Pattern Recognition With Cluster Data

Dragoş Constantinescu[1,2]

Summary:

Fitting techniques

- Mirror Mode pattern
- Example

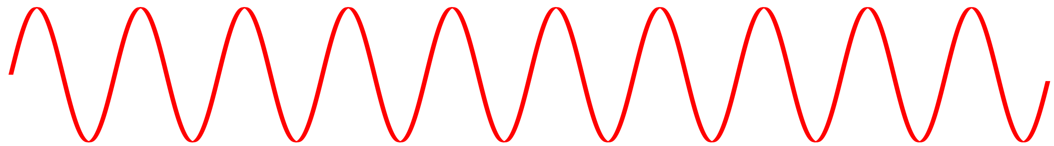
Virtual interference techniques

- General idea
- Beamformer and applications
- Minimum variance and applications

1: Institut für Geophysik und extraterrestriche Physik, TU Braunschweig, Germany
2: Institute for Space Sciences, Bucharest, Romania

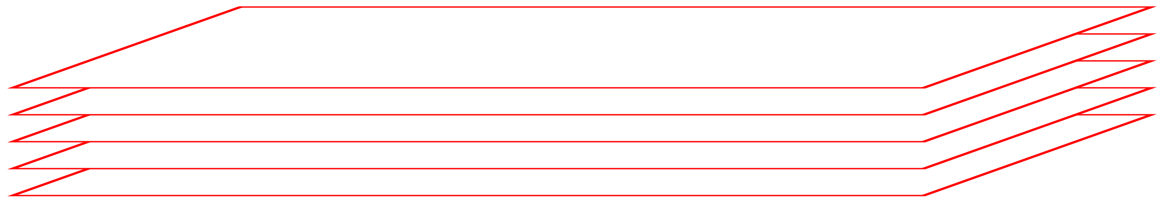
Pattern Examples

- Fourier



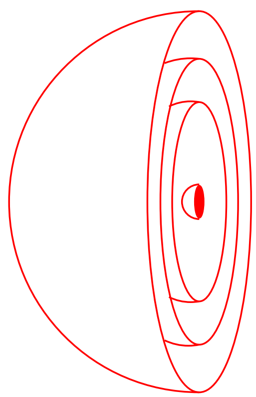
(1D)

- Plane wave



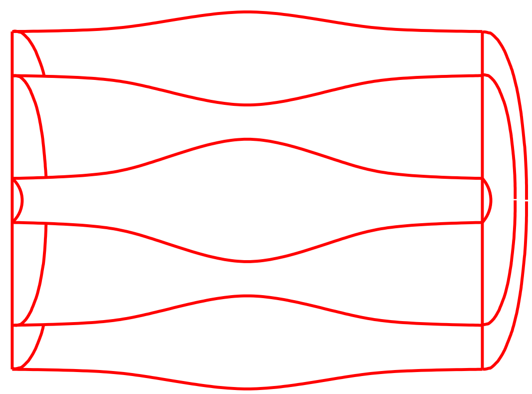
(3D)

- Spherical wave



(4D)

- Cylindrical wave



(6D)

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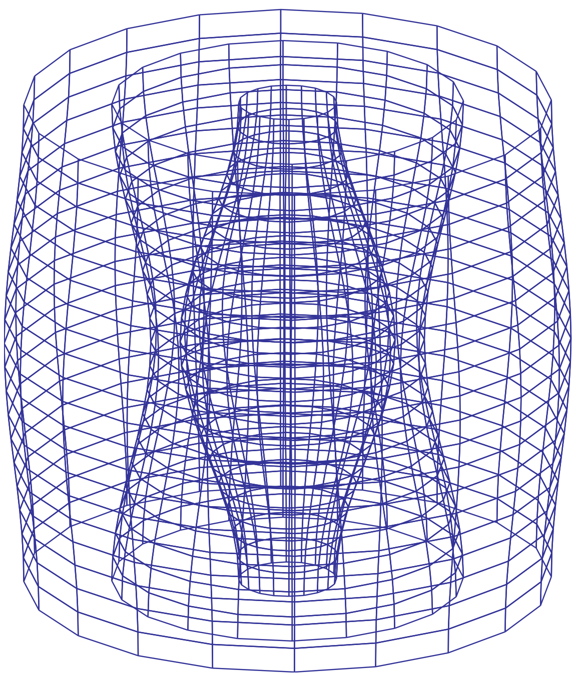
Mirror Mode Pattern

Magnetic field perturbation:

