Magnetic Mirror Modeling

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Preview

Short description of magnetic mirrors

- What are the MM?
- Where can MM be found?

Analytical model

- Assumptions and basic relations
- Conditions of existence and stability

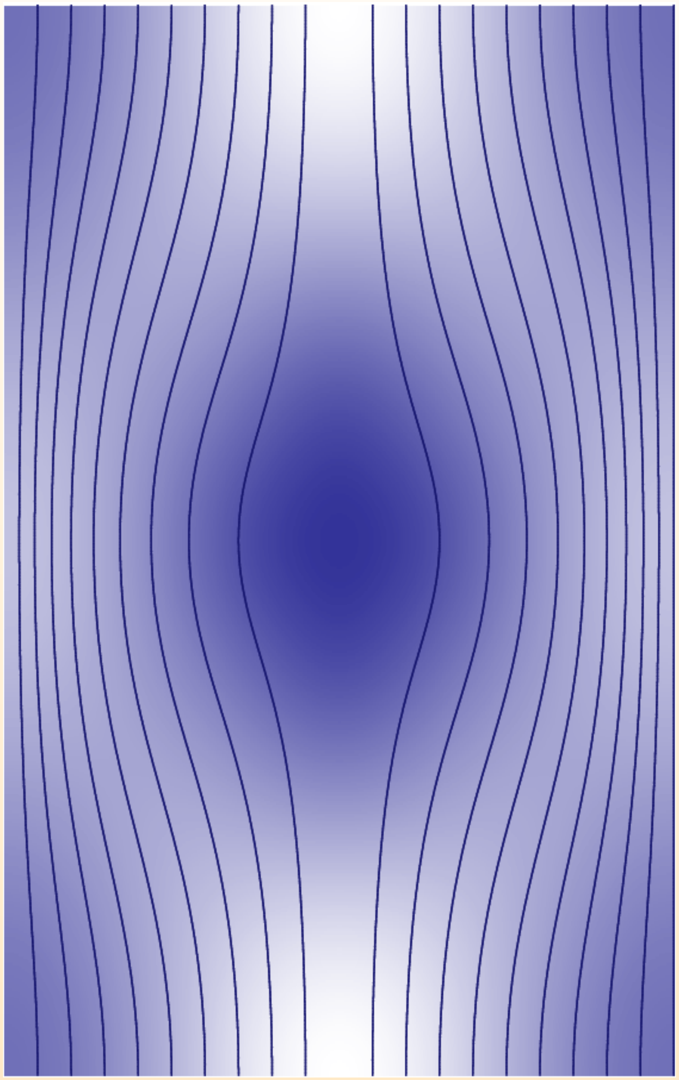
Looking for MM in Cluster data

- Procedure
- Results

Conclusions

Characteristics

- Fundamental plasma instability
- Needs temperature anisotropy ($T_\perp > T_\parallel$) in order to develop
- Non propagating (purely imaginary frequency), strongly compressive mode
- Magnetic field is anticorrelated with plasma density
- Very common in Earth magnetosheath but also in many other space plasmas



