

Location of Wave Sources Using Cluster as a Sensor Array

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Outline

Method description

- Wave telescope
- Source locator

Motion effects

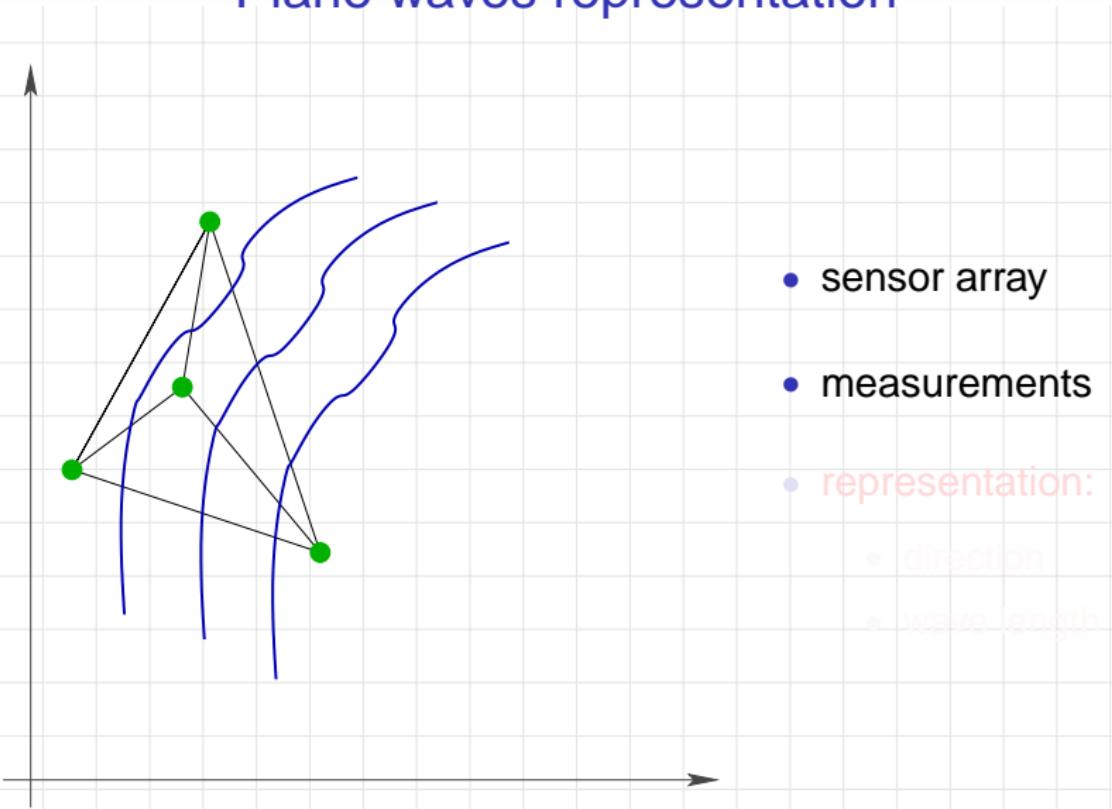
- Static case
- Sensor motion and plasma flow
- Effect of sensor motion
- Effect of plasma flow

Application to Cluster data

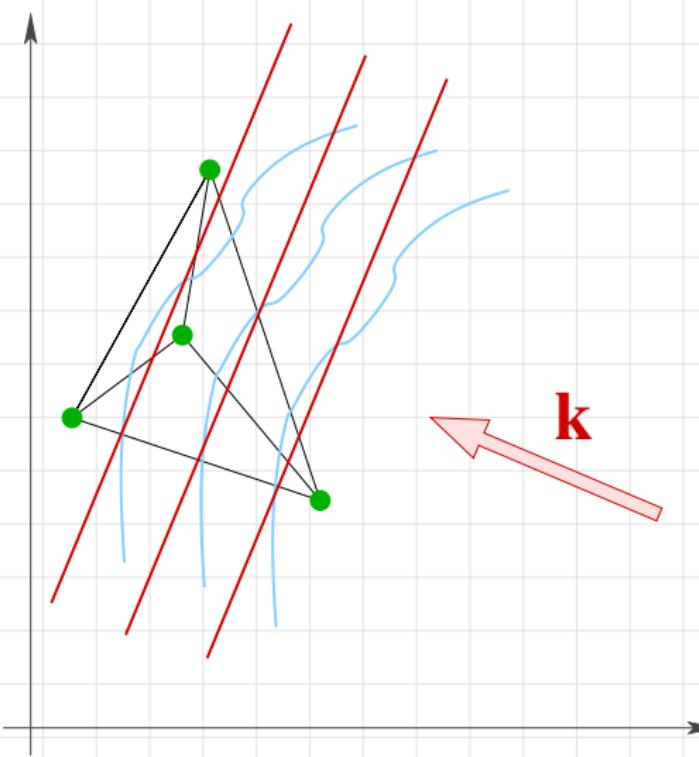
- Measured data
- Source locator results

Conclusions

Plane waves representation

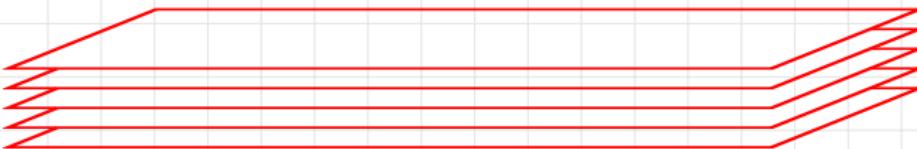


Plane waves representation



- sensor array
- measurements
- representation:
 - direction
 - wave length

Wave telescope



elementary wave

$$w_{\text{sensor}}(\mathbf{k}') = C e^{i \mathbf{k}' \cdot \mathbf{r}_{\text{sensor}}}$$

