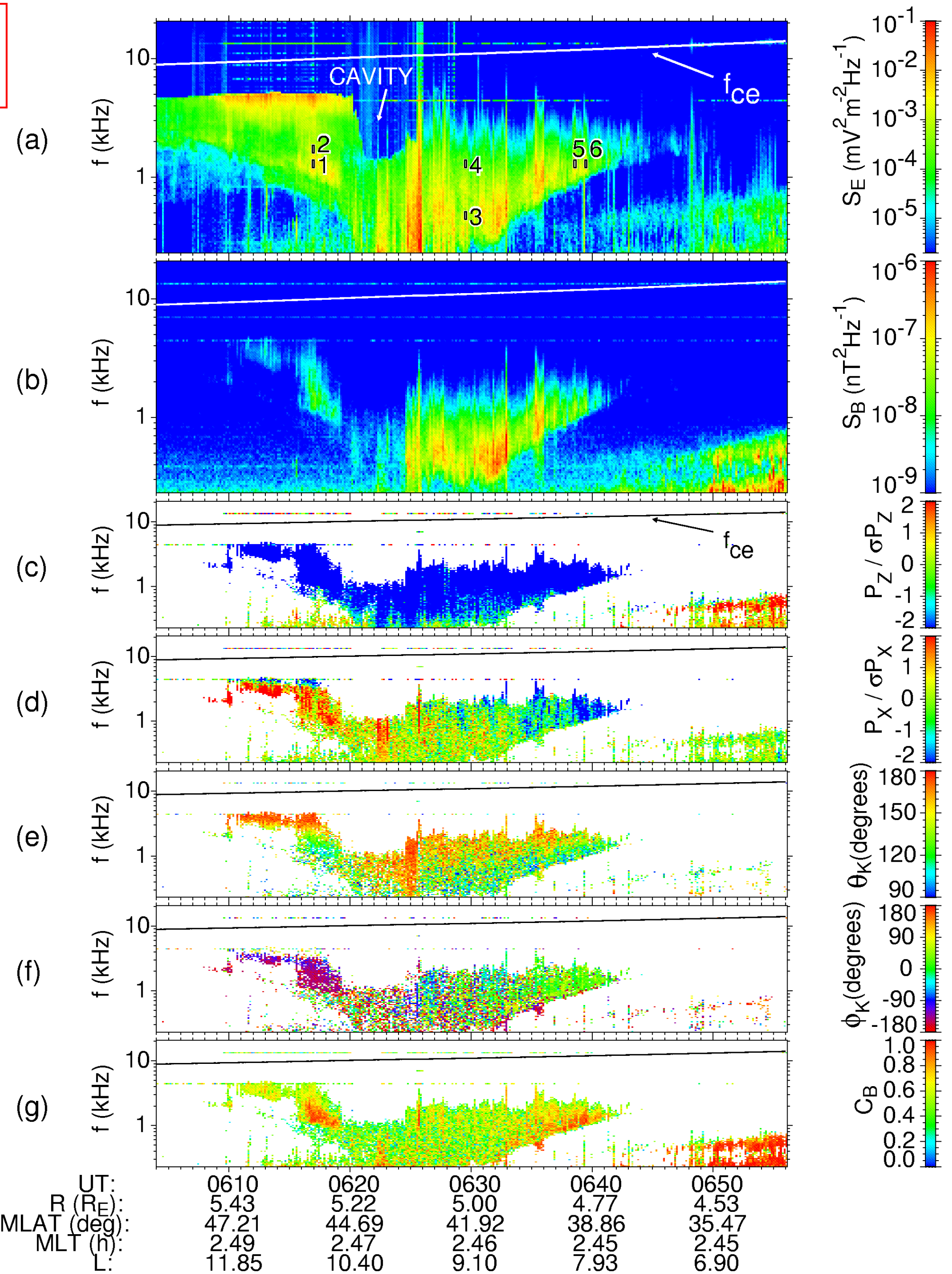
- Wentzel-Krammers-Brillouin (WKB) approximation or limit of geometric optics: dispersive properties of the medium are slowly varying functions of space and time compared to the wavelength and wave period. Fails: sharp gradients and/or rapid changes of the refractive index
- Need of realistic description of the medium