Assumptions

Small perturbations

- ▷ $\delta B << B$

Time-independent magnetic field

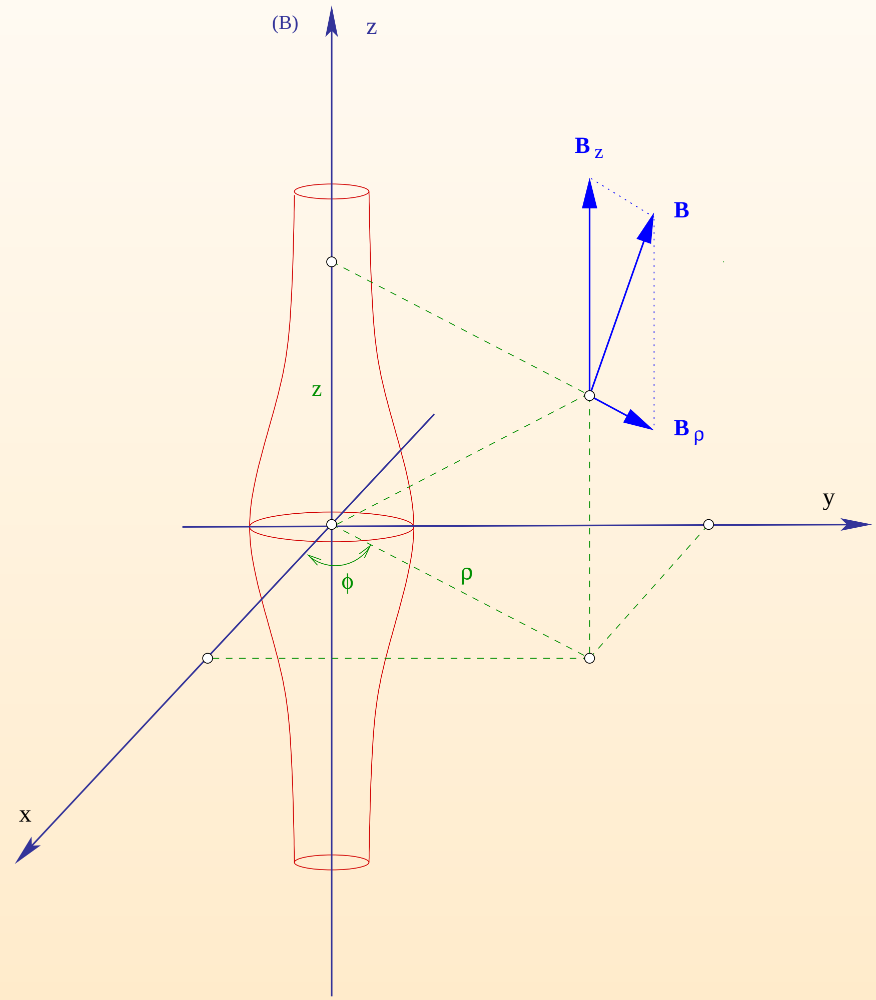
- ▷ $B \neq B(t)$

Symmetry around z-axis

- ▷ $B \neq B(\varphi)$

Periodicity along z-axis

- ▷ $B(\rho, z + 2L) = B(\rho, z)$



Basic relations

Pressure equilibrium:

- $$\nabla\left(p_\perp + \frac{B^2}{2\mu_0}\right) + \nabla\left[\left(p_\parallel - p_\perp - \frac{B^2}{\mu_0}\right)\frac{\underline{\underline{BB}}}{B^2}\right] = 0$$

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Anisotropy for bi-Maxwellian distribution:

- $$A(\rho,z) = \frac{T_\perp(\rho,z)}{T_\parallel(\rho,z)} = \frac{p_\perp(\rho,z)}{p_\parallel(\rho,z)} = \left[1 - \left(1 - \frac{1}{A_0}\right)\frac{B_0}{B(\rho,z)}\right]^{-1}$$

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Magnetic field:

- $B(\rho, z) = B_0 e_z + \delta B(\rho, z)$
- $B_\varphi = 0$

Deriving the magnetic field

Fourier series:

- $$\delta B_\rho(\rho, z) = \sum_{n=-\infty}^{\infty} \delta B_\rho^n(\rho) e^{-in\frac{\pi z}{L}}$$
- $$\delta B_z(\rho, z) = \sum_{n=-\infty}^{\infty} \delta B_z^n(\rho) e^{-in\frac{\pi z}{L}}$$
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Bessel equations:

- $$\rho^2 \frac{d^2}{d\rho^2} \delta B_\rho^n + \rho \frac{d}{d\rho} \delta B_\rho^n + \left[\left(\frac{n\alpha\rho}{L} \right)^2 - 1 \right] \delta B_\rho^n = 0$$