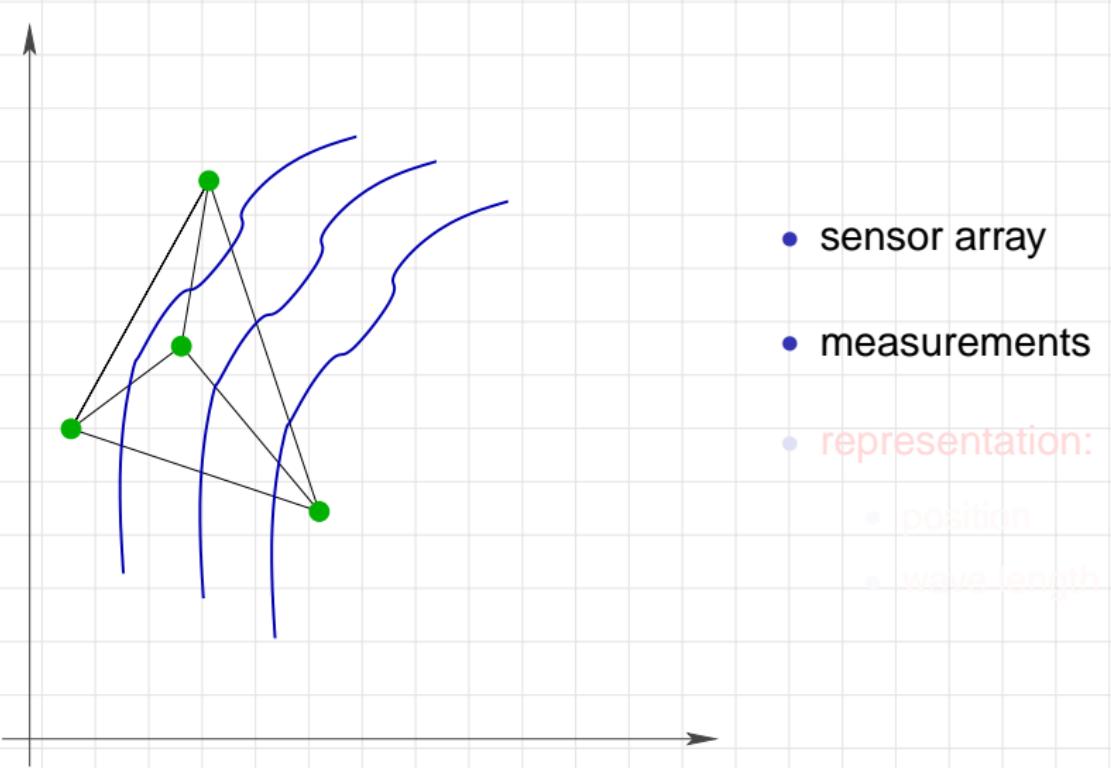
array output matrix

$$\mathcal{B}_{ij}(\omega) = B_i(\omega) B_j(\omega)$$

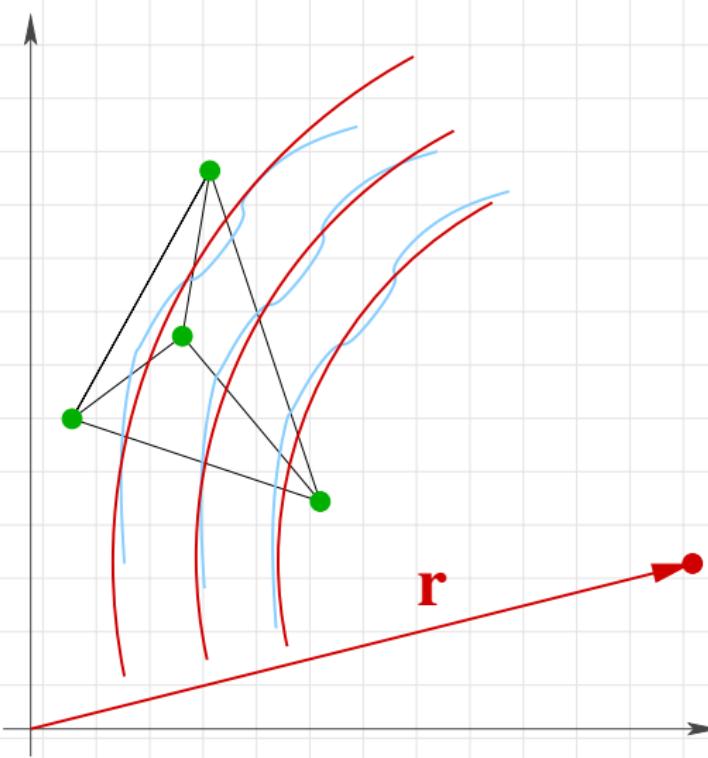
output power

$$P(\mathbf{k}') = [\mathbf{w}^\dagger(\mathbf{k}') \mathcal{B}^{-1}(\omega) \mathbf{w}(\mathbf{k}')]^{-1}$$

Spherical waves representation



Spherical waves representation



- sensor array
- measurements
- representation:
 - position
 - wave length

Source locator



elementary wave

$$w_{\text{sensor}}(k', \mathbf{r}') = C \frac{1}{\rho'_{\text{sensor}}} e^{ik' \rho'_{\text{sensor}}}$$

array output matrix

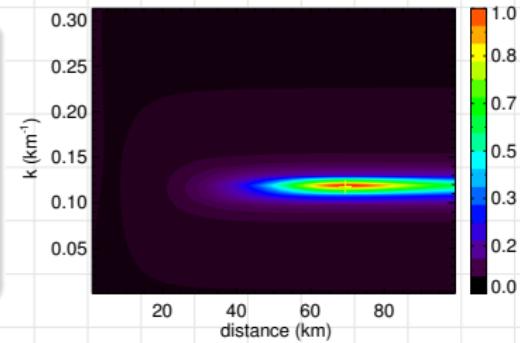
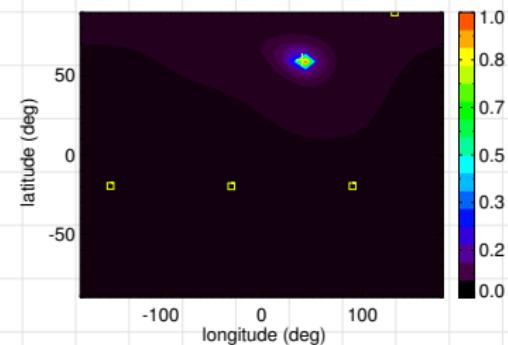
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Static case: artificial data

- one spherical wave
- random noise (10%)
- regular tetrahedron
- 10 km separation



	given	found
dist.	70 km	69 km
long.	40°	43°
lat.	60°	59°
k	0.11 km^{-1}	0.10 km^{-1}

Sensor motion and plasma flow

- General wave field determined by the phase $\Phi(\mathbf{r}, t)$

$$B \propto e^{-i\Phi(\mathbf{r}, t)}$$

frequency: $\omega(\mathbf{r}, t) \doteq \frac{\partial \Phi}{\partial t}$

wave vector: $\mathbf{k}(\mathbf{r}, t) \doteq -\nabla \Phi$

- Observer motion \Rightarrow Doppler effect $\Rightarrow \omega$ change:

$$\omega' = \frac{d\Phi}{dt} = \frac{\partial \Phi}{\partial t} + \mathbf{v} \cdot \nabla \Phi = \omega - \mathbf{k} \cdot \mathbf{v}$$

- Plasma flow \Rightarrow Pattern deformation $\Rightarrow \mathbf{k}$ change:

$$\mathbf{k}' = -\nabla \Phi'$$

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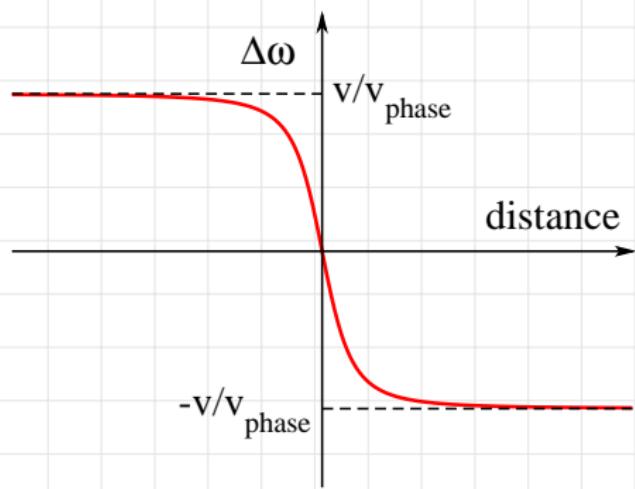
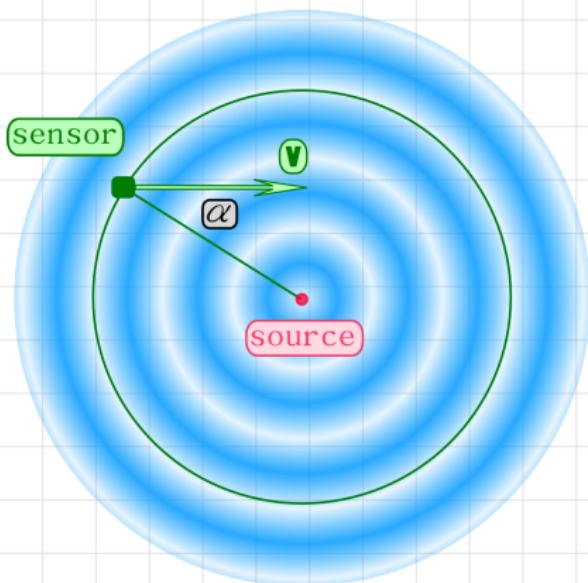
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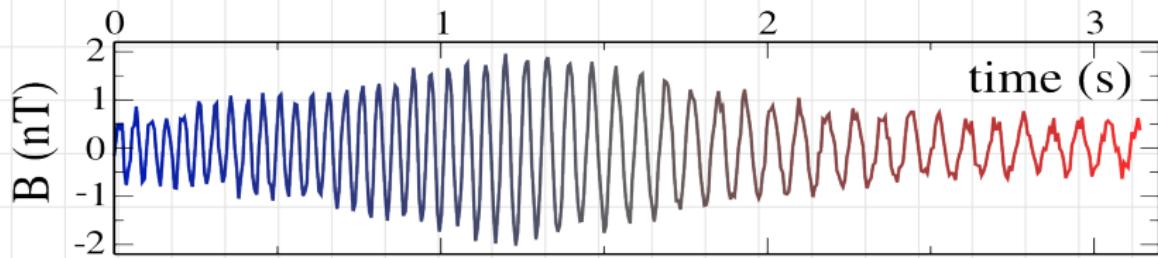
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Doppler effect for spherical wave

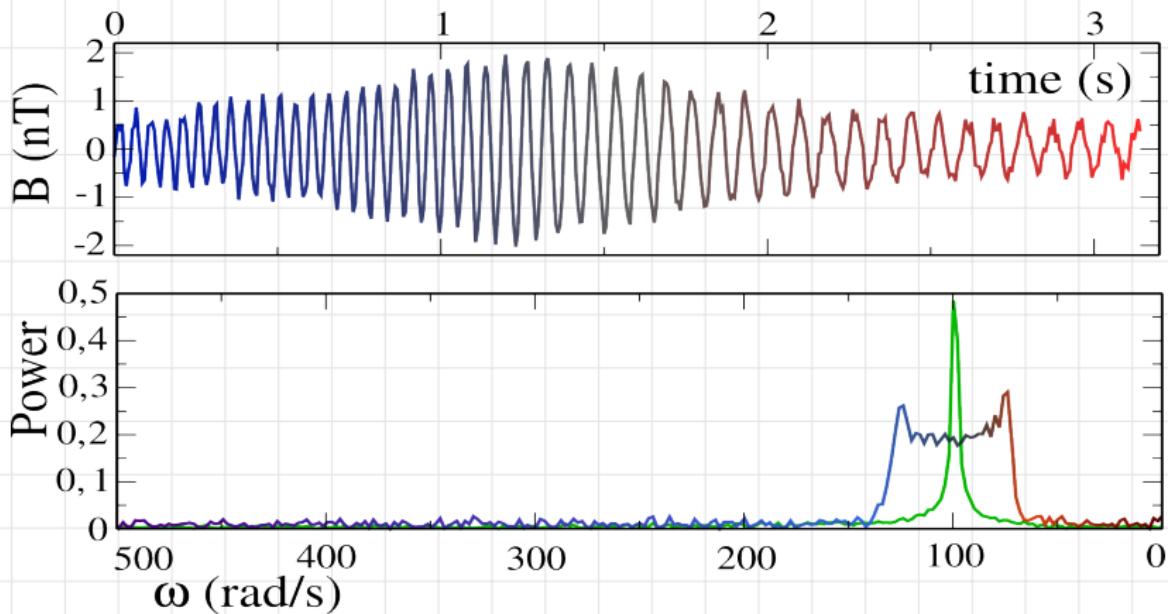
$$\omega(t) = \omega_0 \left[1 - \frac{v}{v_{phase}} \cos\alpha(t) \right] ; \quad \lambda = \lambda_0$$