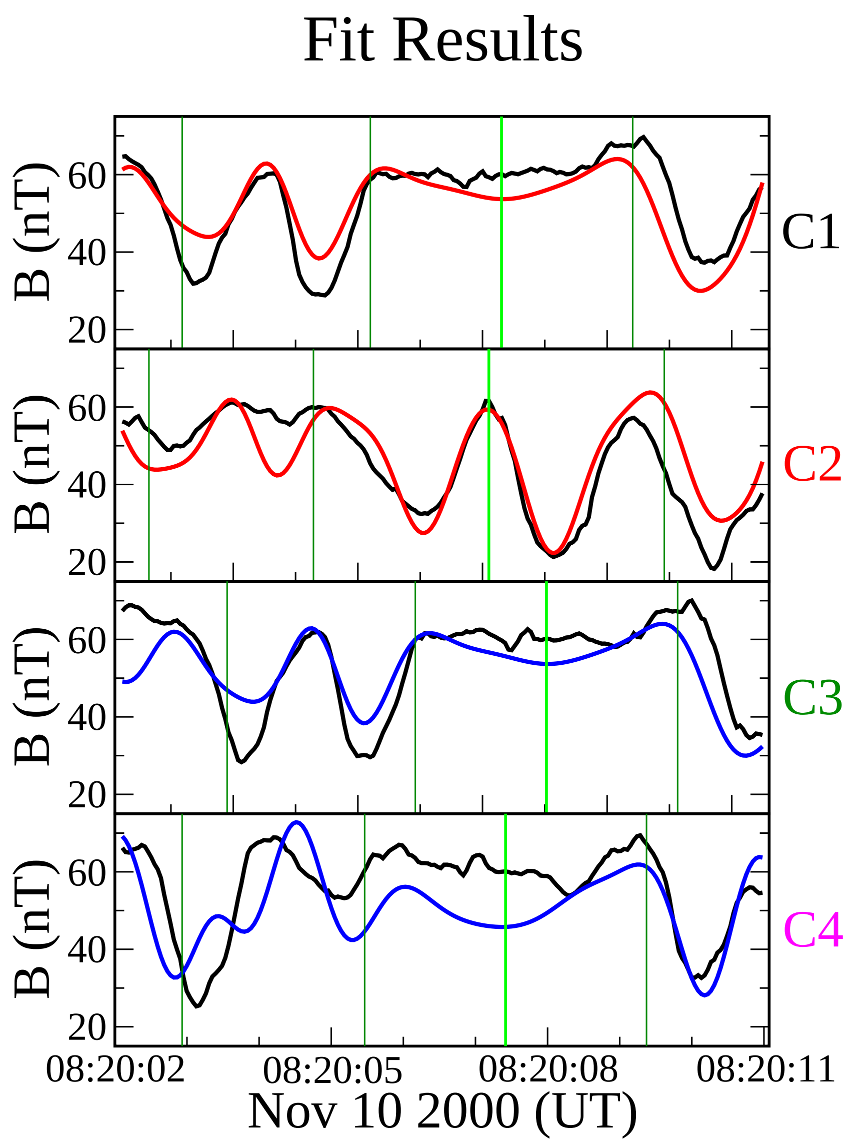
- $$\delta B_\rho(\rho, z) = \frac{2\pi}{\alpha} \sum_{n=1}^{\infty} J_1\left(\frac{n\alpha\rho}{L}\right) \left[a_n \sin\left(\frac{n\pi z}{L}\right) - b_n \cos\left(\frac{n\pi z}{L}\right)\right]$$
- $$\delta B_z(\rho, z) = 2 \sum_{n=1}^{\infty} J_0\left(\frac{n\alpha\rho}{L}\right) \left[a_n \cos\left(\frac{n\pi z}{L}\right) + b_n \sin\left(\frac{n\pi z}{L}\right)\right]$$



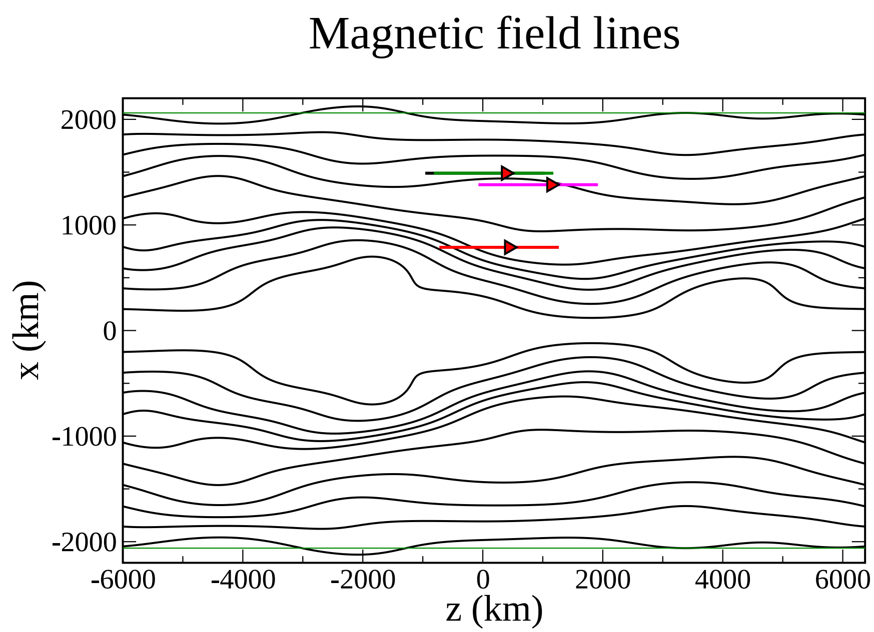
- Multi-layer structure
- Central structure is the classical image of magnetic mirror
- Multiple magnetic field minima belong to one structure
- In real world only inner layers will survive

From JASTP Vol 64, Constantinescu, Self-consistent model of mirror structures

Application to Cluster data



- fit on data from C1 and C2
- C3 and C4 are witness spacecraft
- Resulting dimensions:
  - ▷ $L = 6186$ *km*
  - ▷ $R = 2051$ *km*



Virtual Interference Techniques

Interference of the measured values from an array of S sensors $\mathbf{B} = (B_1, ...B_S)^T$ with a test pattern $\mathbf{w} = (w_1, ..., w_S)^T$ depending on the parameters $\mathbf{q} = (q_1, ..., q_n)$.

Method:

- Construct the output power P as a combination between $\mathbf{B}$ and $\mathbf{w}$
- no unique way to do that but the guideline is ...
- The power should maximize when the parameters are chosen such way that the test pattern is closest to the pattern present in the data

Beamformer Technique

- Power: $P_{BF} = \mathbf{w}^{+}\mathcal{M}\mathbf{w}$ where we define the sensor output matrix as $\mathcal{M} = \mathbf{B}\mathbf{B}^{+}$

- The test pattern $\mathbf{w}$ is chosen depending on the problem to study

Example: Plane waves representation

- measurements: $B_m = \sum_q B_{0q} e^{i(\mathbf{k}_q \mathbf{r}_{mq} - \omega t + \varphi_q)}$

- test pattern: $w_m = \frac{1}{\sqrt{S}} e^{i\mathbf{k}^{test}\mathbf{r}_m}$

- in this case $\mathbf{w} = \mathbf{w}(\mathbf{k}^{test})$ so we can determine the wave vector $\mathbf{k}$