Deriving the magnetic field

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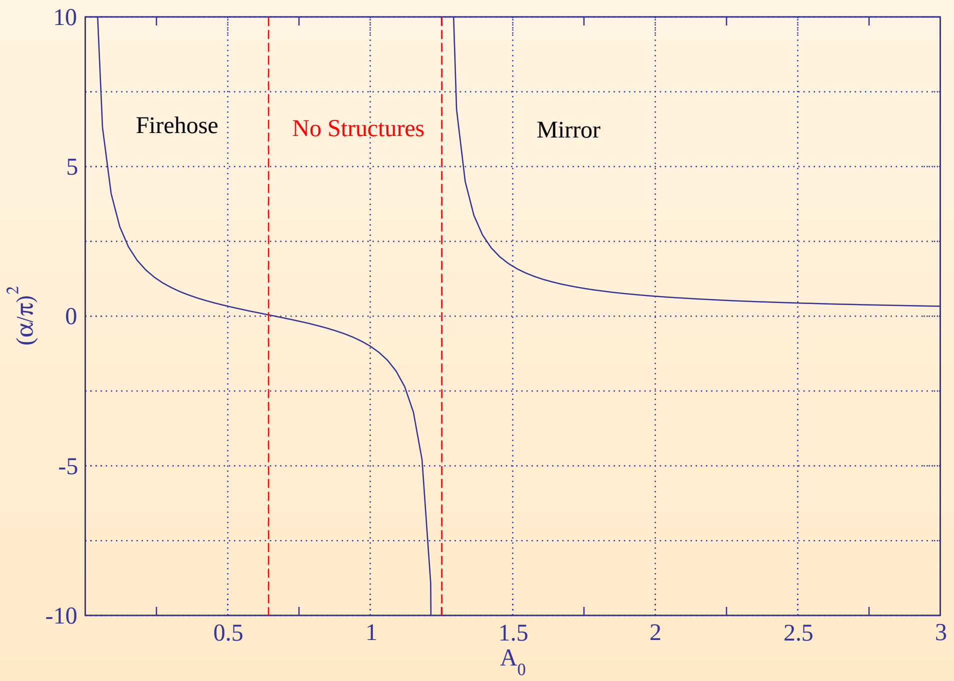
- α parameter:
  - ▷ $$\alpha = \pi \sqrt{\frac{\frac{1}{2}\left(1 - \frac{1}{A_0}\right) + \frac{1}{\beta_{0\perp}}}{A_0 - 1 - \frac{1}{\beta_{0\perp}}}}$$

Physical solution of Bessel equation

$B \neq \infty$ condition:

- $\alpha^2 > 0$
  - $A_0 > 1 + \dfrac{1}{\beta_{0\perp}}$ $MIRROR$
  - $A_0 < \dfrac{\beta_{0\perp}}{\beta_{0\perp} + 2}$ $FIREHOSE$



Solution

Magnetic field perturbation:

- $$\delta B_\rho(\rho, z) = \frac{2\pi}{\alpha} \sum_{n=1}^{\infty} J_1\left(\frac{n\alpha\rho}{L}\right)\left[a_n \sin\left(\frac{n\pi z}{L}\right) - b_n \cos\left(\frac{n\pi z}{L}\right)\right]$$
- $$\delta B_z(\rho, z) = 2 \sum_{n=1}^{\infty} J_0\left(\frac{n\alpha\rho}{L}\right)\left[a_n \cos\left(\frac{n\pi z}{L}\right) + b_n \sin\left(\frac{n\pi z}{L}\right)\right]$$