Auroral region

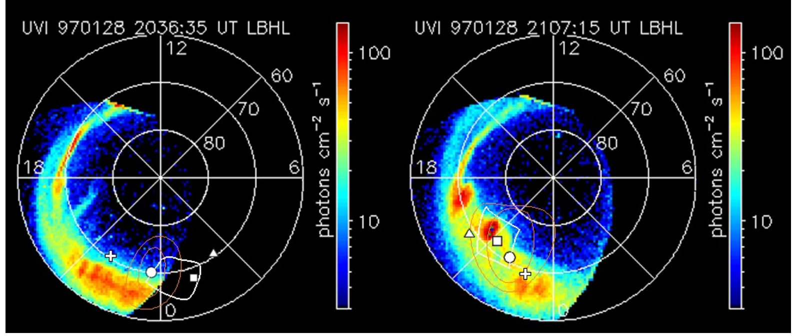


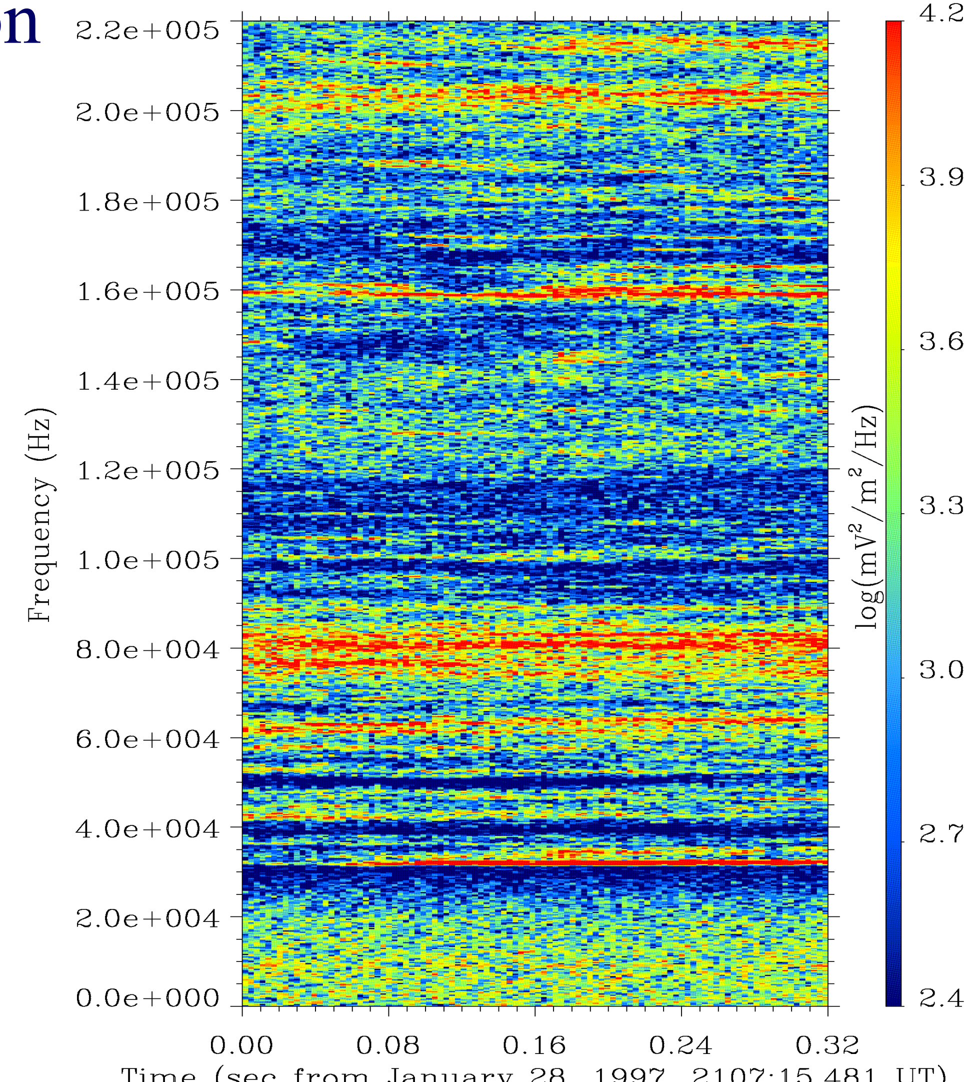
Propagation of auroral hiss

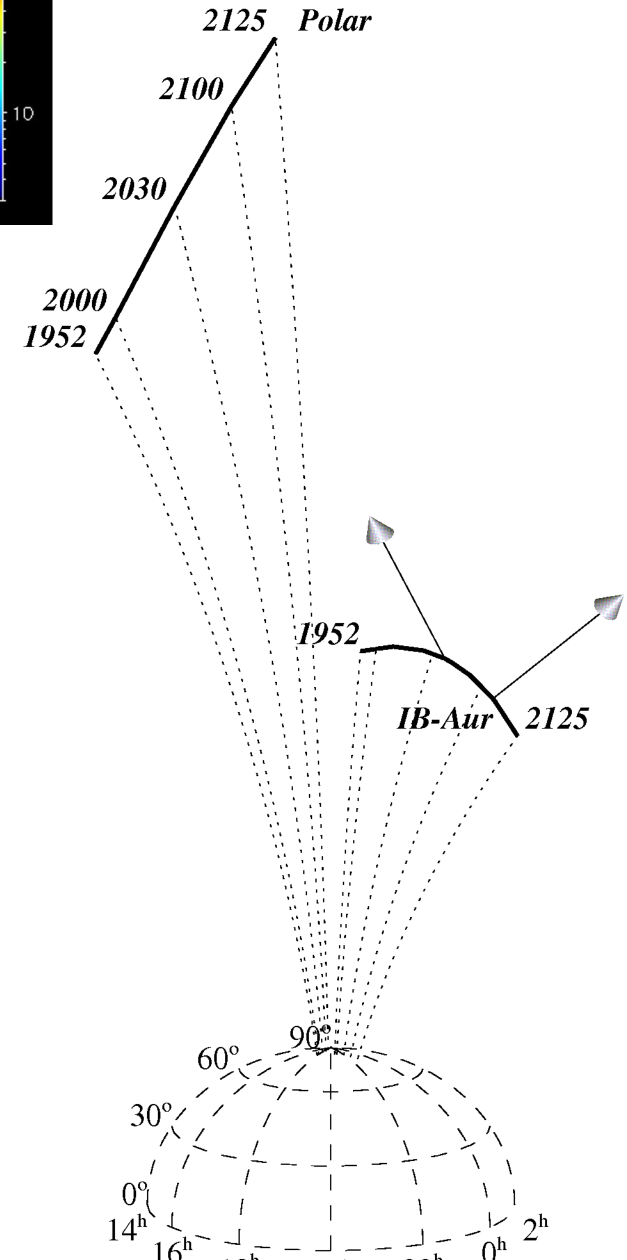




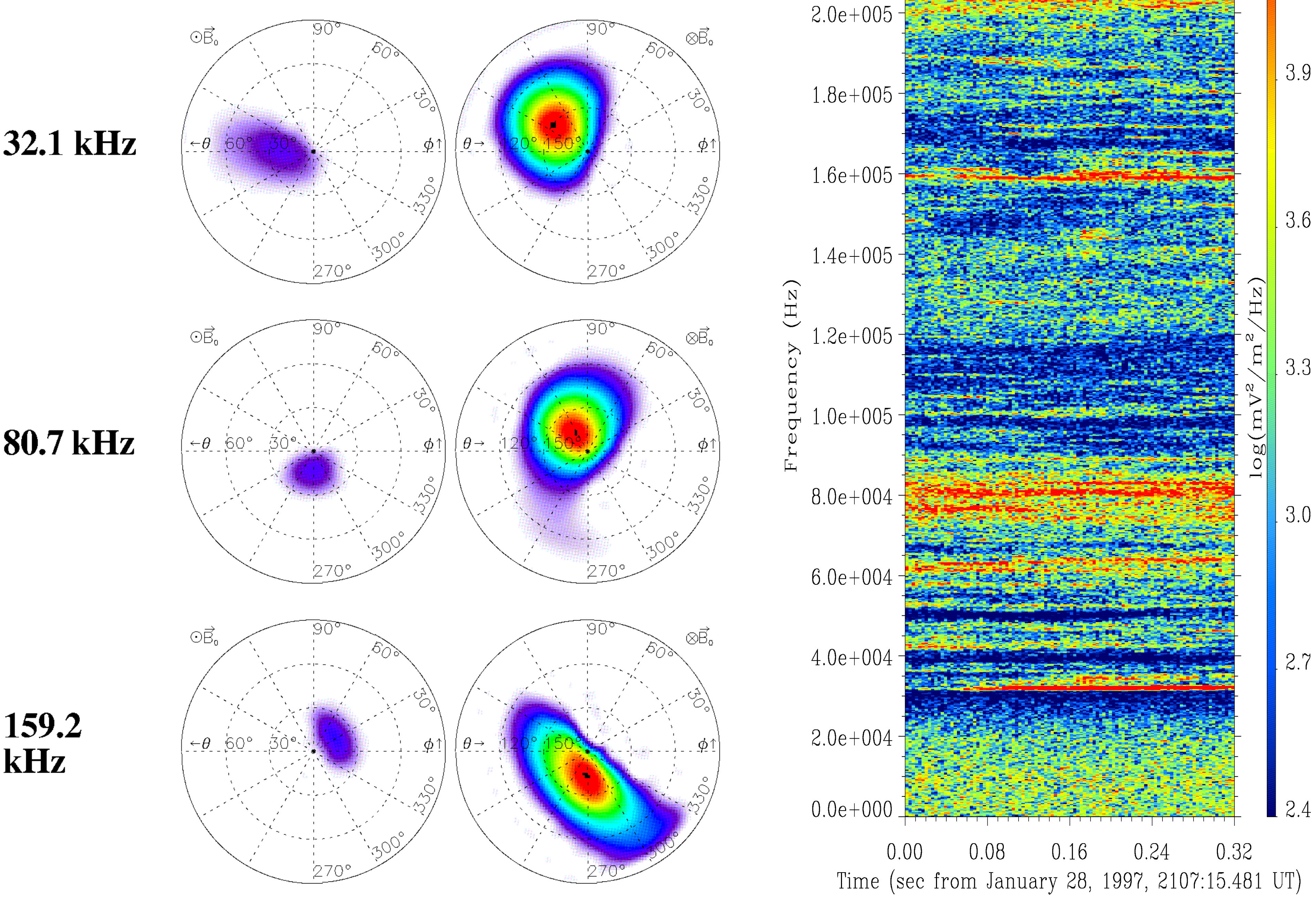
Propagation of the auroral kilometric radiation (AKR)