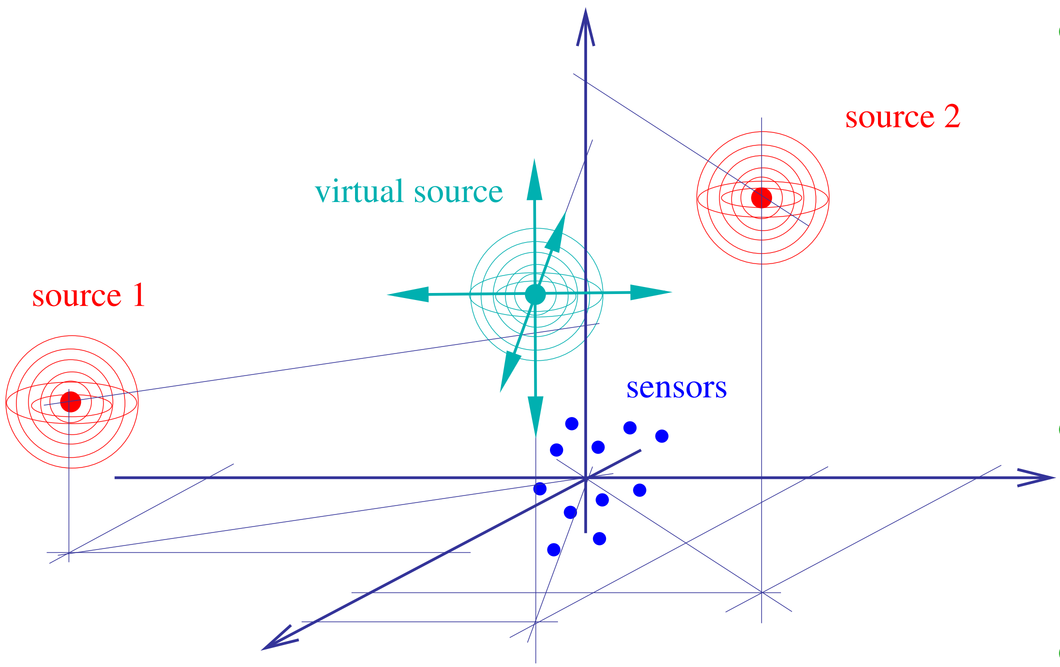
- eg. only one source $B_m = B_0 e^{i\mathbf{k}\mathbf{r}_m}$

  ▷ $P_{BF} = \frac{1}{S} |B_0|^2 \left| \sum_m e^{-i(\mathbf{k} - \mathbf{k}_{test})\mathbf{r}_m} \right|$

  ▷ P_{BF} *is maximum when* $\mathbf{k} = \mathbf{k}^{test}$

Beamformer for Spherical Waves

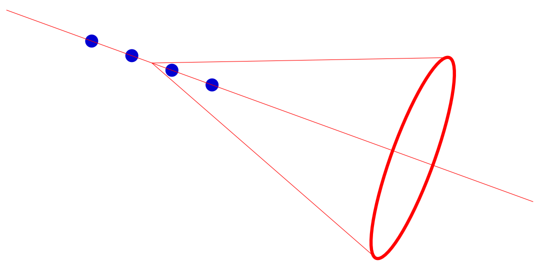
- measurements: $B_m = \sum_q \frac{1}{\rho_q} B_{0q} e^{i(k_q \rho_{mq} - \omega_q t + \varphi_q)}$
- test pattern: $w_m = \left(\sum_n (\rho_n^{test})^{-2}\right)^{-1} \frac{1}{\rho_m^{test}} e^{ik^{test} \rho_m^{test}}$

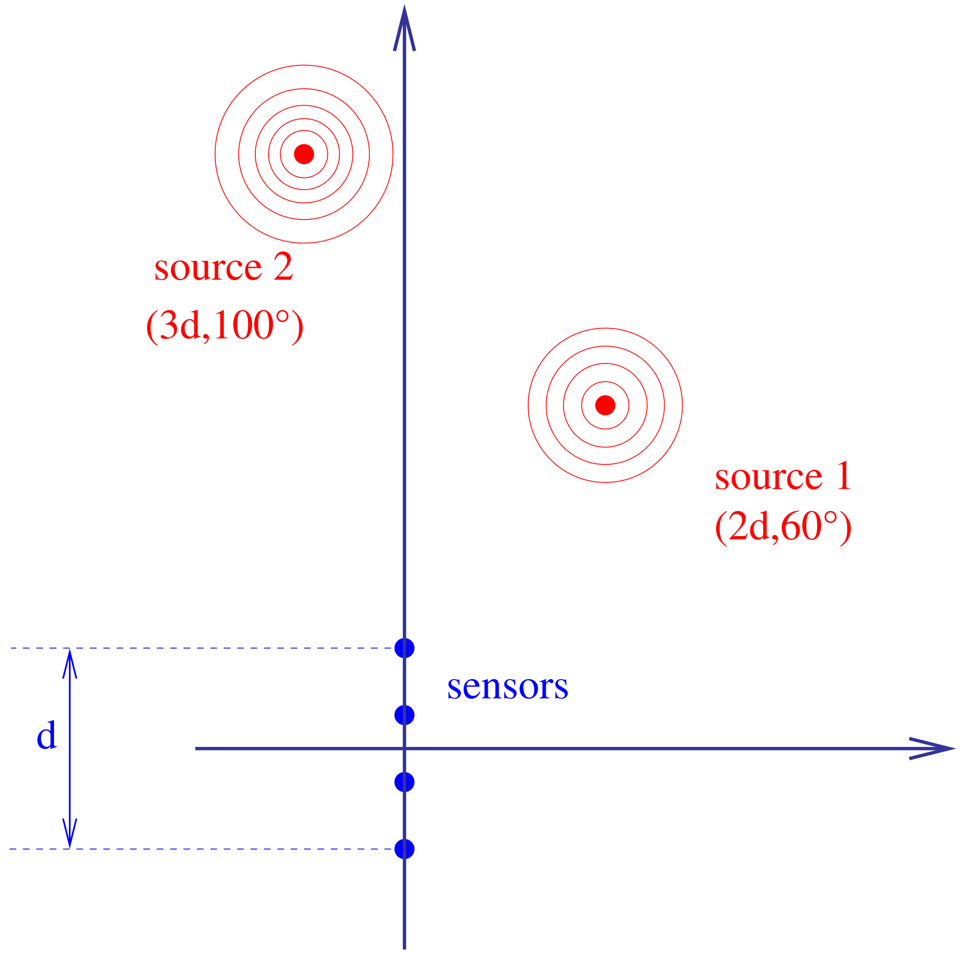


- $\rho^{test} = \rho^{test}(\mathbf{r}^{test})$
  - ▷ *in this case* $\mathbf{w} = \mathbf{w}(\mathbf{r}^{test}, k^{test})$
  - ▷ *we can determine:*
    - * *source position*
    - * k *vector magnitude*
- difficulty:
  the test-space dimension has increased from 3 to 4
- in what follows we consider k known

