Searching wave source locations using Cluster data

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Summary:

Using patterns in data analysis

Capon technique

Source location examples

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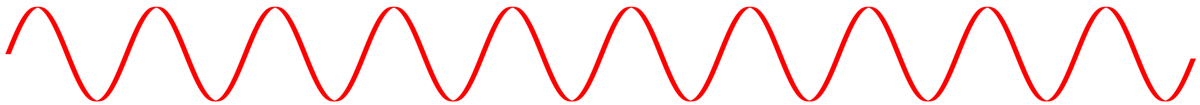
Pattern analysis

Assume that wave field $\boldsymbol{B}$ is a sum of elementary waves $\boldsymbol{w}$ $\rightarrow$ pattern

$$\left.\begin{array}{ll}\text{measured values} & \boldsymbol{B}(\boldsymbol{q}') \\ \text{test pattern} & \boldsymbol{w}(\boldsymbol{q})\end{array}\right\} \Rightarrow \quad \text{Power} \quad P(\boldsymbol{q})$$

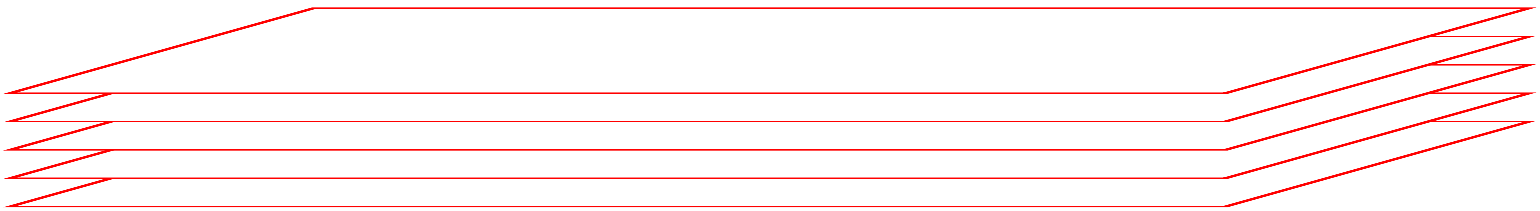
Goal: construct power in such a way that $P(\boldsymbol{q})\big|_{\boldsymbol{q}=\boldsymbol{q}'} = \text{max}$

Fourier analysis



wave field	$B_j = B_n e^{-i\omega_n t_j}$
pattern	$w_j(\omega) = Ce^{-i\omega t_j}$
array output	$b(\omega) = w_j^*(\omega) B_j$
power	$P(\omega) = \|b(\omega)\|^2$

Beamformer for plane waves



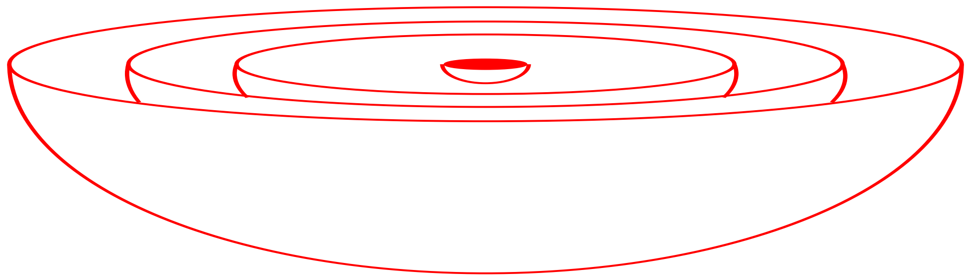
wave field $B_{sj} = B_n e^{i(\boldsymbol{k}_n \cdot \boldsymbol{r}_s - \omega_n t_j)}$

pattern $w_{sj}(\boldsymbol{k}, \omega) = C e^{i(\boldsymbol{k} \cdot \boldsymbol{r}_s - \omega t_j)}$

array output $b(\boldsymbol{k}, \omega) = w^*_{sj}(\boldsymbol{k}, \omega) B_{sj}$