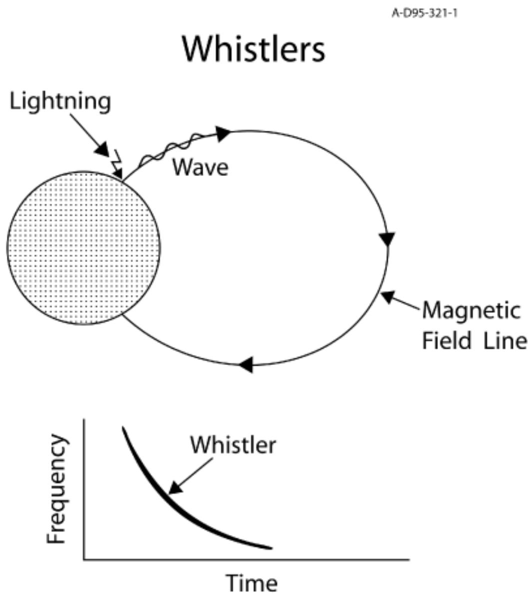




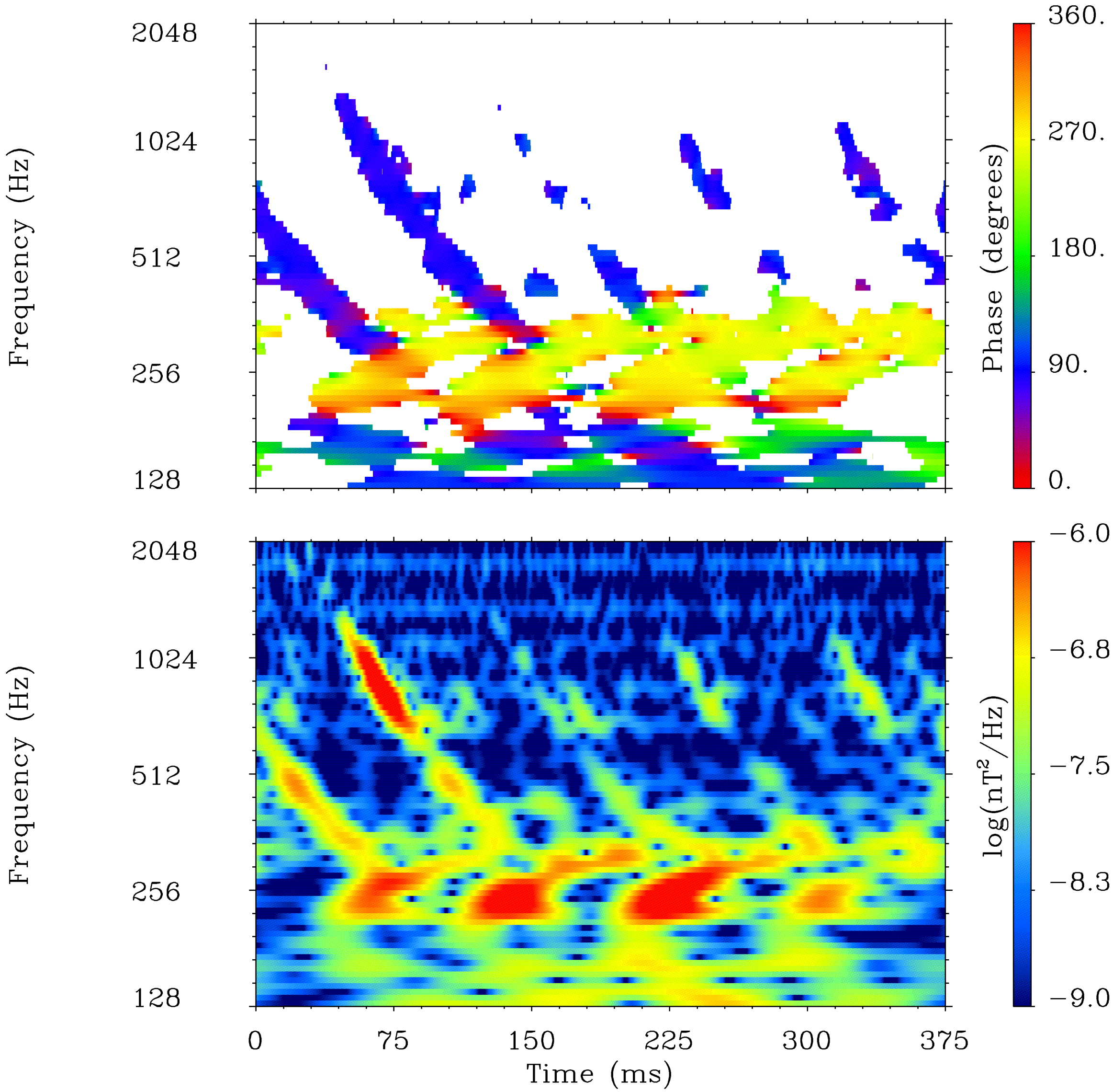
Wave distribution functions of AKR



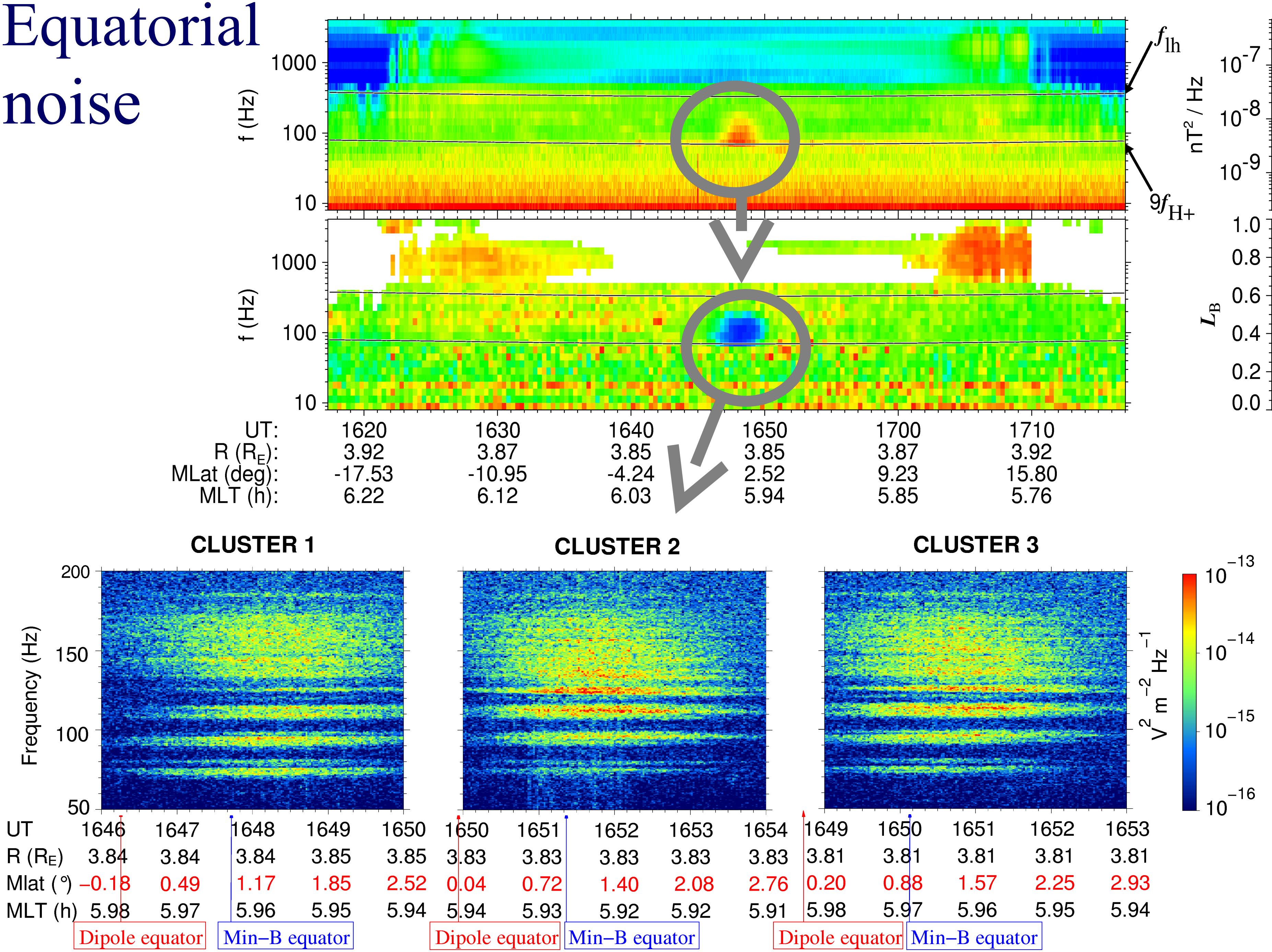
Whistlers and proton whistlers



Storey, 1953



Equatorial noise



March 31, 2001

- Major interplanetary disturbance.
- Pushing the bow shock inside the geosynchronous orbit [Ober *et al.*, 2002].
- Producing geomagnetic activity, $K_P = 9-$, Dst = –360 nT [Skoug *et al.*, 2003].