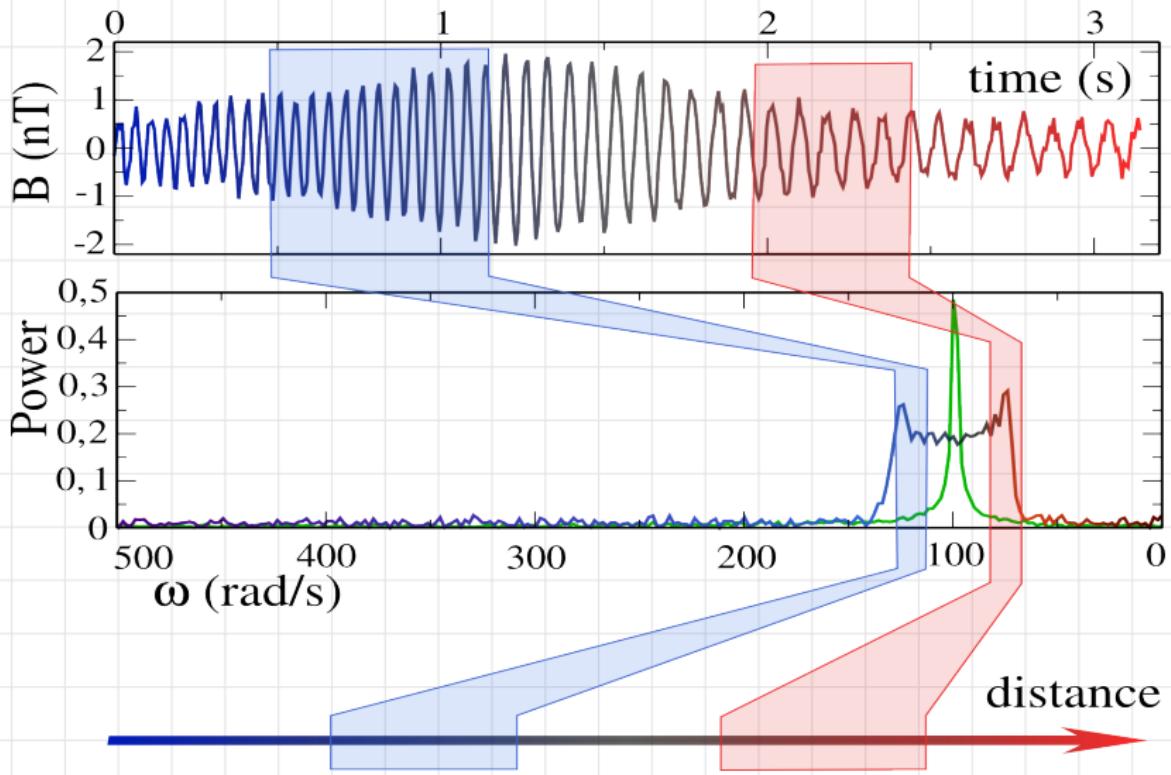
Frequency – position mapping



Frequency – position mapping

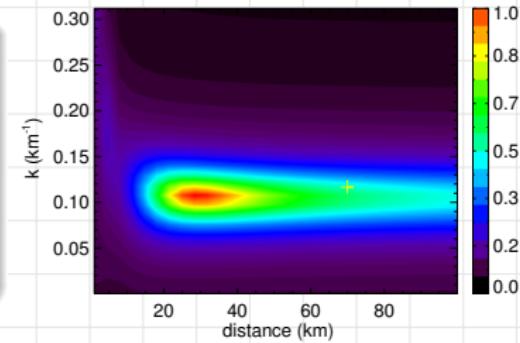
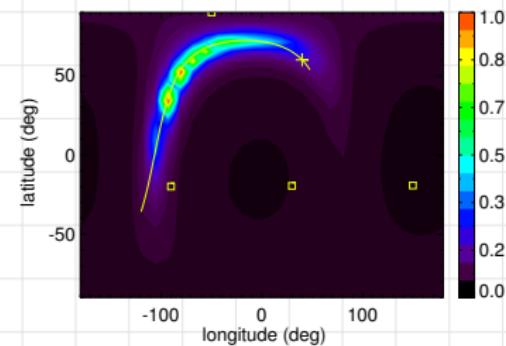


Frequency – position mapping



Moving sensor: artificial data

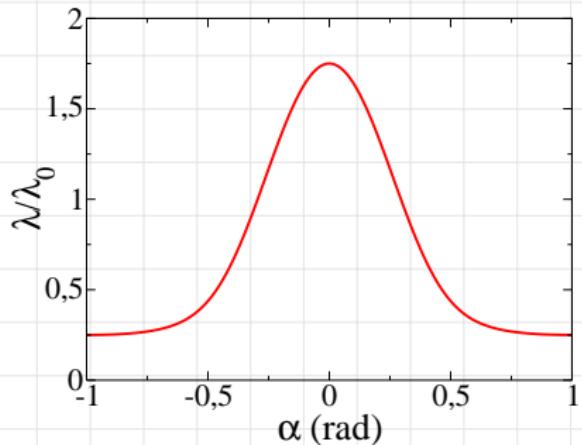
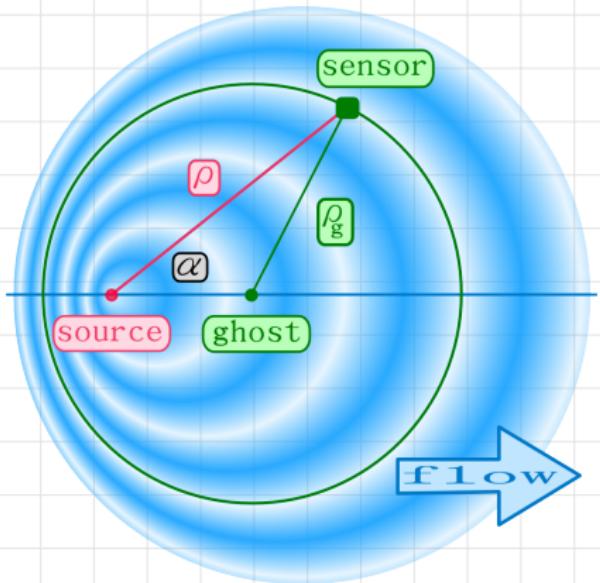
- one spherical wave
- random noise (10%)
- regular tetrahedron
- 10 km separation



	closest	found
dist.	21 km	28 km
long.	-91°	-93°
lat.	39°	34°
k	0.11 km^{-1}	0.10 km^{-1}

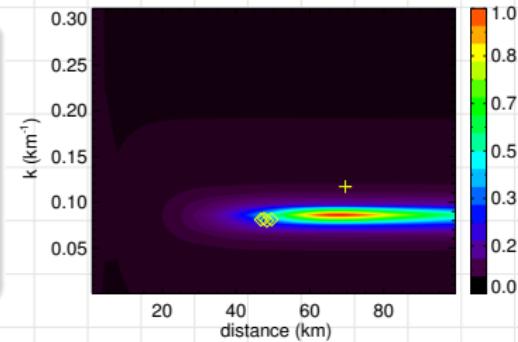
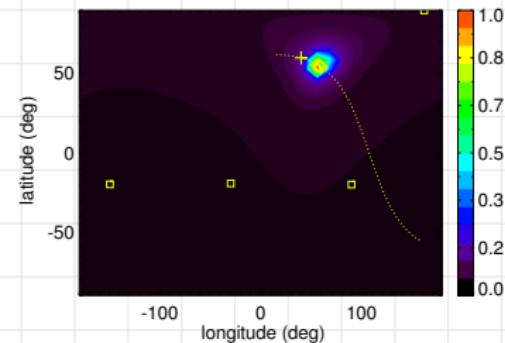
Effect of plasma flow

$$\omega = \omega_0 \quad ; \quad \lambda = \lambda_0 \frac{\rho}{\rho g} \sqrt{1 - M^2 \sin^2 \alpha}$$