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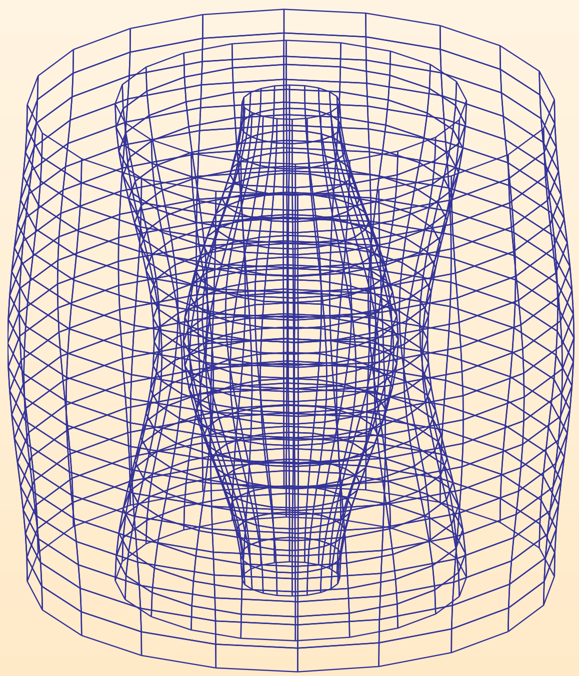

The current density

$$j_B(\rho, z) = \frac{2\pi^2}{\mu_0 \alpha L}\left[1 + \left(\frac{\alpha}{\pi}\right)^2\right] \sum_{n=1}^{\infty} n J_1\left(\frac{n\alpha\rho}{L}\right)\left[a_n \cos\left(\frac{n\pi z}{L}\right) + b_n \sin\left(\frac{n\pi z}{L}\right)\right] e_\varphi$$

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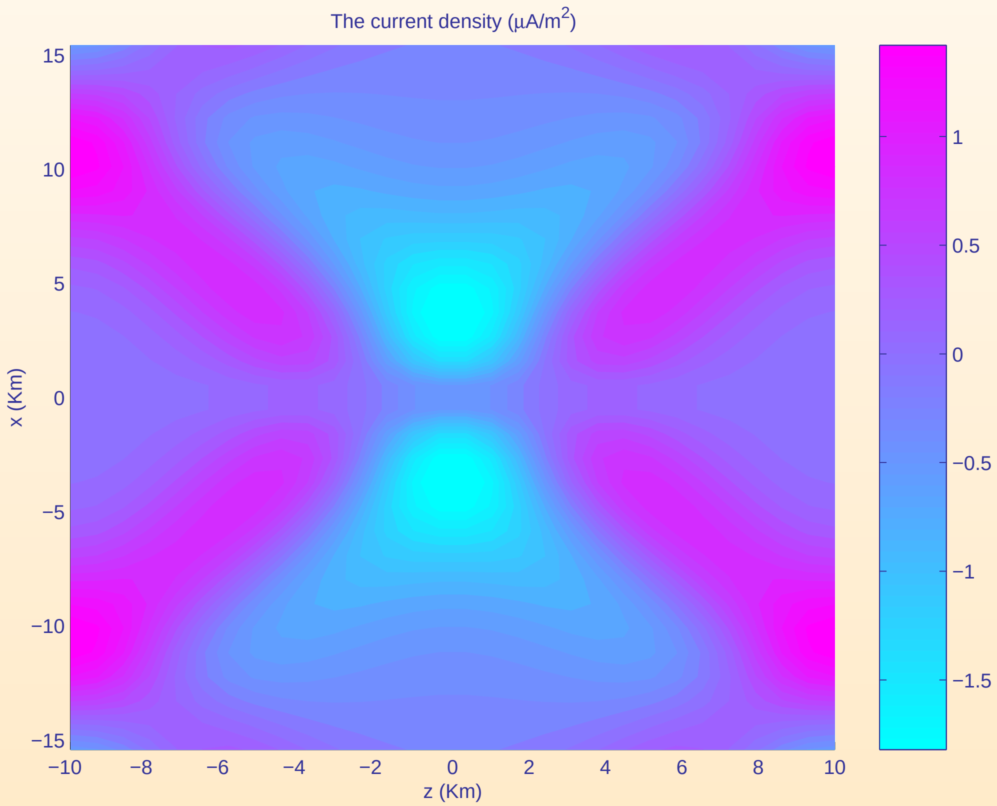