- Dispersionless injection of energetic electrons in the pre-midnight sector, AE = 1200 nT [Baker *et al.*, 2002].

Whistler-mode chorus Cluster 3

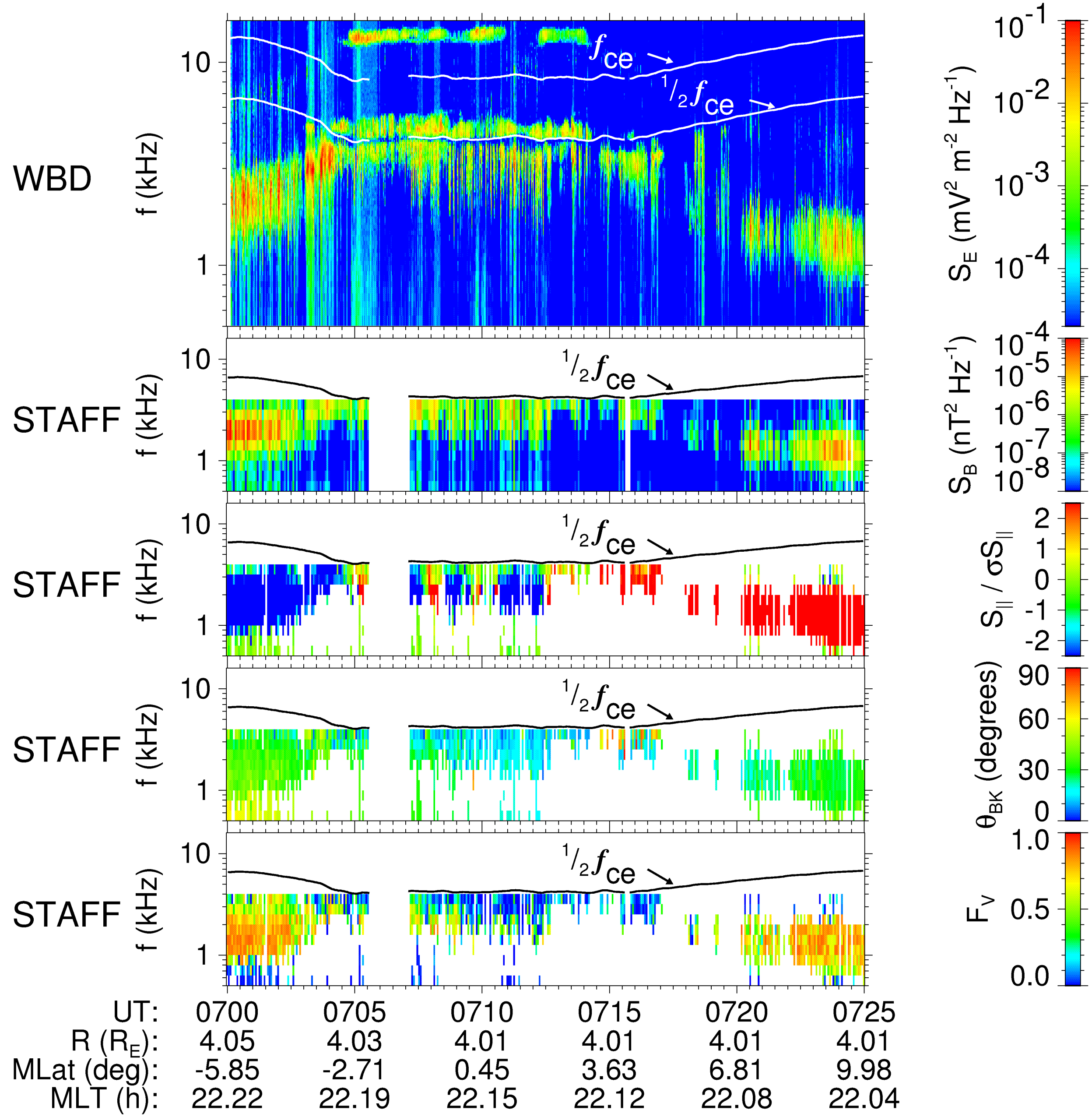
Power spectrogram of magnetic field fluctuations.

Power spectrogram of magnetic field fluctuations.

Parallel component of the Poynting vector normalized by its standard deviation.

Angle deviation of the wave vector form the static magnetic field.