Plasma flow: artificial data

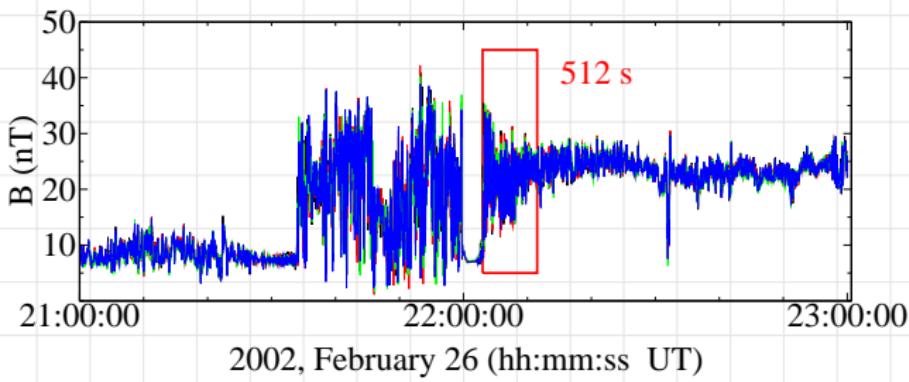
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	ghost	found
dist.	48 km	68 km
long.	58°	56°
lat.	55°	53°
k	0.08 km^{-1}	0.08 km^{-1}

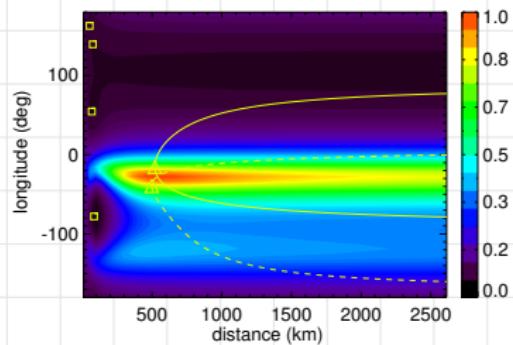
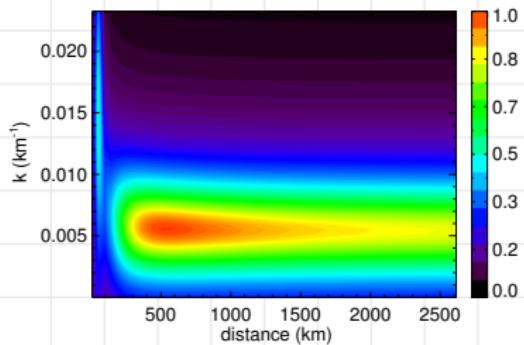
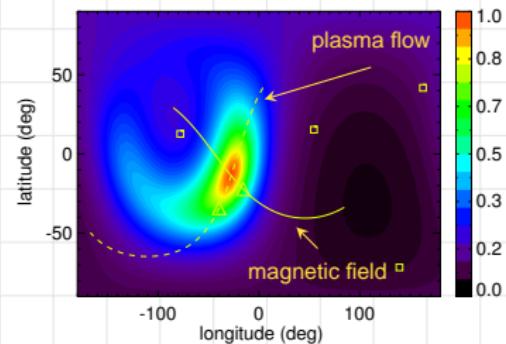
Cluster data

Time interval: 2002 February 26, 22:03 – 22:11 UT
Location: magnetosheath
Plasma flow velocity: 140 km/s [-97, -14, -101] km/s
Spacecraft separation: between 87 and 135 km
Maximum variance: parallel with mean magnetic field



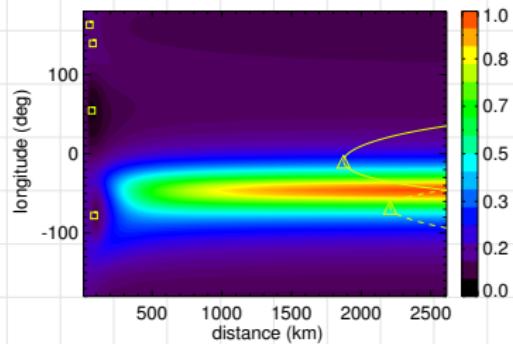
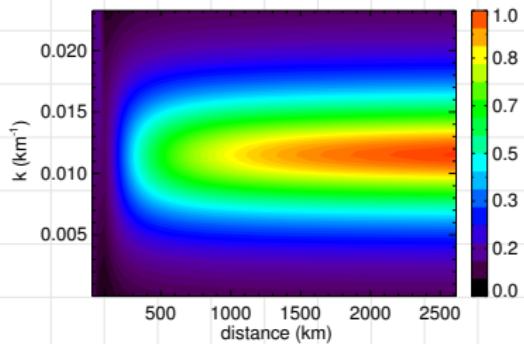
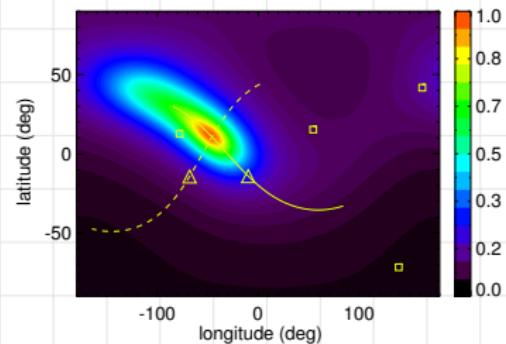
Cluster data: close moving source

frequency: 66 mHz
wave length: 1142 km
distance: 538 km
longitude: -27°
latitude: -13°



Cluster data: distant elongated source

frequency: 181 mHz
wave length: 551 km
distance: > 2500 km
longitude: -45°
latitude: -10°



Conclusions

1. **Source locator** is the generalization of the wave telescope technique to **spherical waves**.
2. Source locator determines a virtual source: Local curvature center of the detected wave fronts.
3. A virtual source is a representation of a real wave source.
4. Source locator provides information about the motion and shape of the wave sources.
5. We have identified a wave source in the magnetosheath, close to the shock.
6. Future work needs to take into account wave mode information.