power $P(\boldsymbol{k}, \omega) = \|b(\boldsymbol{k}, \omega)\|^2$

Beamformer for spherical waves



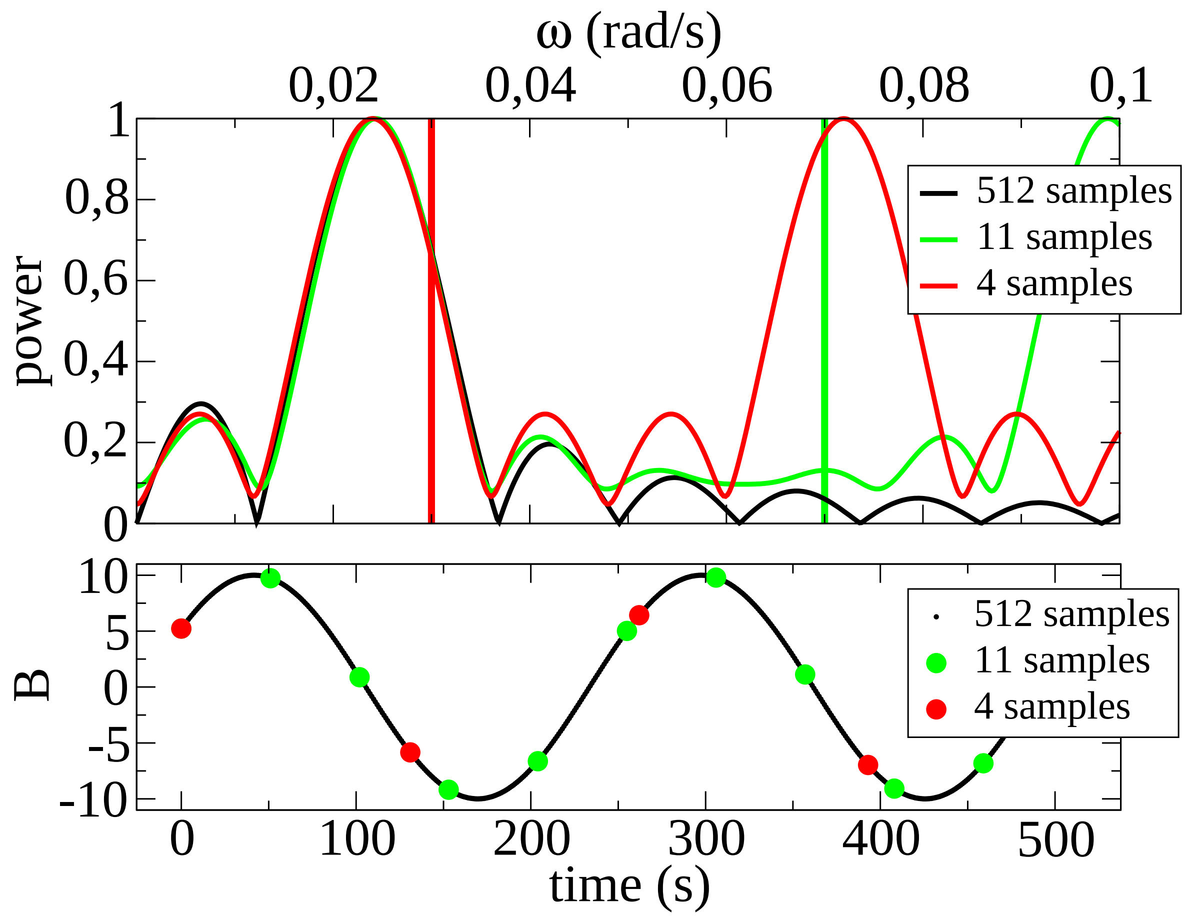
wave field $$B_{sj} = B_n \frac{1}{\rho_s} e^{i(k_n \rho_s - \omega_n t_j)}$$

pattern $$w_{sj}(k, \omega, \boldsymbol{r}_{\text{source}}) = C \frac{1}{\rho_s} e^{i(k \rho_s - \omega t_j)}$$

array output $$b(k, \omega, \boldsymbol{r}_{\text{source}}) = w^*_{sj}(k, \omega, \boldsymbol{r}_{\text{source}}) B_{sj}$$

power $$P(k, \omega, \boldsymbol{r}_{\text{source}}) = \|b(k, \omega, \boldsymbol{r}_{\text{source}})\|^2$$