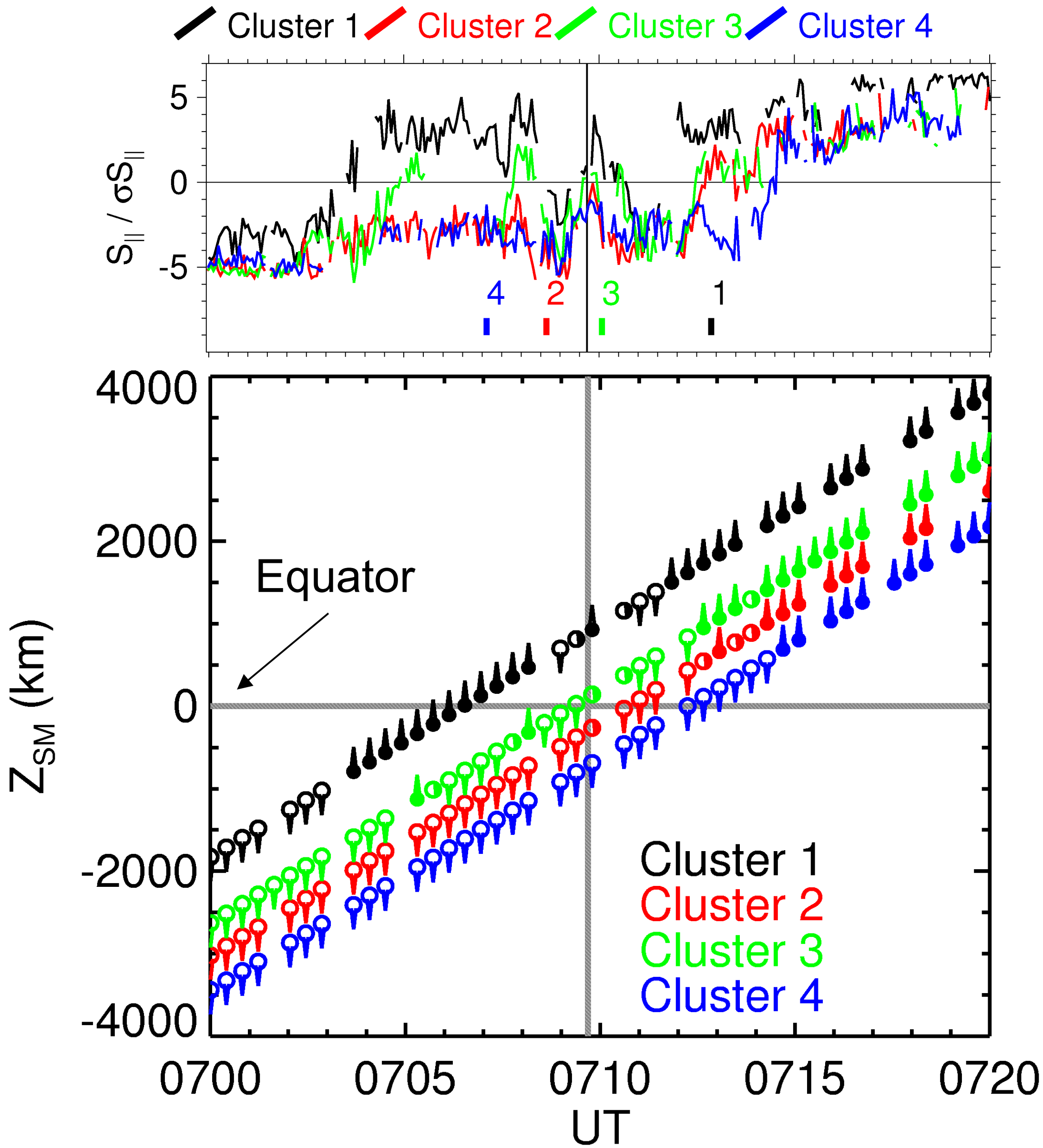
Electromagnetic planarity.



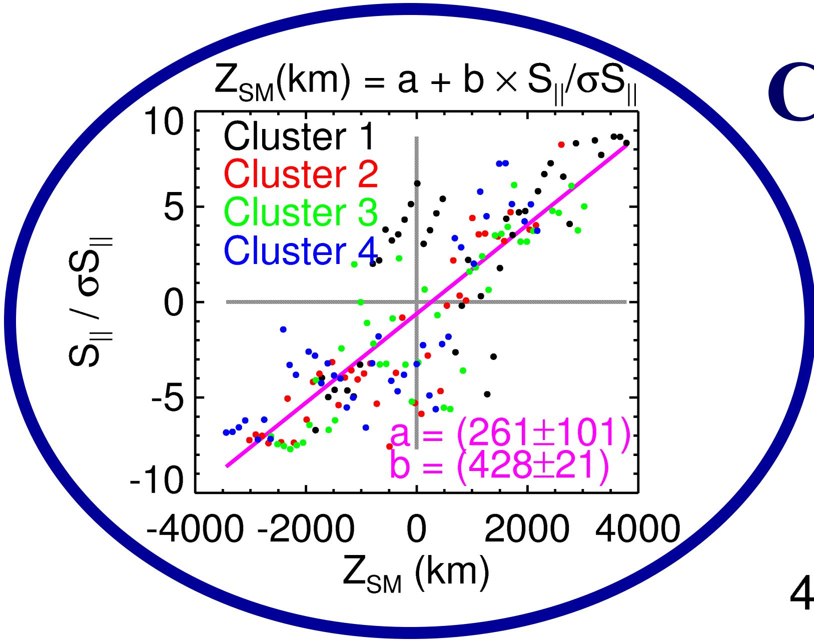
Multipoint measurements

Parallel component of the Poynting vector normalized by its standard deviation.

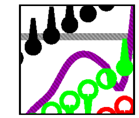
Z_{SM} coordinate (perpendicular to the geomagnetic equatorial plane) of all the four spacecraft.