Particles trajectories

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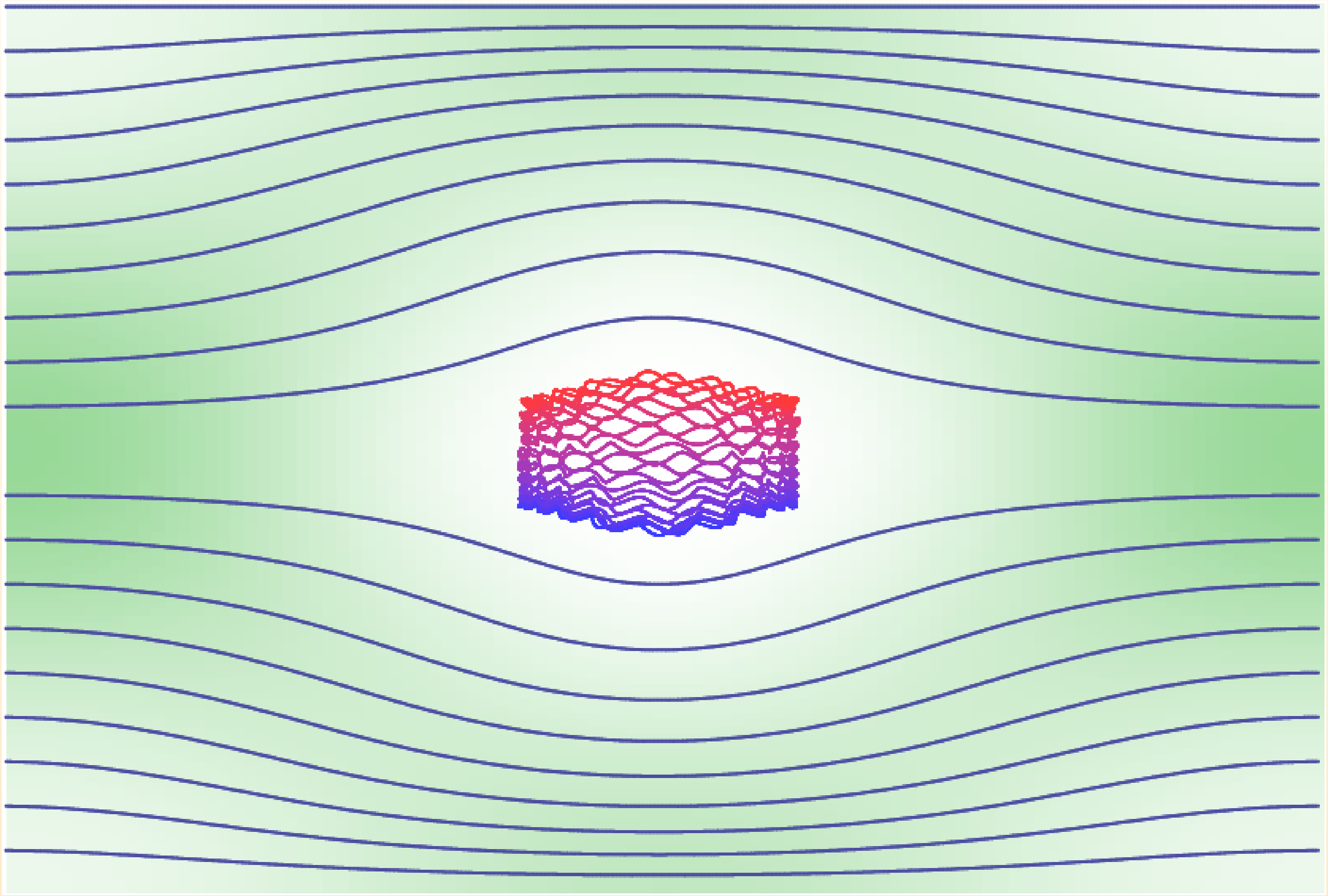