Low number of data points



Nyquist:

$$\tau_{\text{min}} = 2dt$$

N samples:

$$T \leq (N/2)\tau$$

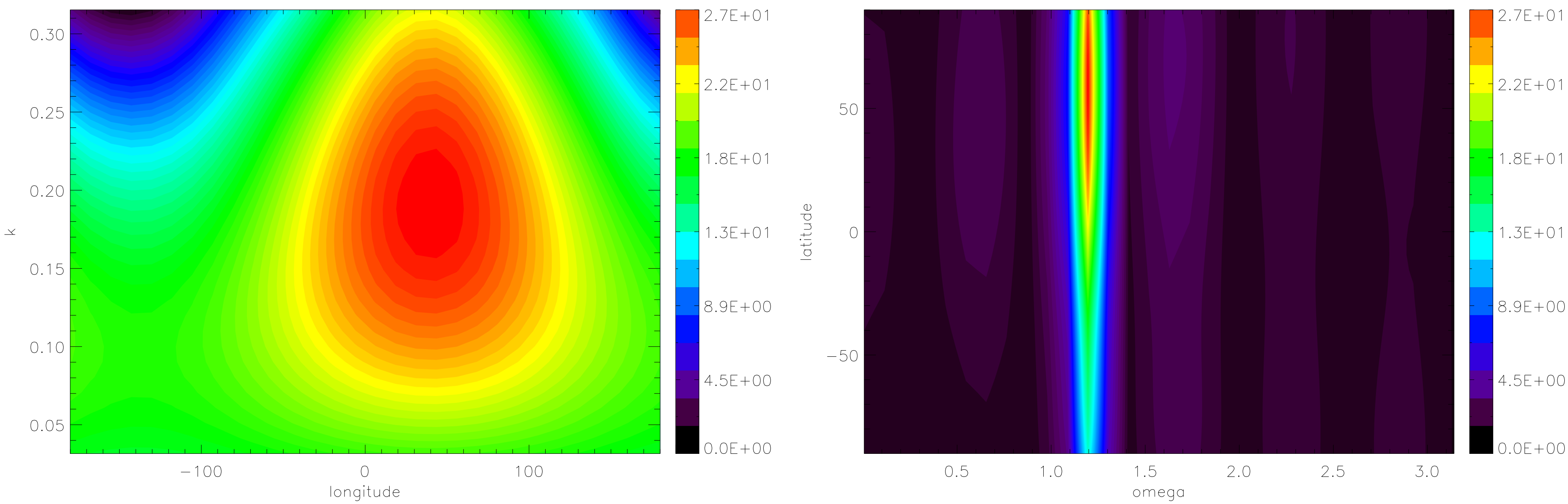
Broadening and side peaks are indirect consequences of the Nyquist theorem

Example: Beamformer for one plane wave

space: 4 points (tetrahedron)

time: 512 points

	given	obtained
longitude (deg)	40	43
latitude (deg)	60	58
frequency (s^{-1})	1.22	1.20
wave nr (km^{-1})	0.19	0.19



Capon Method

Problem:

- the power is too high for wrong parameters

Solution:

- minimize the power for all parameters except the right ones:
  - ▷ $P_{Capon}(\boldsymbol{q}) \leq P(\boldsymbol{q})$
  - ▷ $P_{Capon}(\boldsymbol{q}') = P(\boldsymbol{q}')$
- the result is:
  - ▷ $P_{Capon} = \left(\boldsymbol{w}^{+}\underline{\boldsymbol{B}\boldsymbol{B}}^{-1}\boldsymbol{w}\right)^{-1}$

When the pattern is a plane wave this technique is known as Wave Telescope or k-filtering