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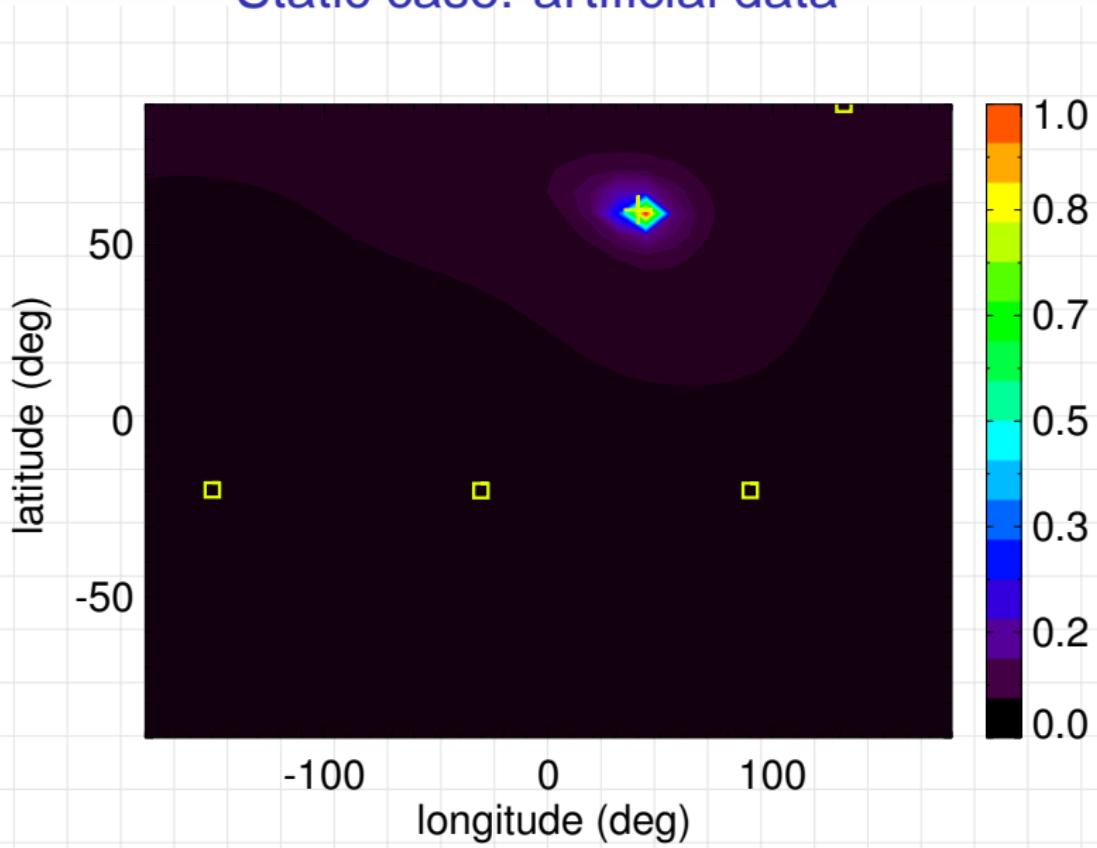
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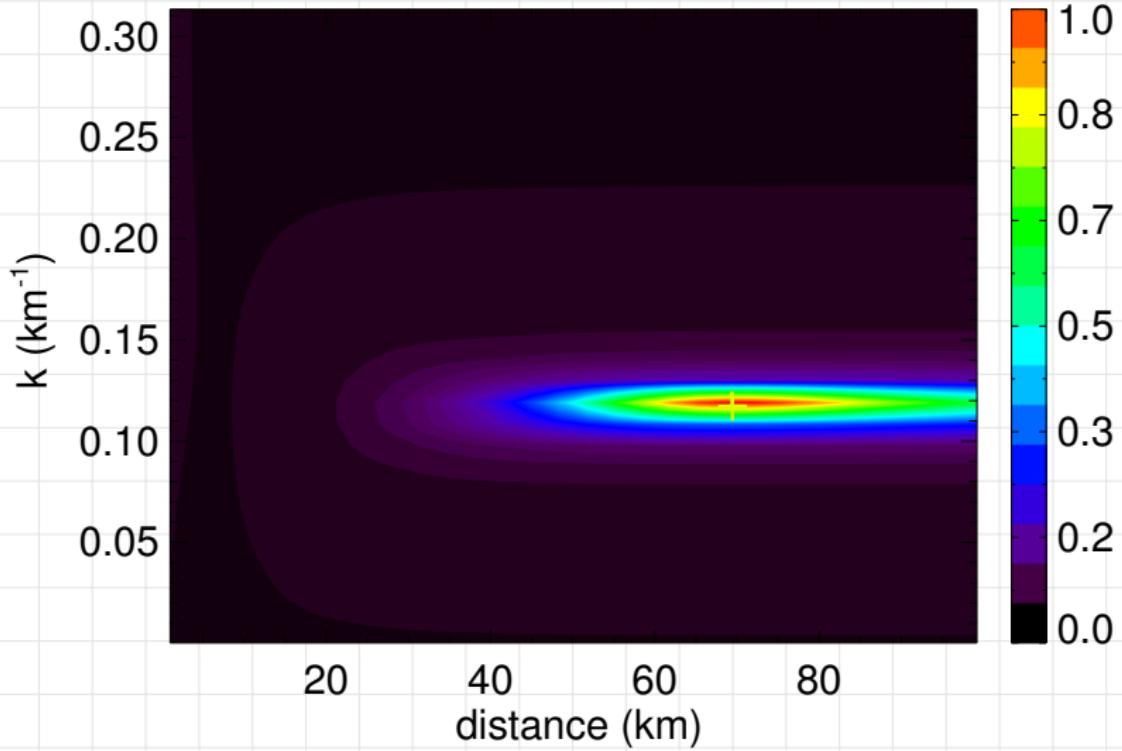
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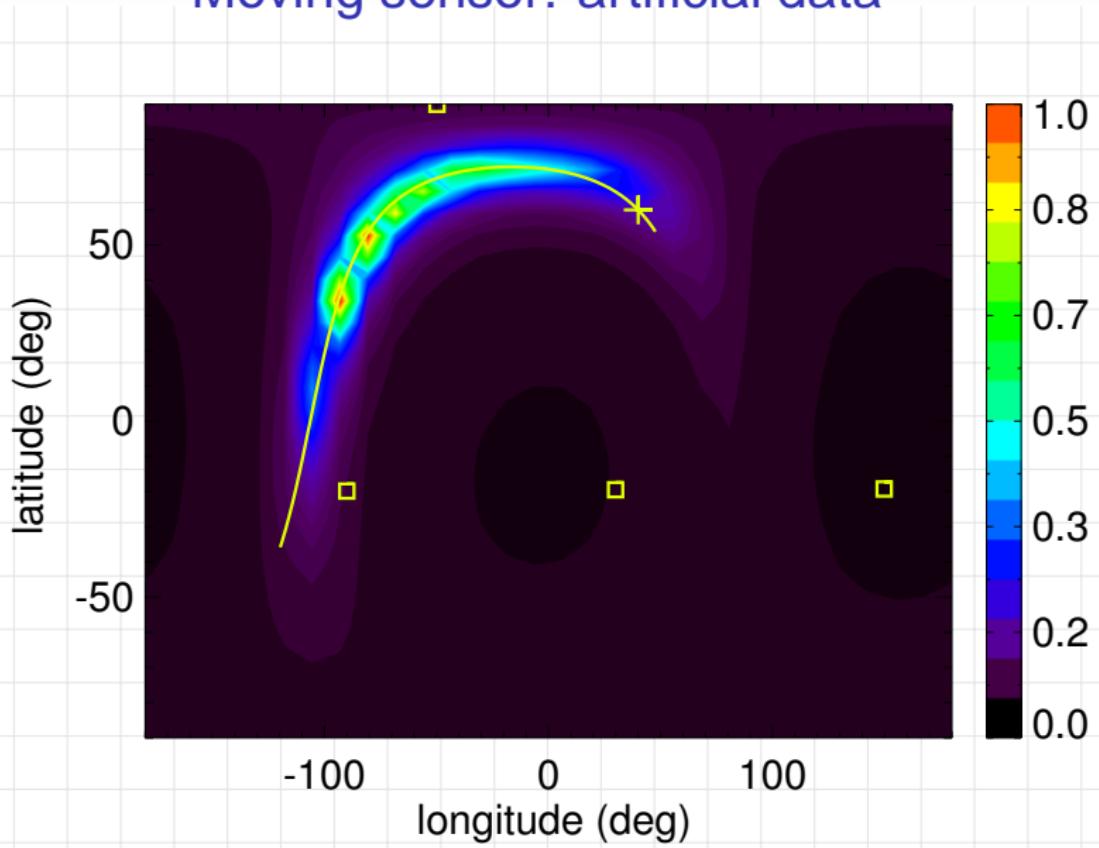
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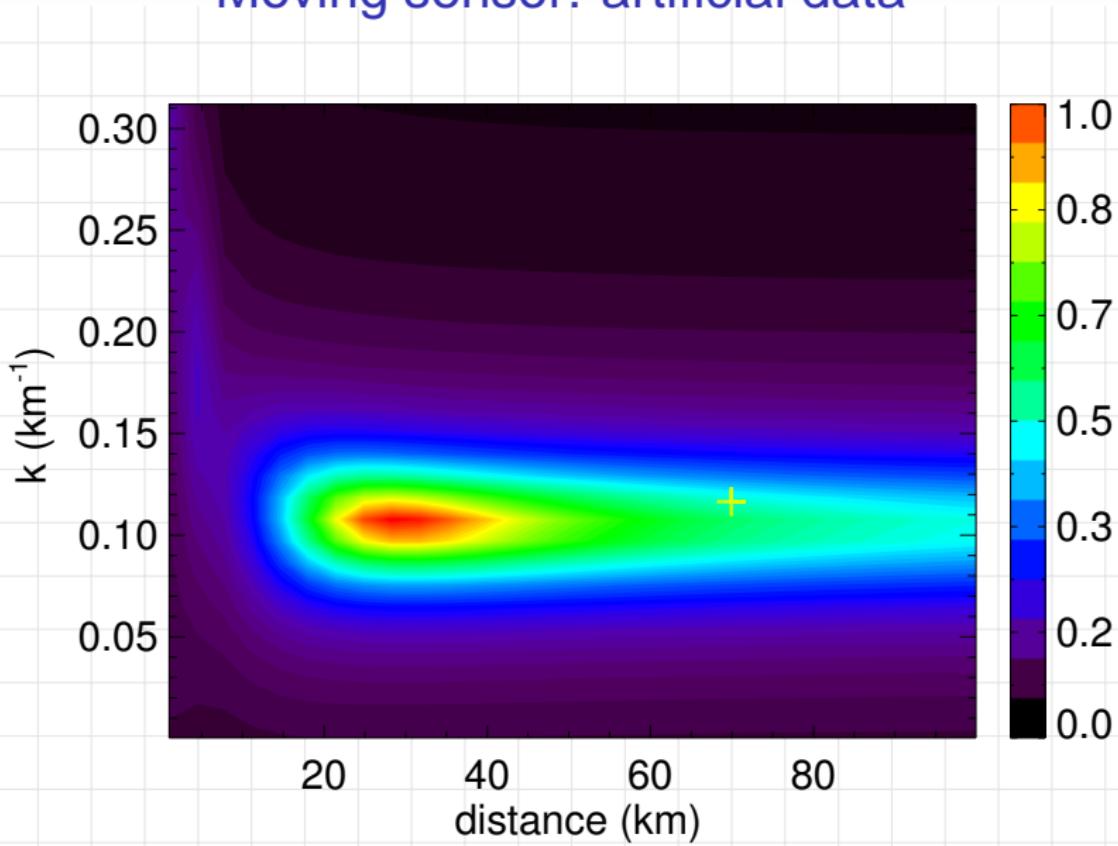
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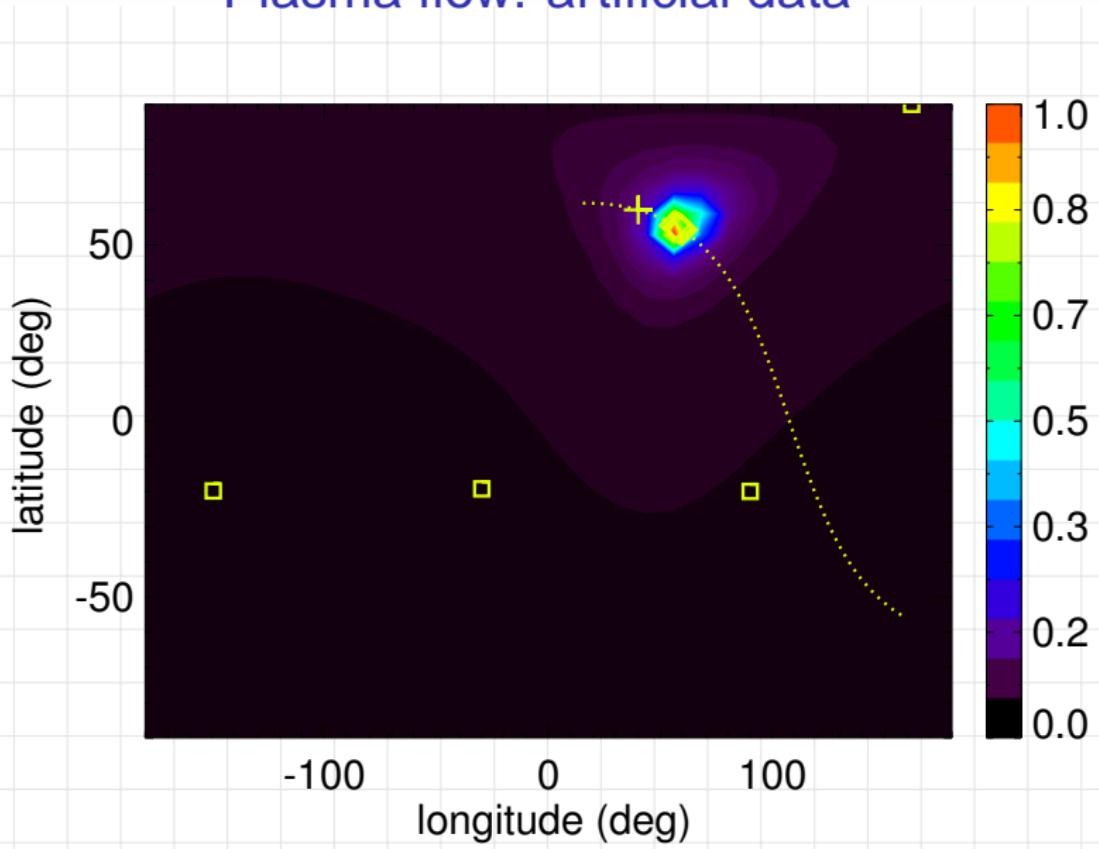
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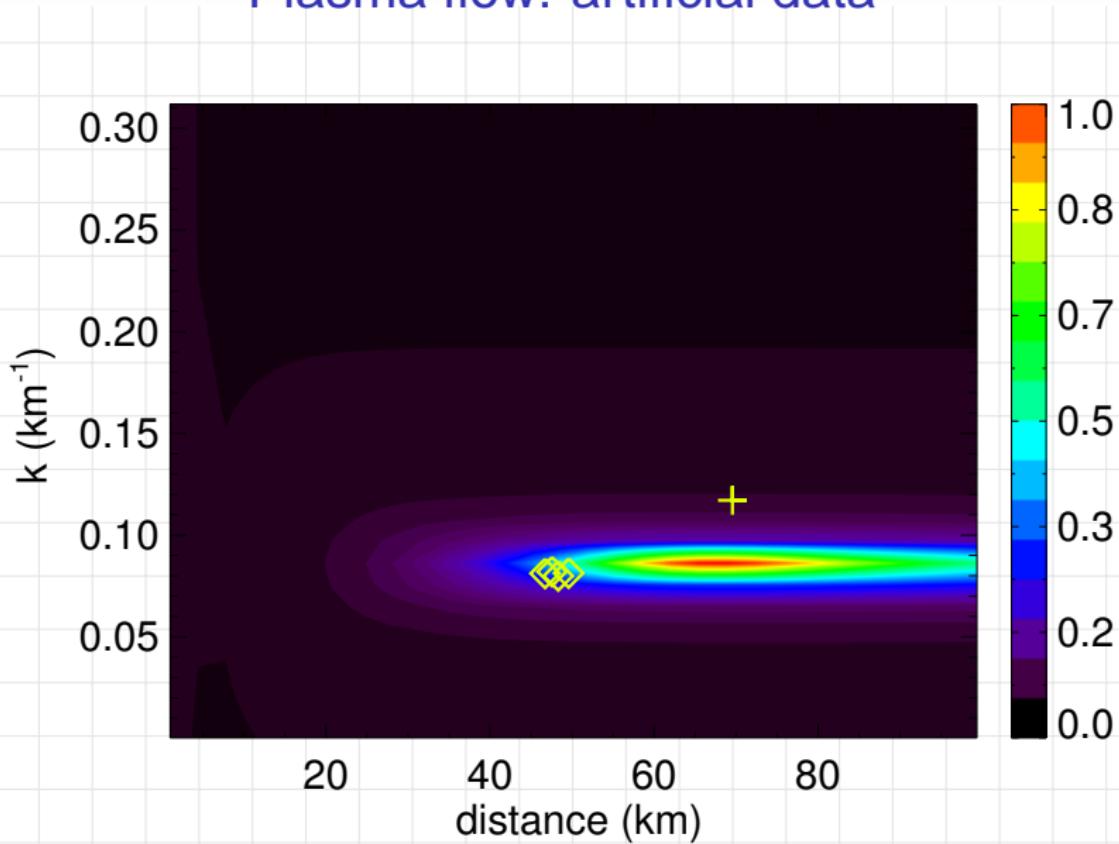
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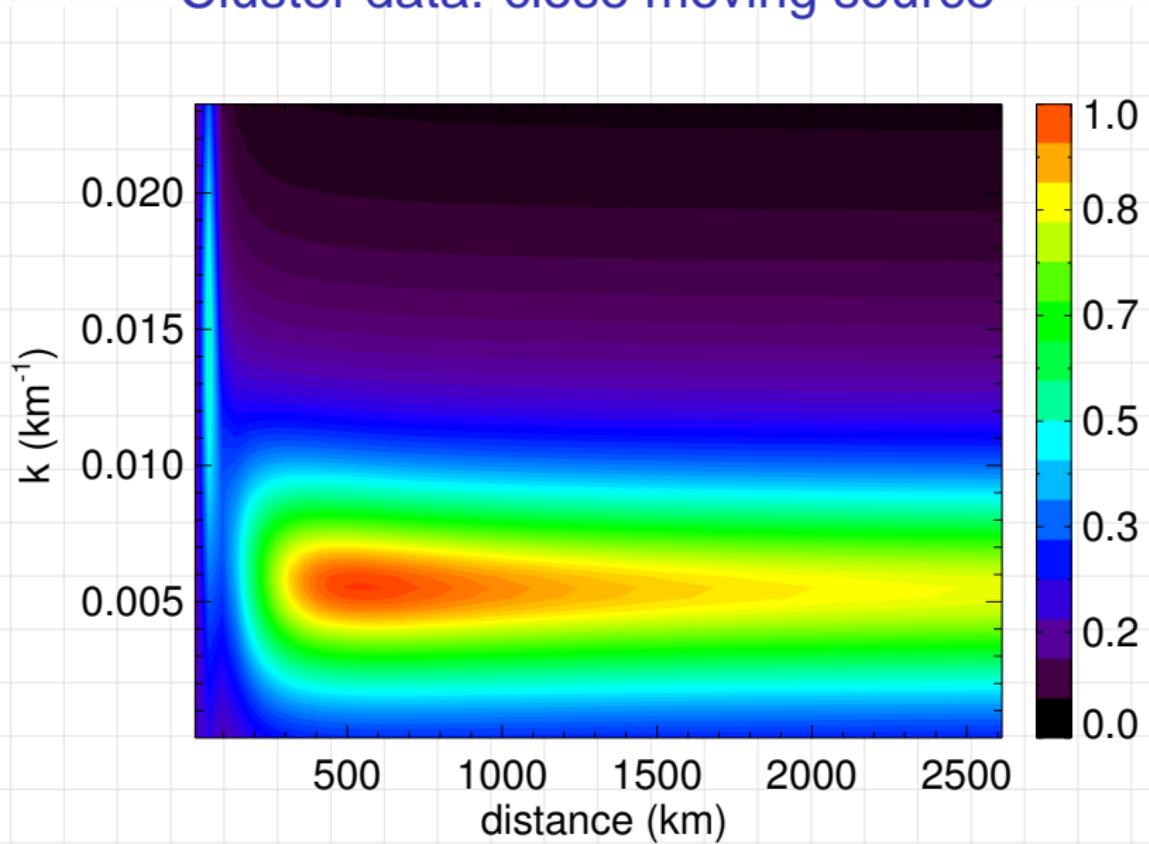
Plasma flow: artificial data