Beamformer for Linear Array Configuration. Data

Because of the symmetry the test-space is reduced to a bi-dimensional space



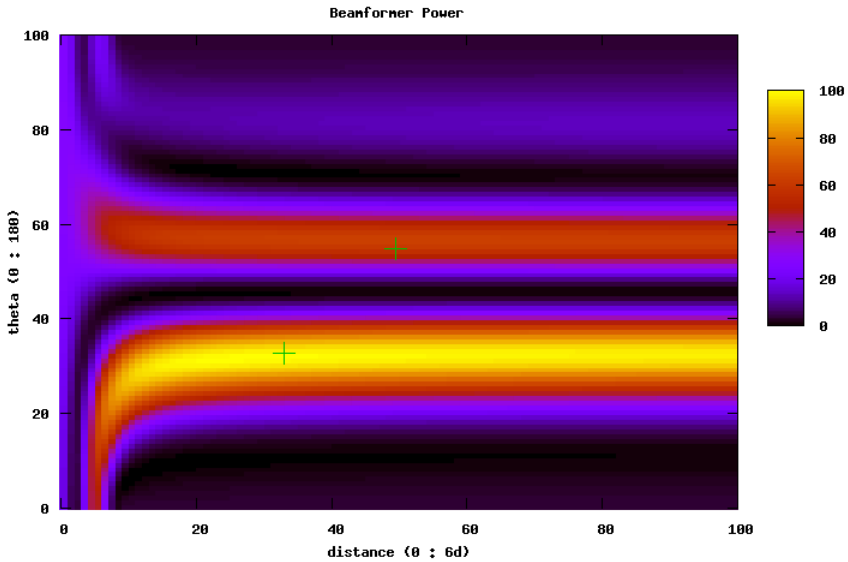


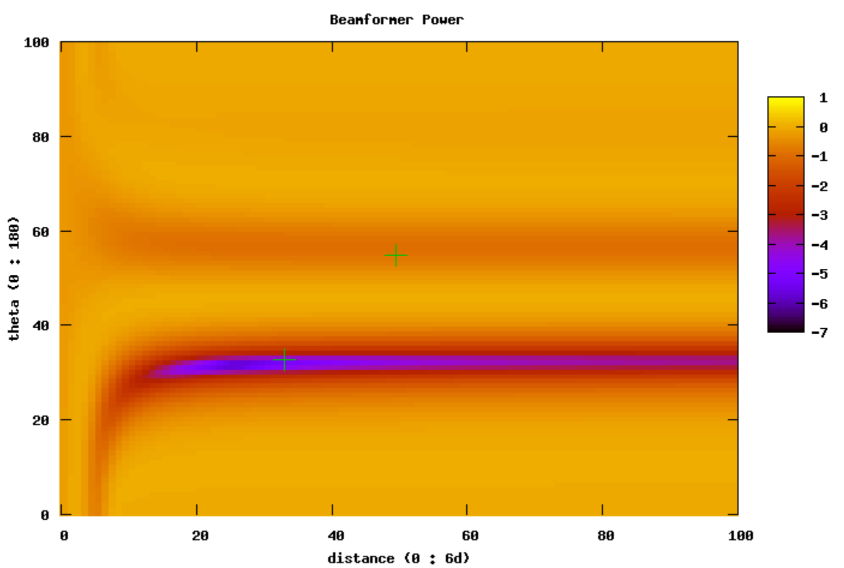
Simulated data:

	θ	ρ
source 1	60^o	$2d$
source 2	100^o	$3d$

- wave length $\lambda = 2d$

Beamformer for Linear Array Configuration. Results





- Results:

	θ	ρ
source 1	60^o	$2d$
result	57^o	$1.5d$

- good direction
- weak signal for 2^{nd} source
- not very good distance resolution
- for non-linear configuration is even worse...
- we need something better

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Minimum Variance Technique

Beamformer problem:

- the power is too high for wrong parameters

Solution:

- construct the power in a different way:
  - $P_{MV} = \left(\mathbf{w}^{+}\mathcal{M}^{-1}\mathbf{w}\right)^{-1}$

    this keeps the beamformer power for the right parameters and minimize the power for the wrong ones

When the pattern is a plane wave this technique is known as Wave Telescope or k-filtering

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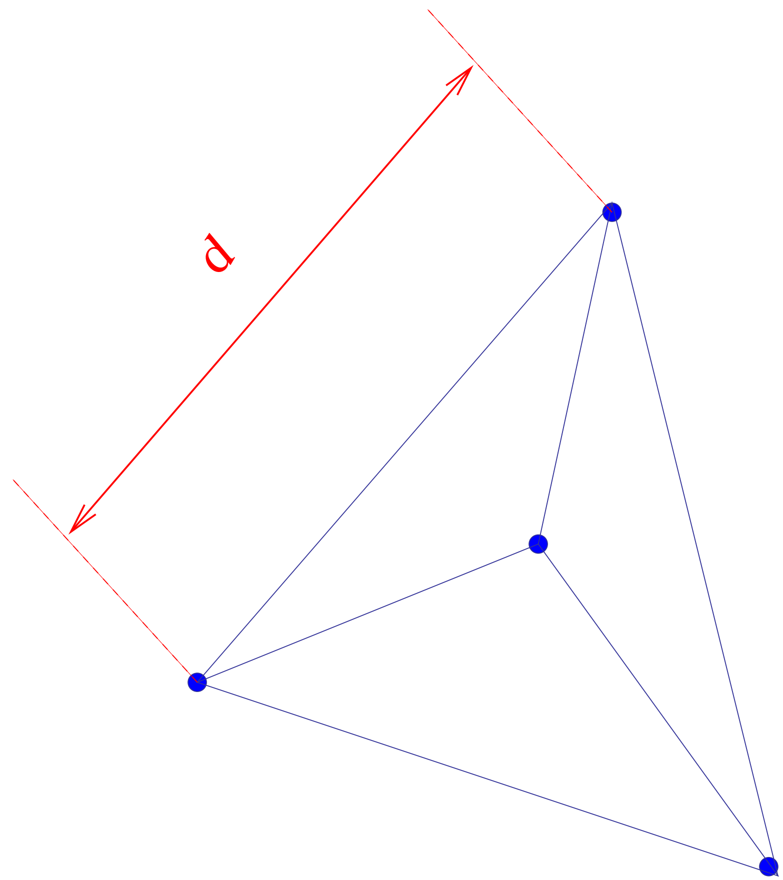
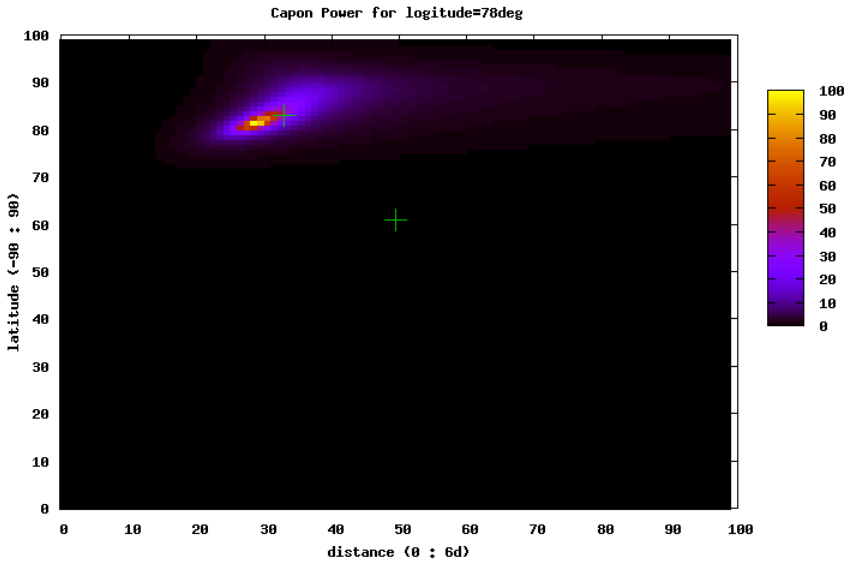
Minimum Variance for Tetrahedron Configuration. Synthetic Data