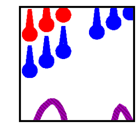
Central position of the source



1. Linear interpolation

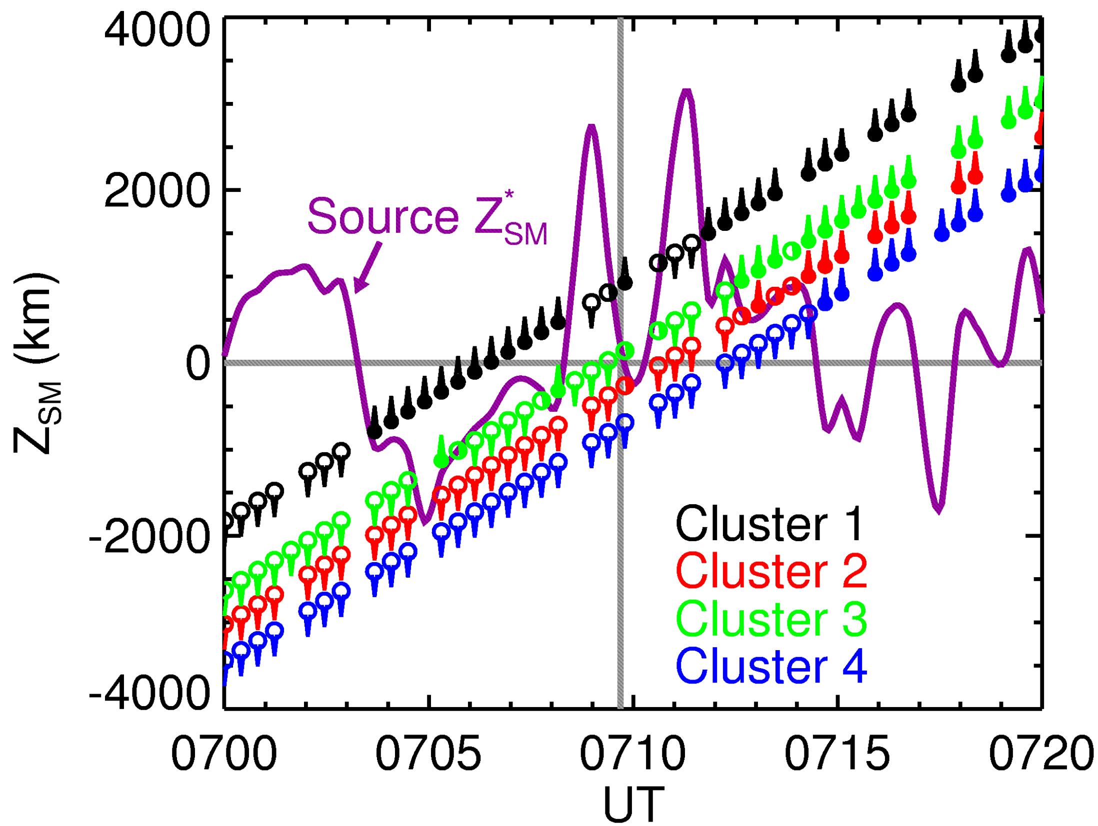


2. Linear extrapolation

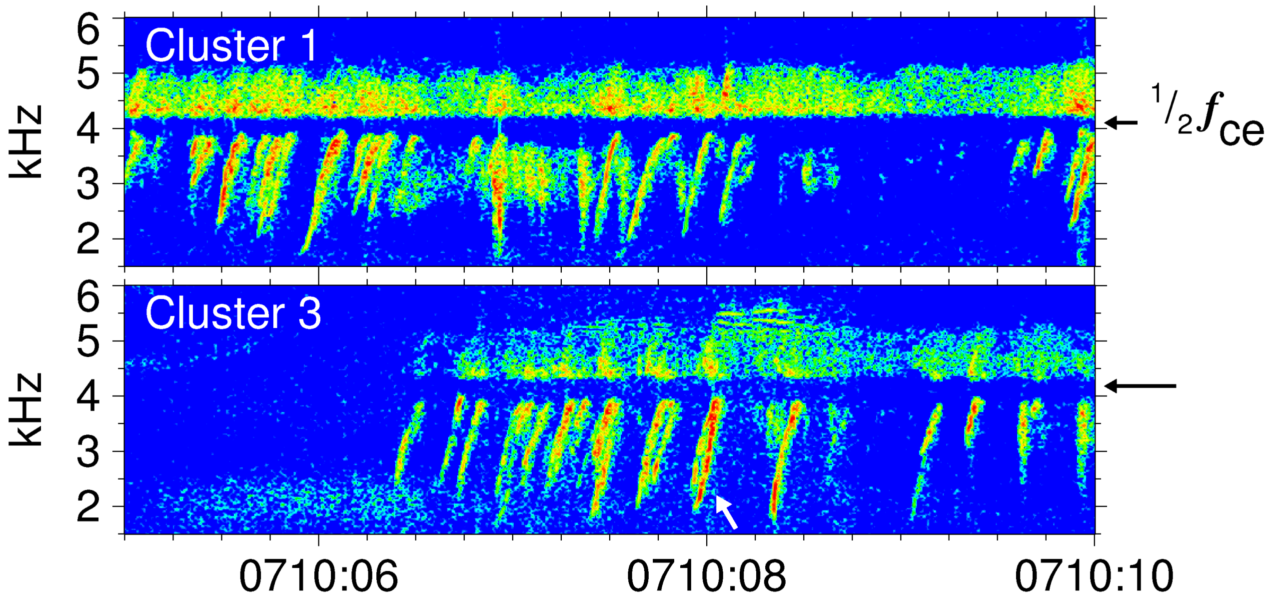


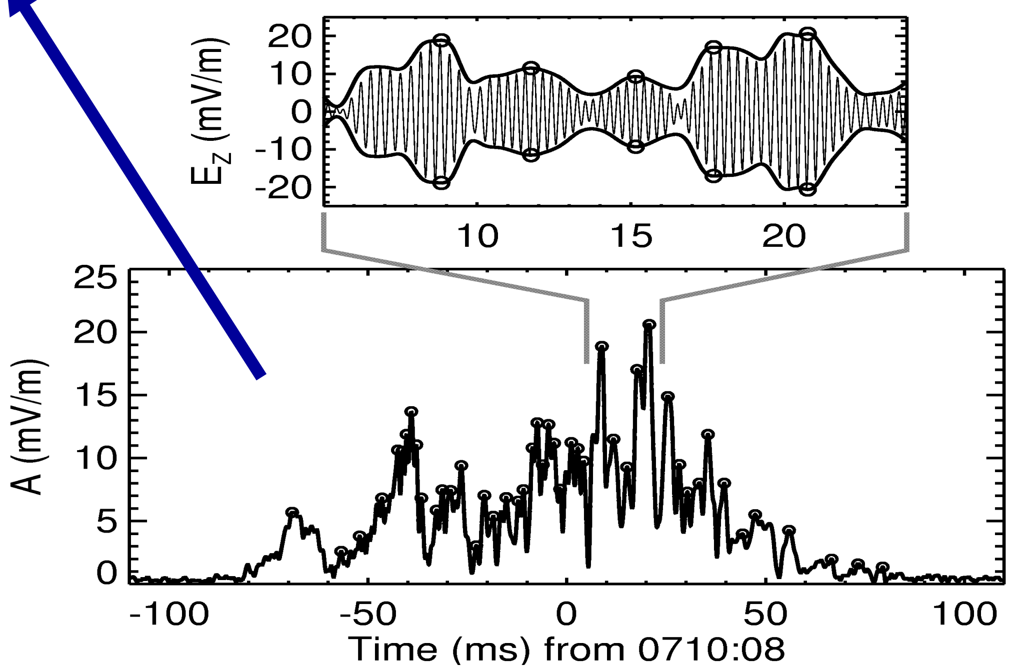
$$Z^*_{SM}(t) = Z^m_{SM}(t) - a\,(S_{||}/\sigma S_{||})_{\min}$$

Z_{SM} coordinate (perpendicular to the geomagnetic equatorial plane) of the four spacecraft.



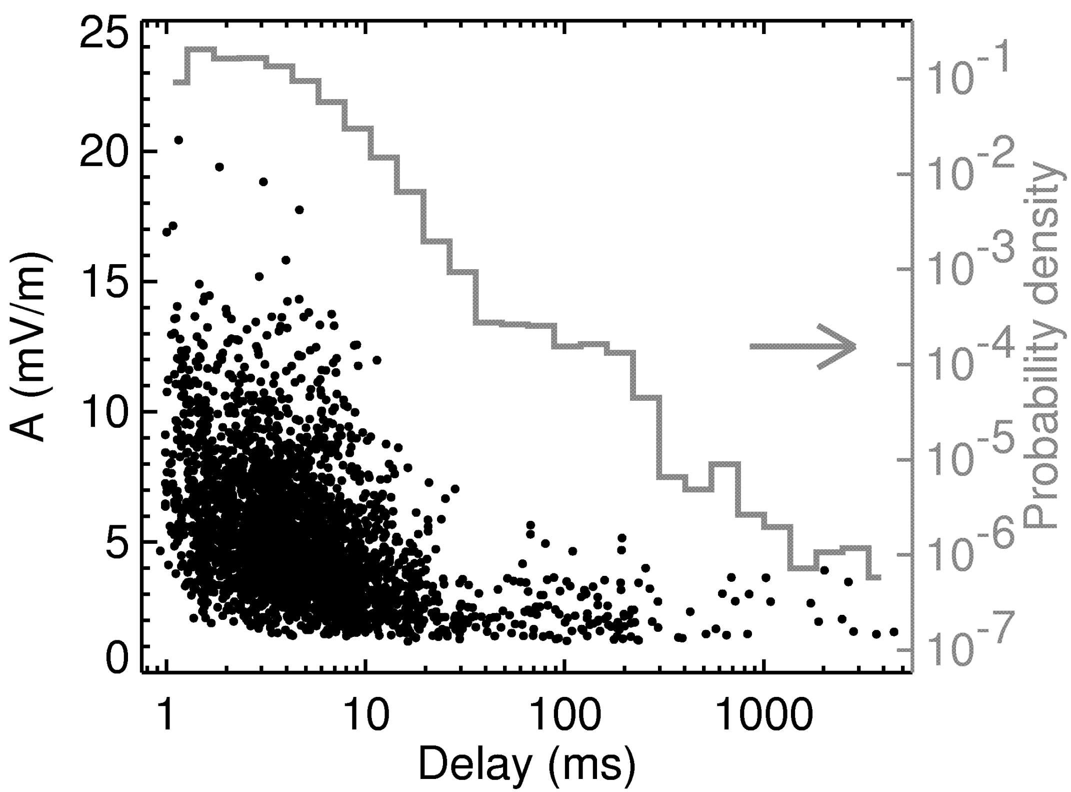
Analysis of wave packets of the whistler-mode chorus