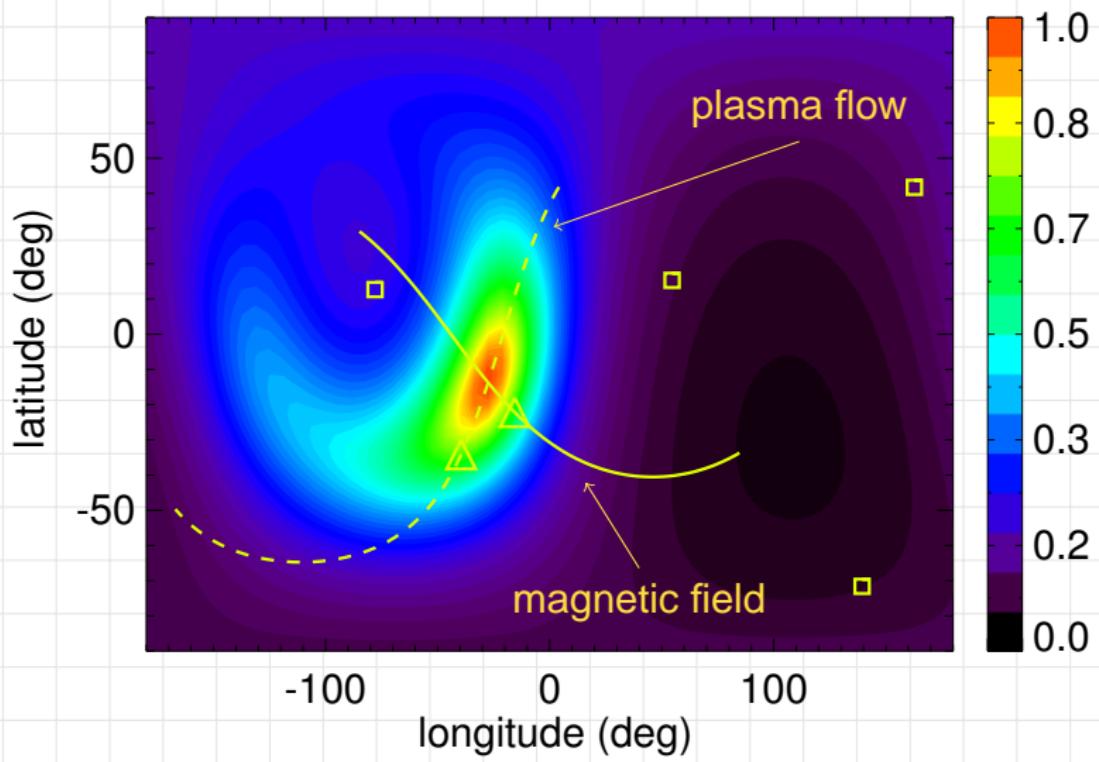
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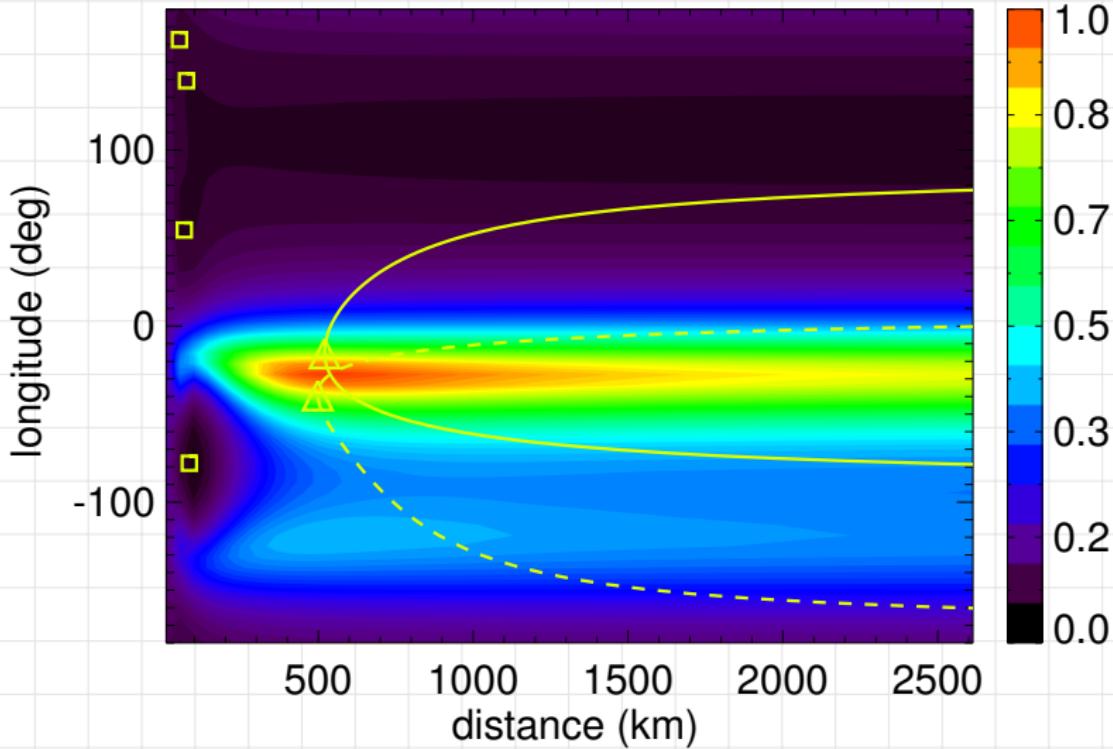
Cluster data: close moving source