- regular tetrahedron
- two sources
- random phase
- random deviation from given frequency
- noise
- same wavelength $\lambda = \frac{2}{3}d$
-

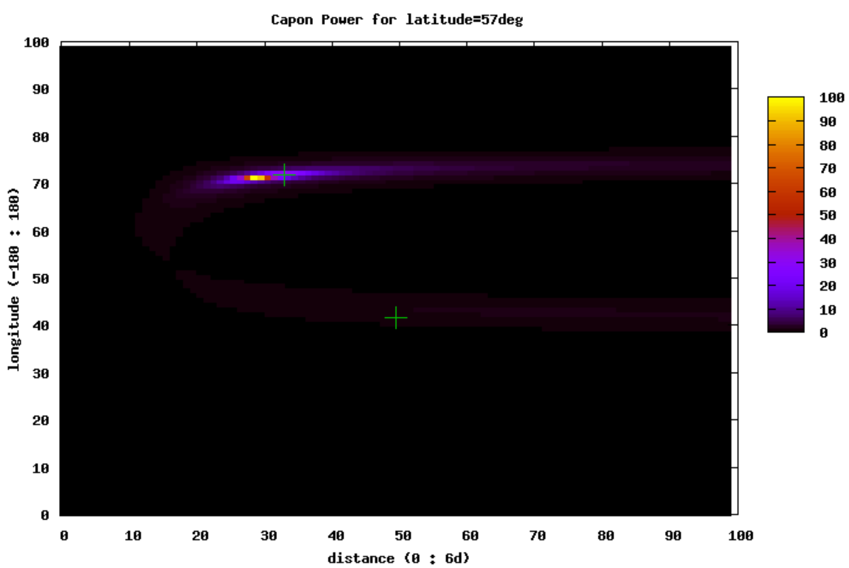
	long	lat	dist
s1	80^o	60^o	$2d$
s2	-30^o	20^o	$3d$