Stability

The current derived from gradient-curvature drift:

- $$j_d = \frac{1}{4}\frac{3\beta_{0\perp} + 2}{2A_0 + 1} j_B$$

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  ▷ *Equilibrium:*

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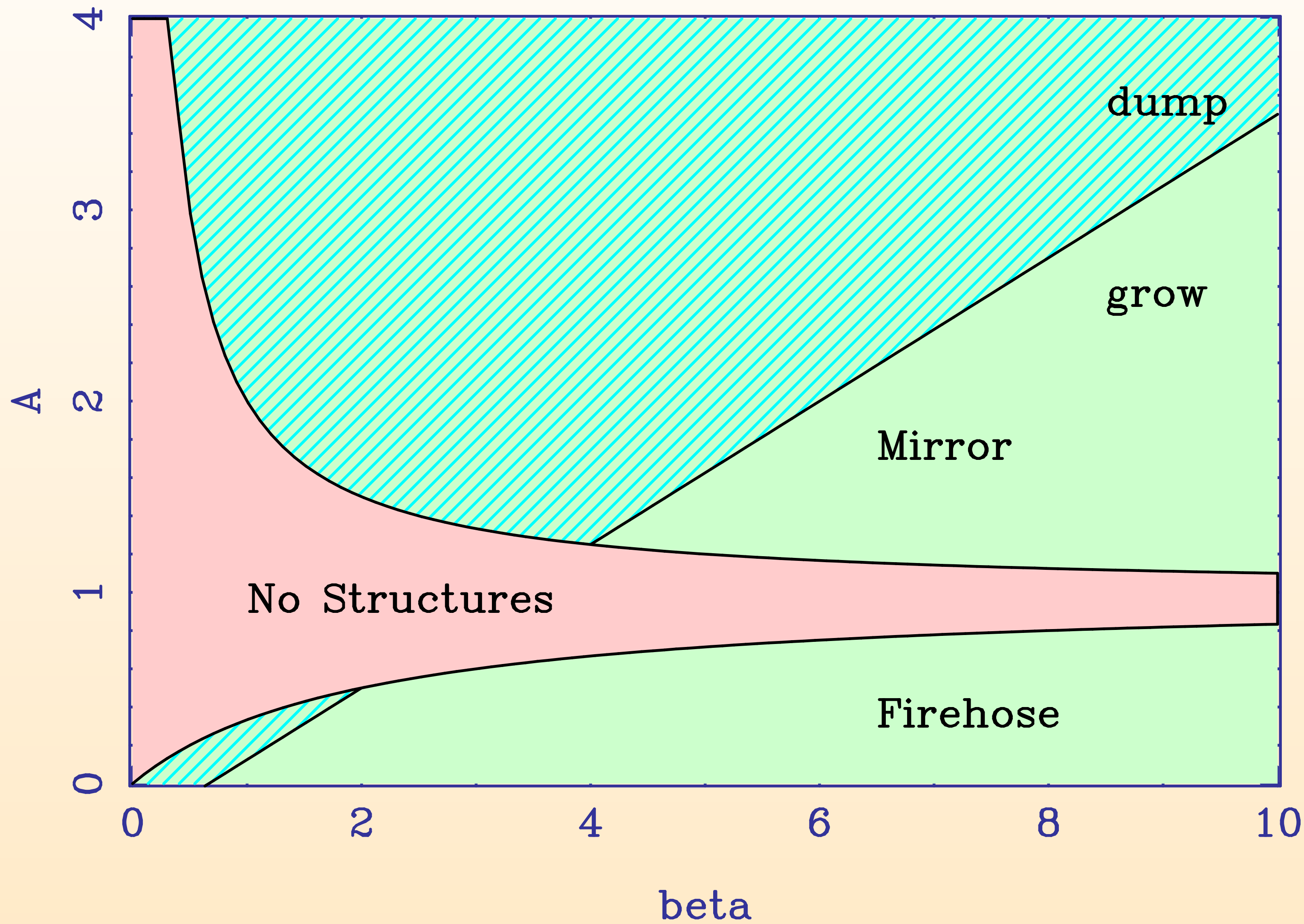
  ▷ *Instability condition:*