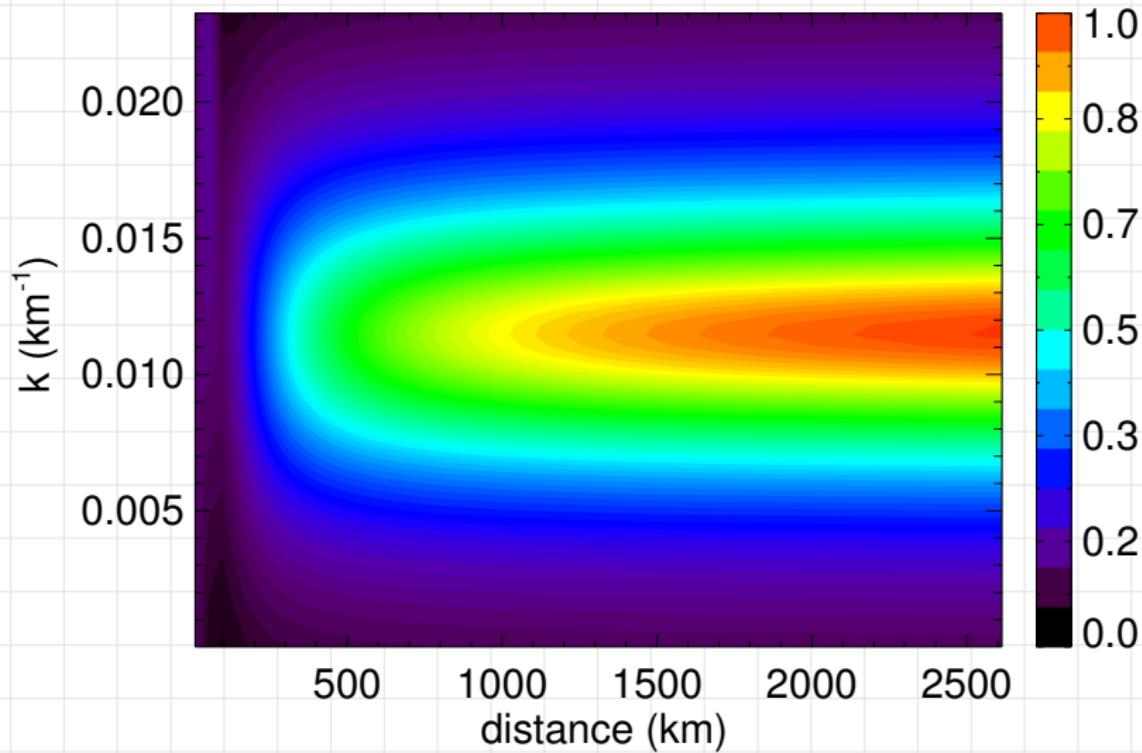
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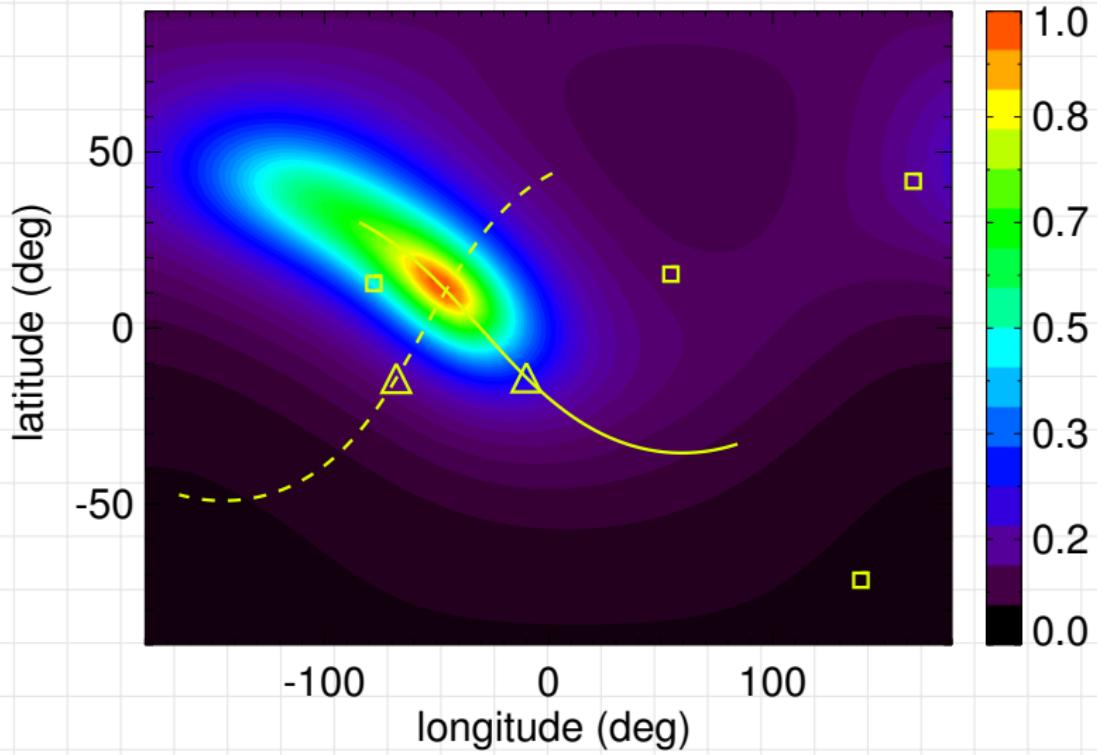
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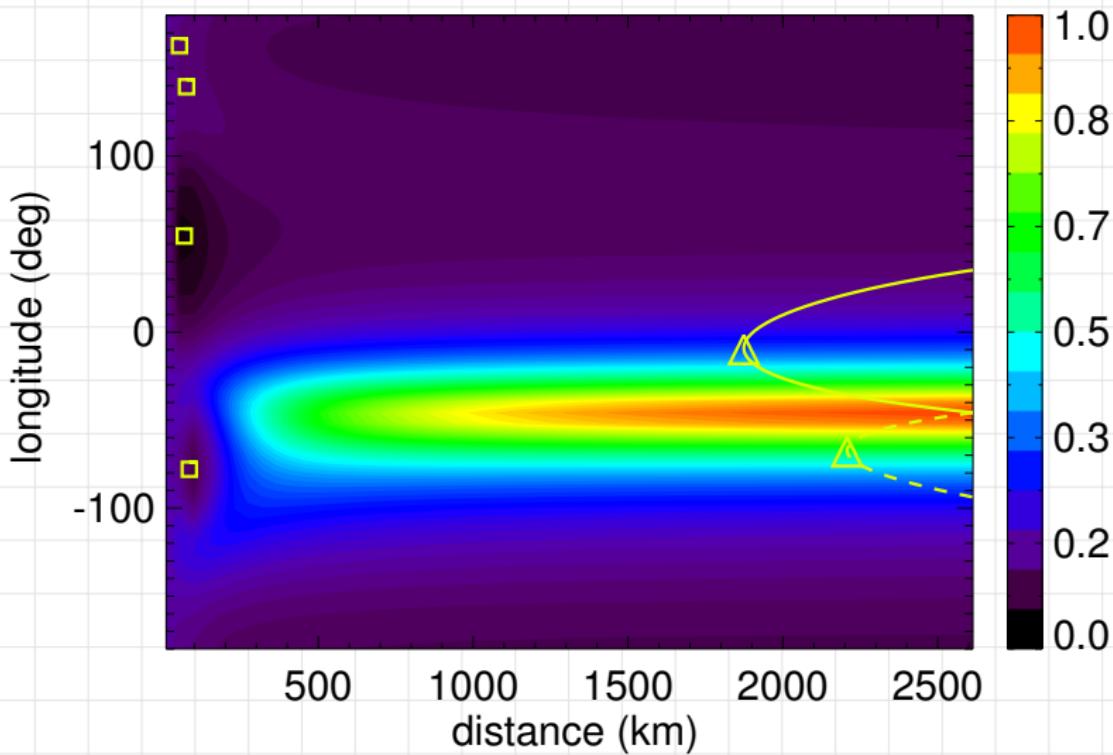
Cluster data: distant elongated source