Spherical wave source location: Setup

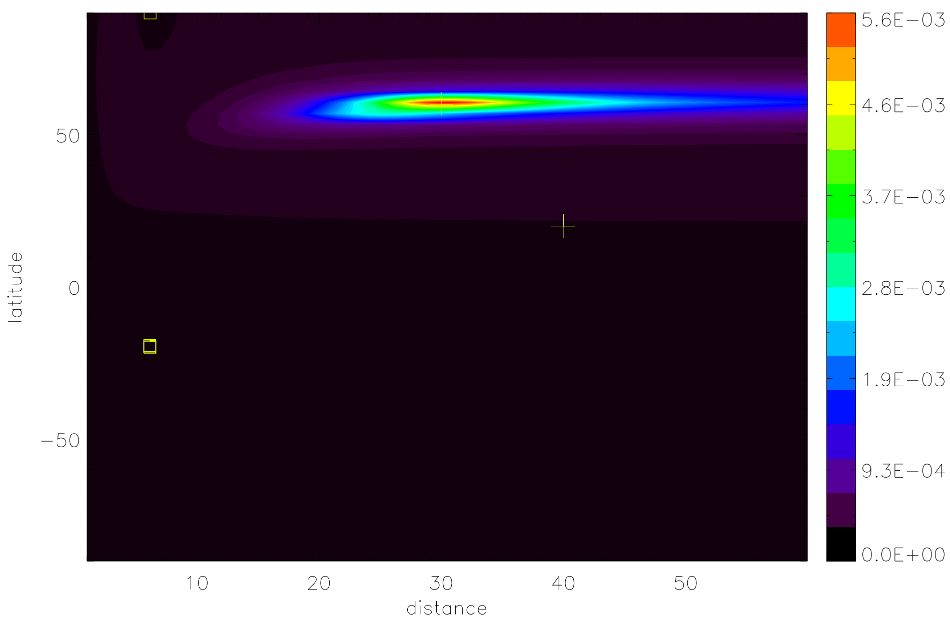
Sensor Array: regular tetrahedron, 10 km

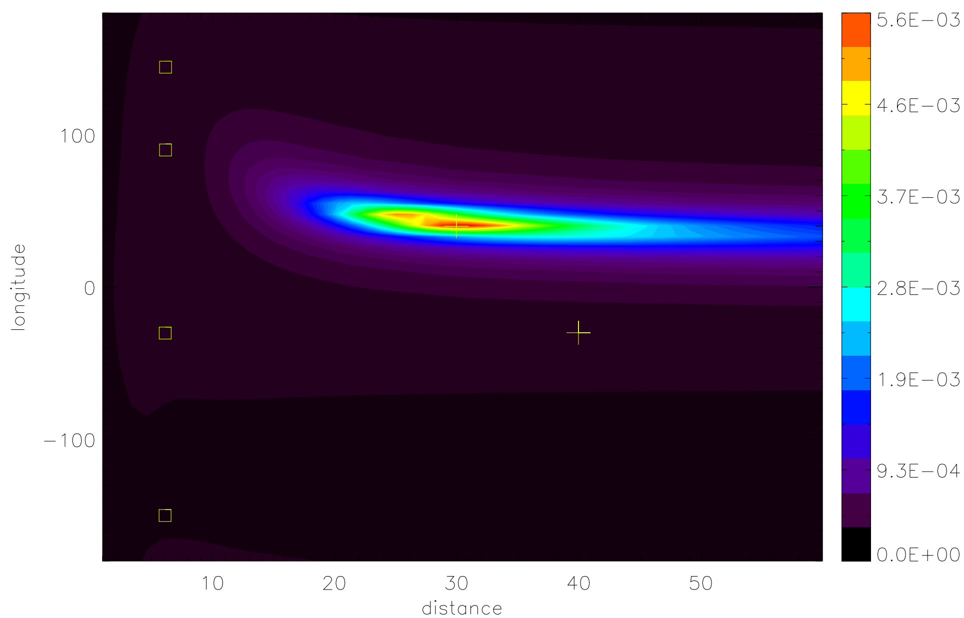
Wave Field: 2 spherical waves + noise

source	ρ (km)	φ (deg)	θ (deg)	amplitude	ν (s^{-1})	λ (km)
1	29.95	40.00	60.00	10.00	99.03	22.96
2	39.93	-30.00	20.00	10.00	98.39	22.96