- FIR filter 2-4 kHz
- Amplitude envelope of the signal
- Local maxima of the sub-structure

Amplitudes of local maxima and time delays between the neighboring maxima



Fine structure **Separate elements**

18th April 2002 Cluster 4

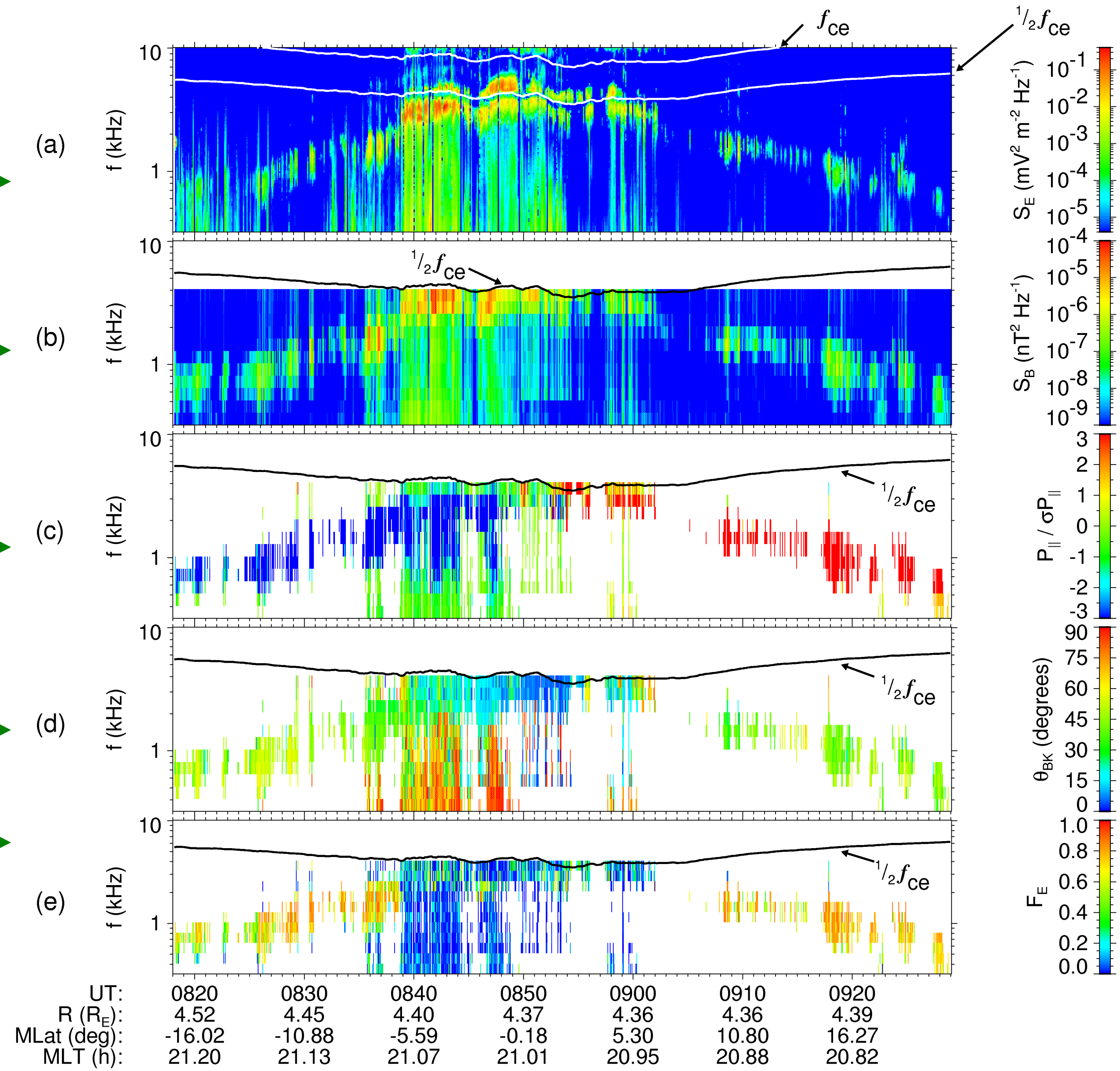
Power spectrogram of the electric field fluctuations.

Power spectrogram of the magnetic field fluctuations.

Parallel component of the Poynting vector normalized by its standard deviation.

Angle deviation of the wave vector from the static magnetic field.

Electromagnetic planarity.