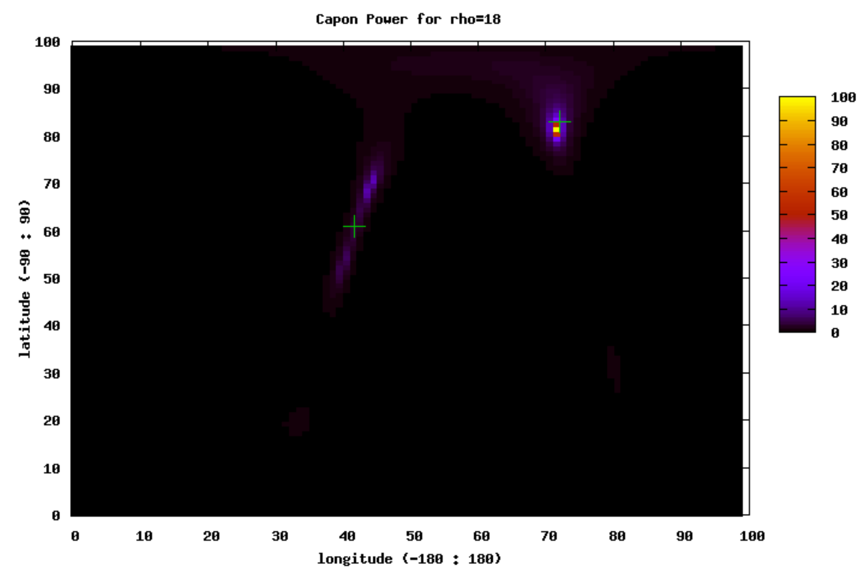


MV for Tetrahedron. Results for Source 1

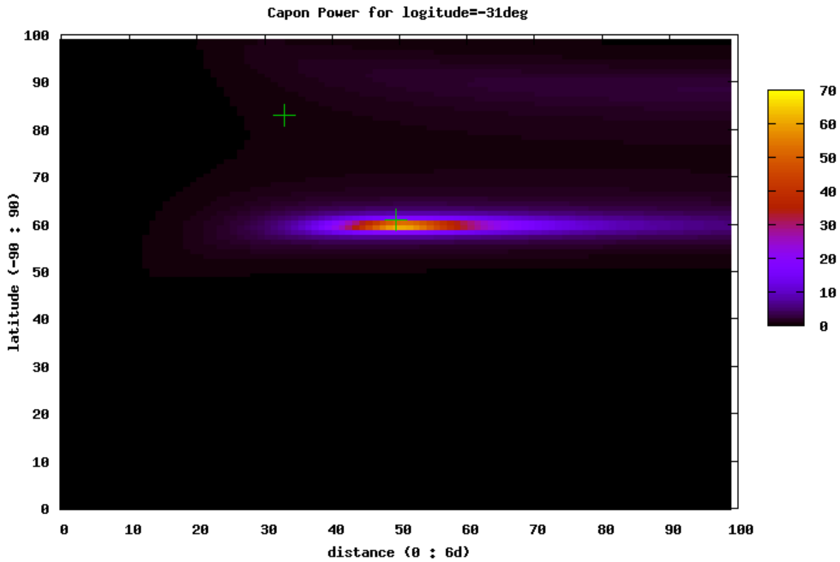


	longitude	latitude	distance
source	80^o	60^o	$2d$
result	78^o	57^o	$1.8d$

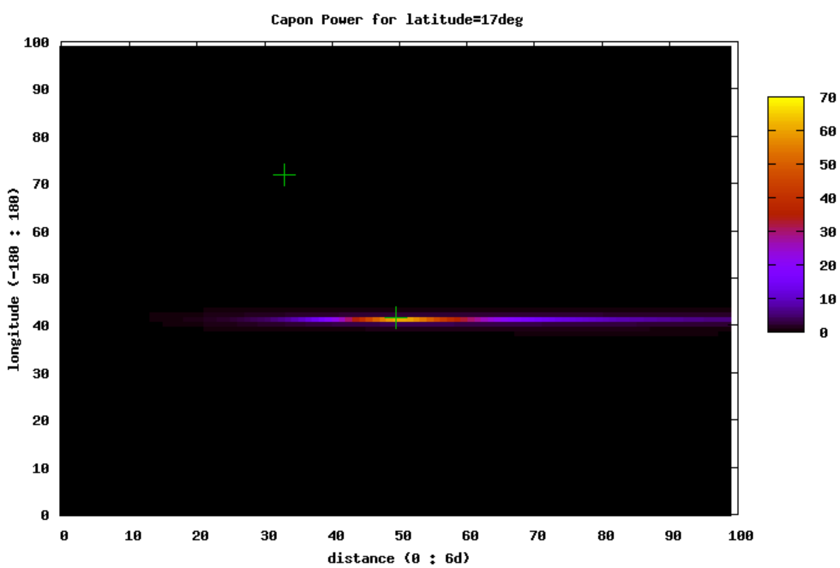


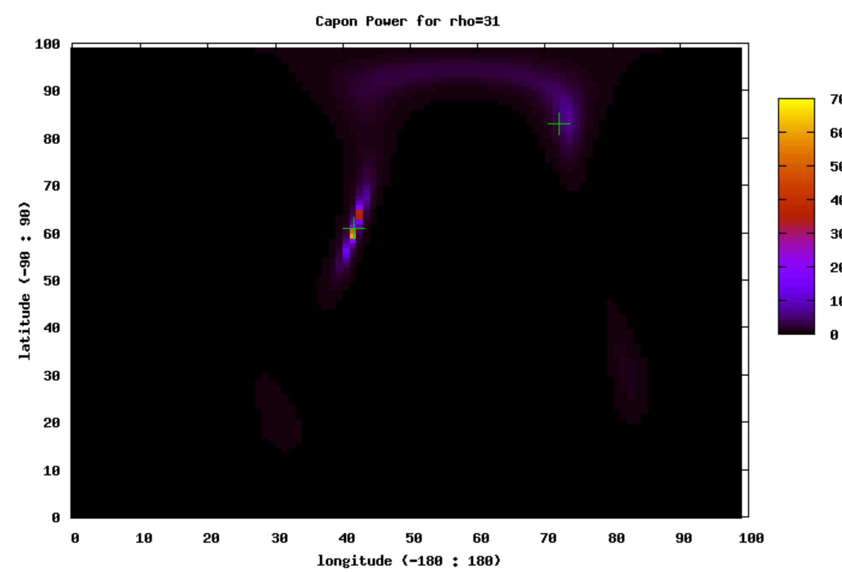


MV for Tetrahedron. Results for Source 2



	longitude	latitude	distance
source	-30^o	20^o	$3d$
result	-31^o	17^o	$3.1d$





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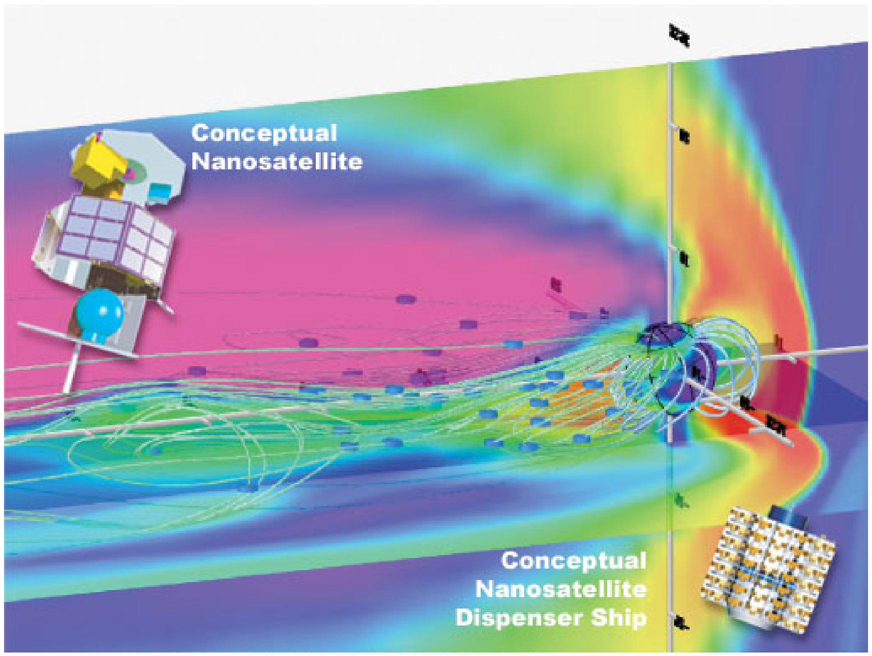
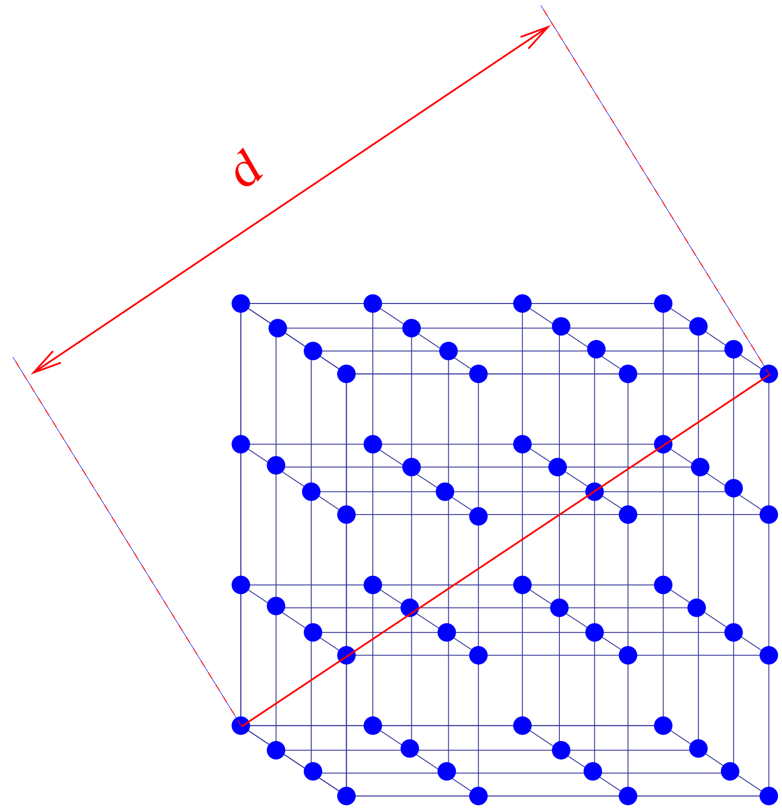
Minimum Variance for Cube Configuration