Spherical wave source location: Source 1

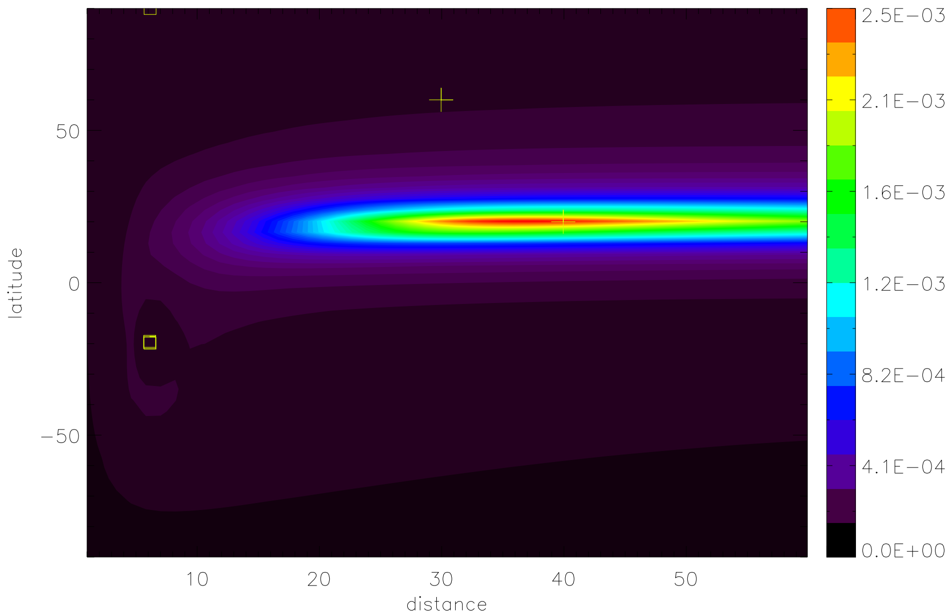
	ρ (km)	φ (deg)	θ (deg)
given	29.95	40.00	60.00
obtained	29.85	40.40	60.61

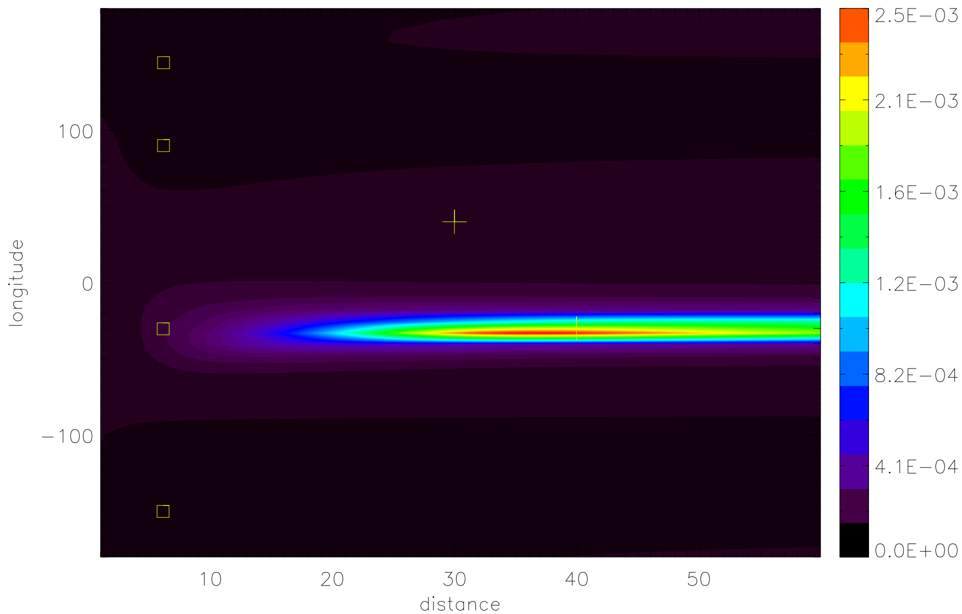




Spherical wave source location: Source 2

	ρ (km)	φ (deg)	θ (deg)
given	39.93	-30.00	20.00
obtained	37.06	-33.06	20.20





Conclusions

Patterns can be used to analyze multi-point measurements

Simple methods fail for low number of samples

Capon technique can be applied for low number of samples

We have a tool for locating the wave sources

Different patterns and arbitrary number of sensors can be used

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