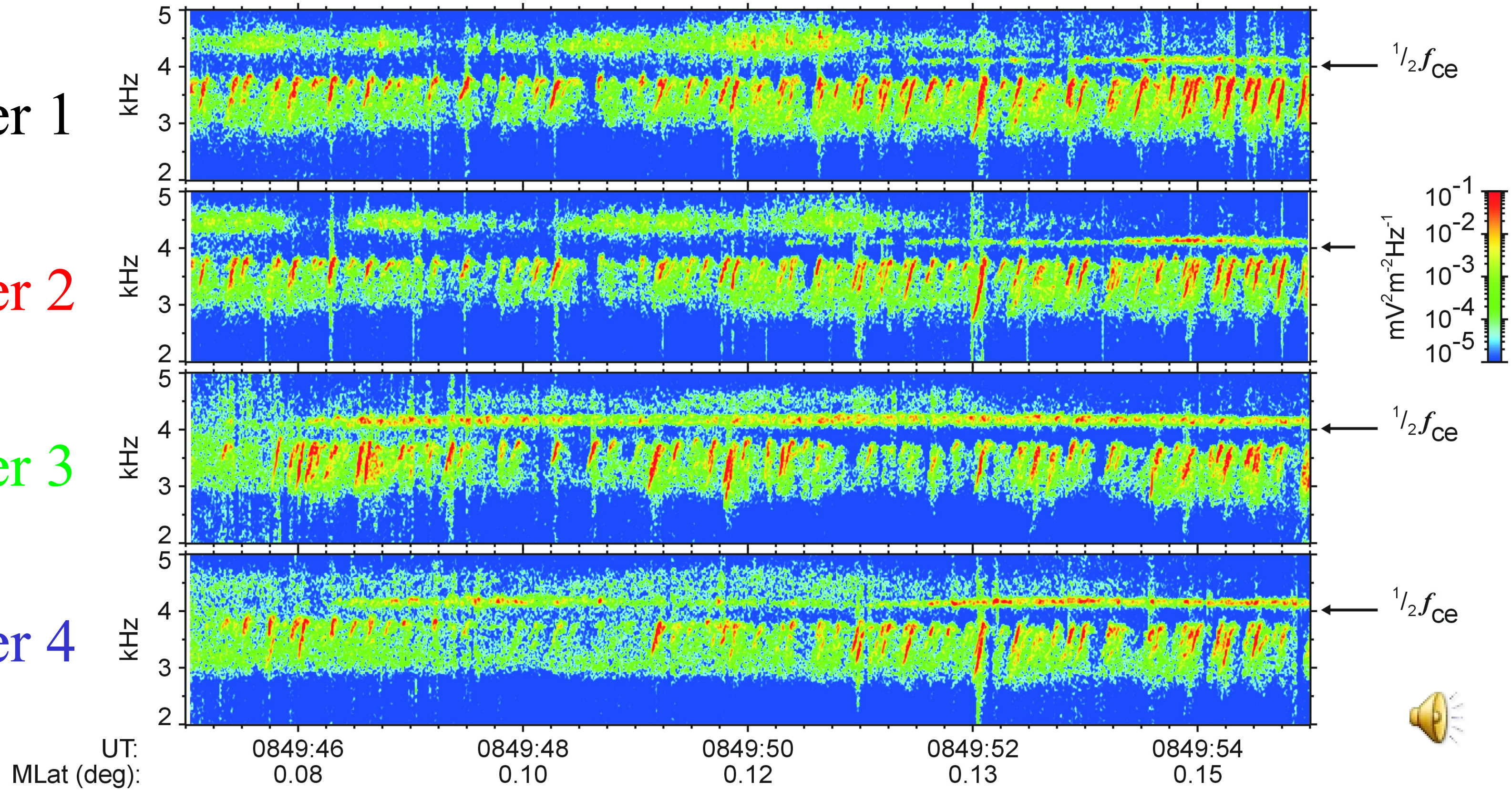
Detailed spectrograms 18th April 2002

Cluster 1

Cluster 2

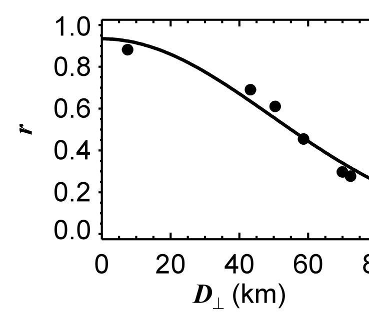
Cluster 3

Cluster 4



CORRELATION COEFFICIENT of elements of the chorus emissions

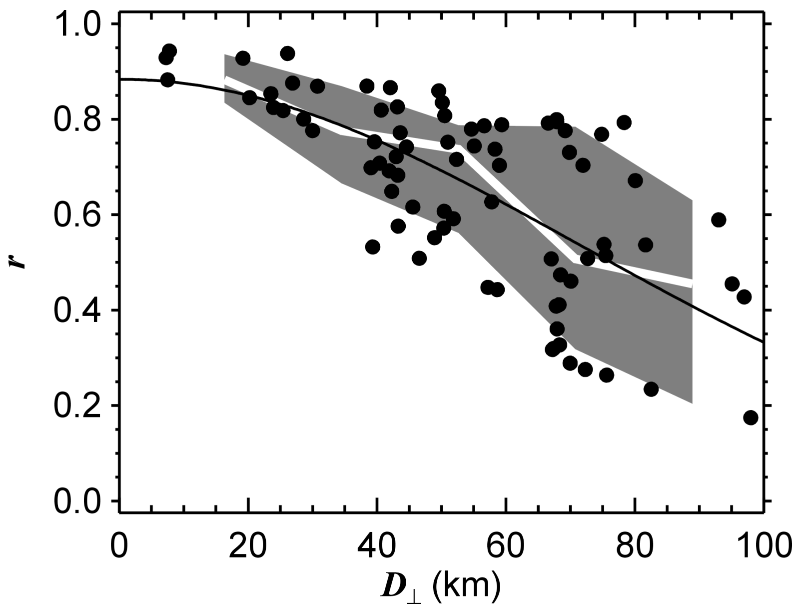
S/C	$D_{\|\|}$	$D_{\perp}$	r
1–2	187 km	7 km	0.88
1–3	258 km	72 km	0.28
1–4	110 km	50 km	0.61
2–3	70 km	69 km	0.30
2–4	-77 km	43 km	0.69
3–4	-148 km	58 km	0.45



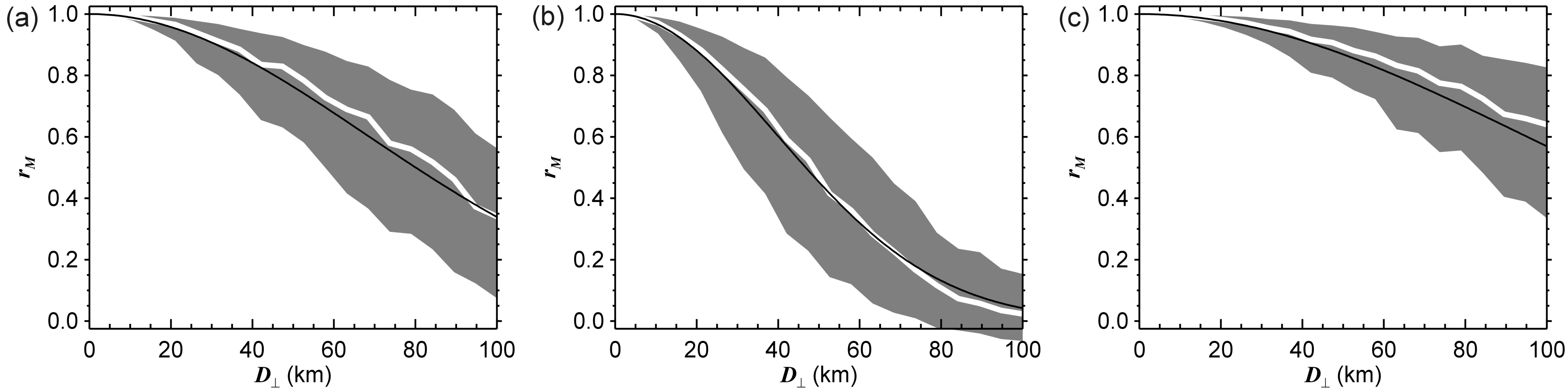
Analysis of more intervals

Correlation coefficient quantiles 0.16, 0.5 a 0.84

Model Gaussian function
$A = 0.88$, $\Delta = 101$ km.



Simulation of the source region



$\Delta_m = 35$ km

$\Delta_m = 20$ km

$\Delta_m = 50$ km

SUMMARY

- Different types of waves propagate in plasmas in the Earth's magnetosphere and in the solar wind. Their propagation and polarization properties can often be explained using the cold plasma theory.
- Wave instruments onboard scientific spacecraft (for example, the STAFF and WBD instruments onboard Cluster) provide us with high-resolution multi-component measurements. These data require special analysis techniques.
- Several examples of such measurement and analysis, in different regions, have been shown:
  - auroral hiss
  - auroral kilometric radiation
  - electron and proton whistlers (*)
  - equatorial noise (*)
  - whistler mode chorus (*)

(*) *will be analyzed during the computer session*