We can dream on...

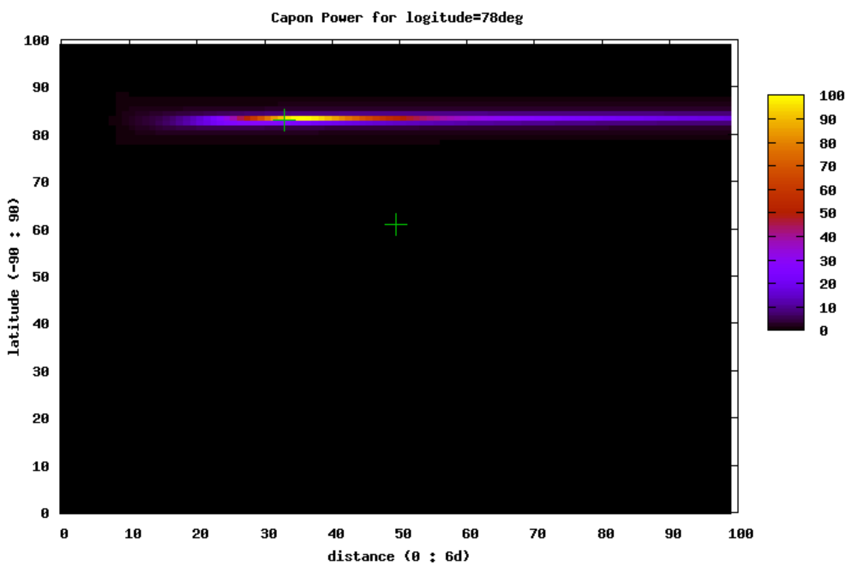


Magnetospheric Constellation

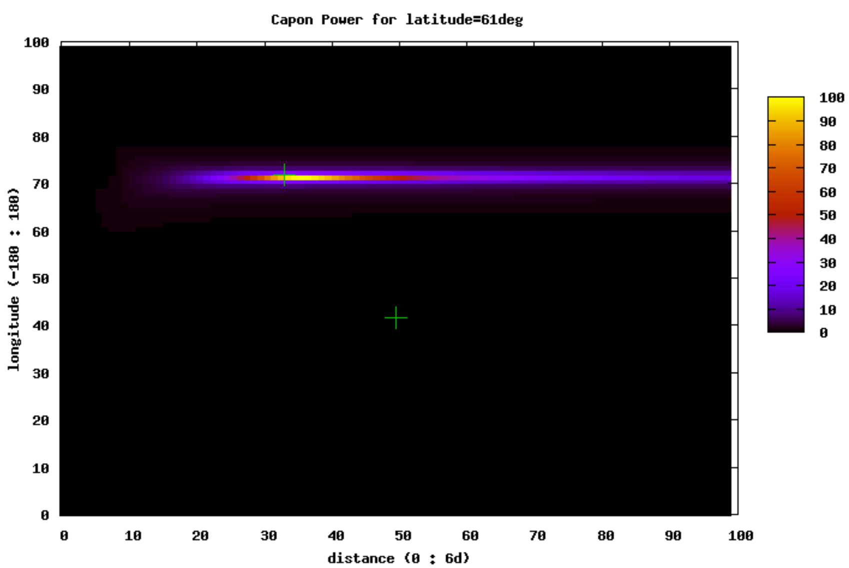


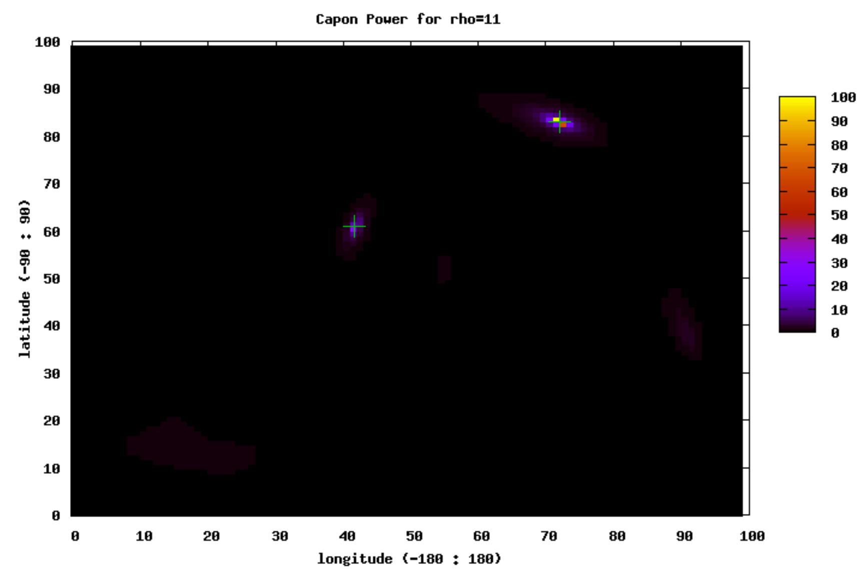
From http://stp.gsfc.nasa.gov/missions/mc/mc.htm