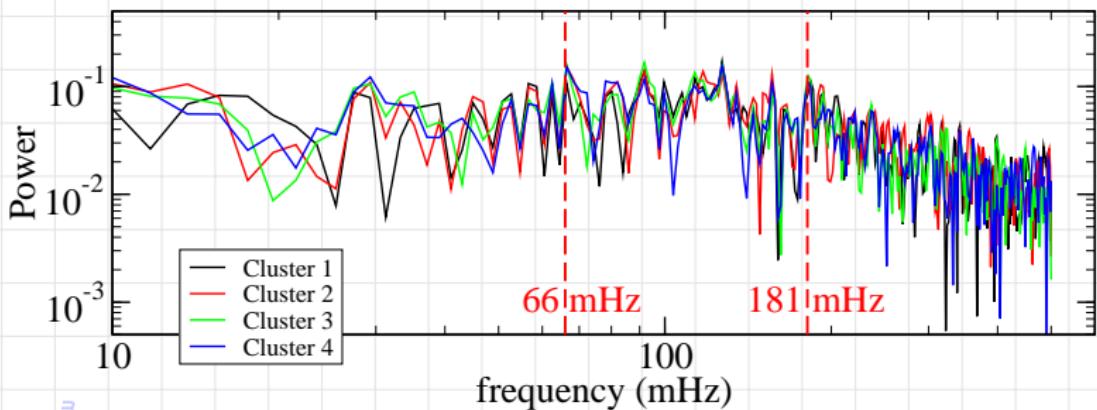
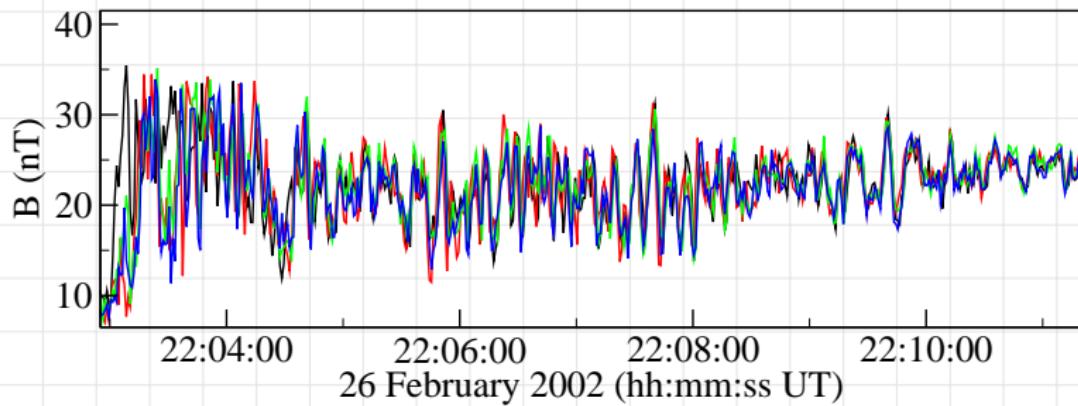
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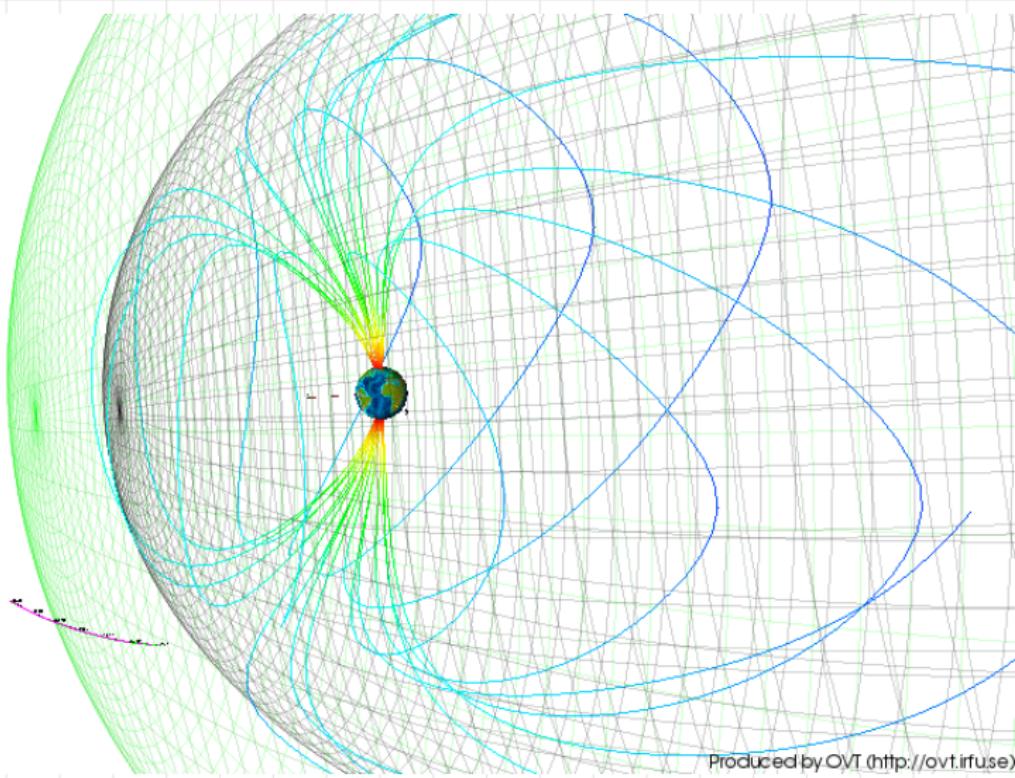
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Cluster data



Orbit, 18:00 – 24:00



Rays for moving source