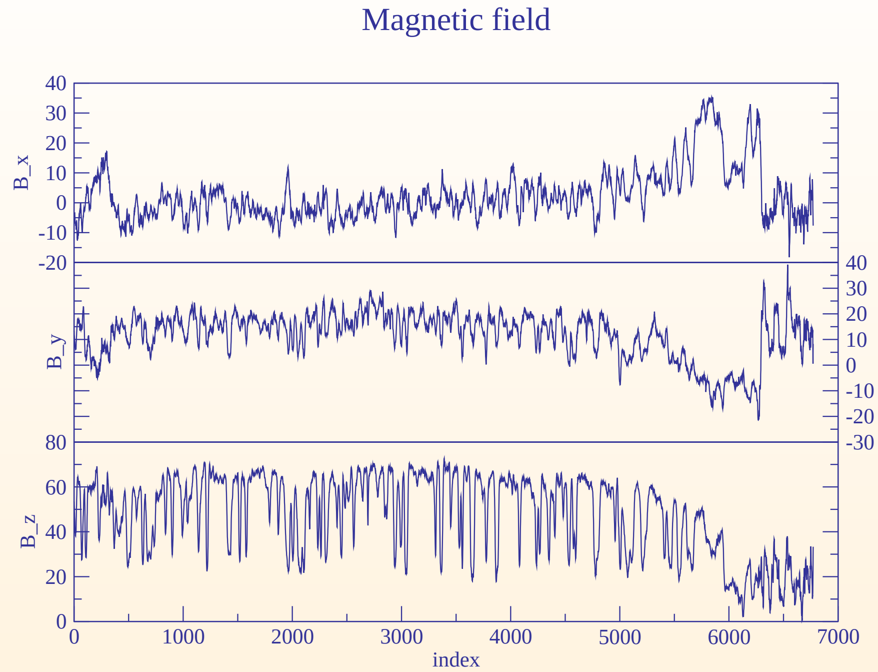
  $$j_d > j_B \qquad \Rightarrow \qquad A_0 < \frac{1}{4}\left(\frac{3}{2}\beta_{0\perp} - 1\right)$$