MV for Cube. Results for Source 1



	longitude	latitude	distance
source	80^o	60^o	$1.9d$
result	78^o	61^o	$2.1d$





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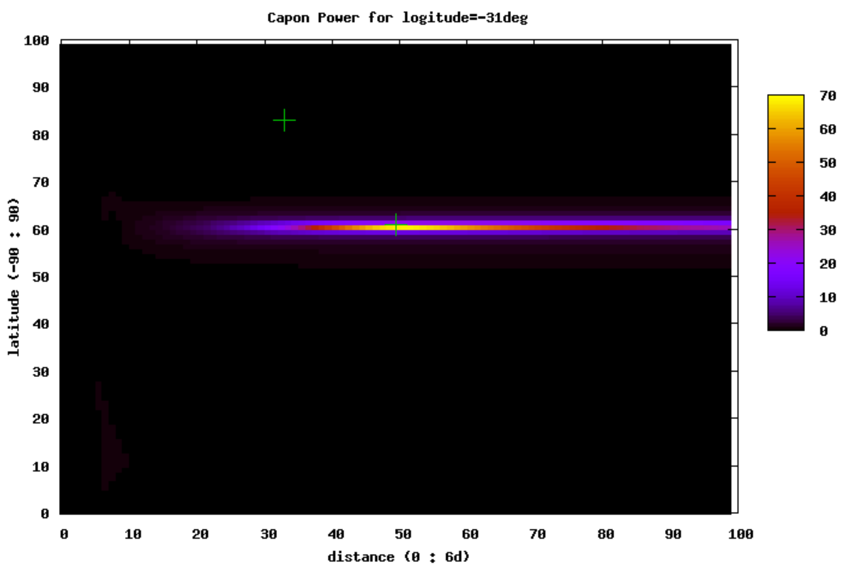
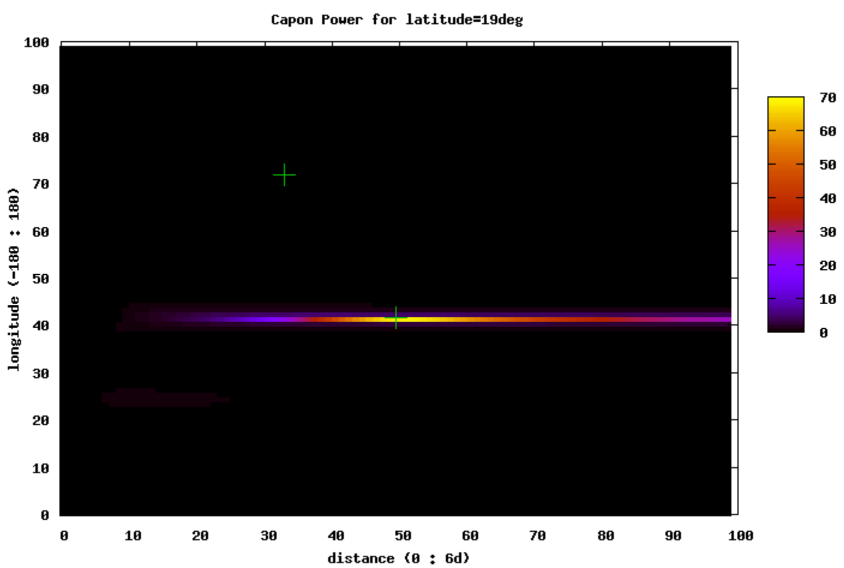
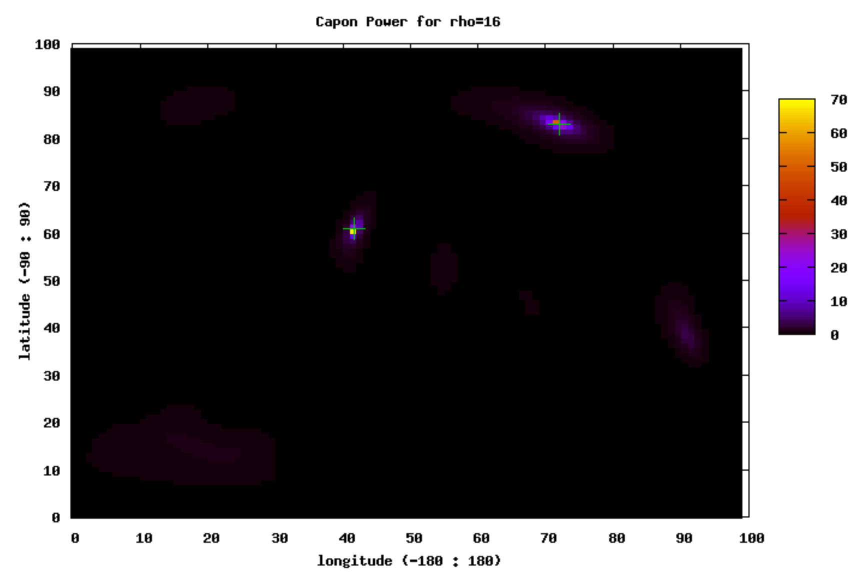
MV for Cube. Results for Source 2



	longitude	latitude	distance
source	-30^o	20^o	$2.8d$
result	-31^o	19^o	$3.1d$





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Conclusions and Further Work

Conclusions

- Virtual interference, in particular minimum variance for plane waves is a proved method for data analysis (Wave Telescope)
- Beamformer with spherical pattern cannot resolve the source distance in a reliable way for a tetrahedron configuration though it might work well for more sensors
- Minimum variance with spherical pattern can be used to determine source locations even for limited number of sensors (4)

Further work

- Find limitations of minimum variance with spherical pattern (distance, wave length)
- Apply the method to real data
- Find spatial arrangement of mirror modes using spherical pattern
- Use mirror mode pattern