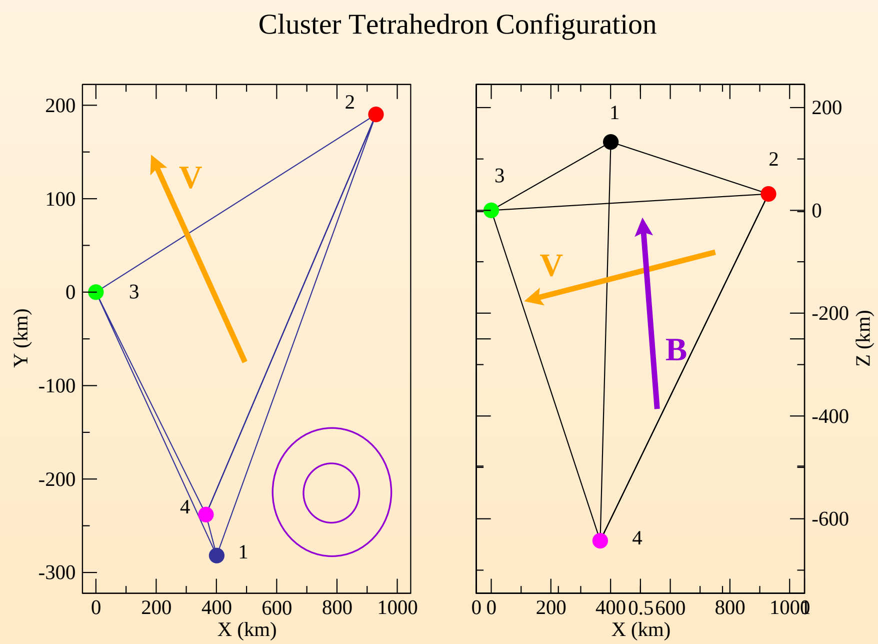
Stability domains in (A_0,β_0)-plane



Data

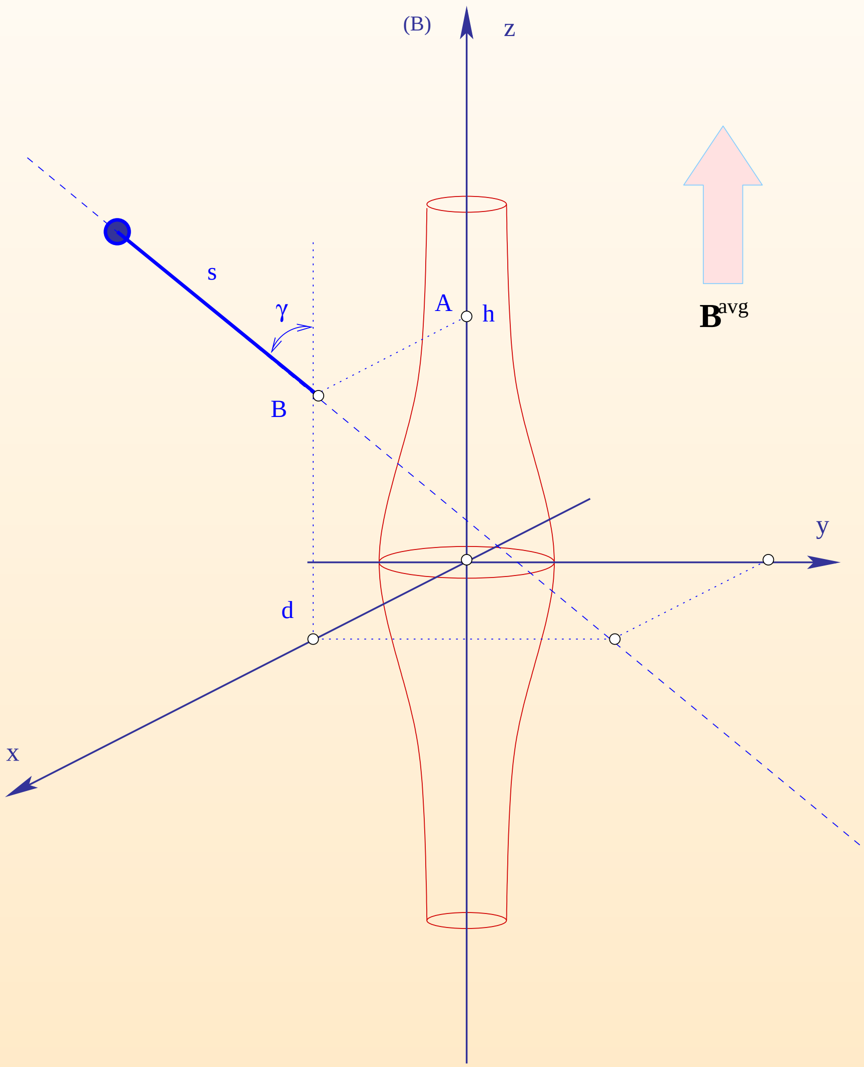
- Date: Nov. 10 2000,
  08:20:00 - 80:25:00 UT
- Data resolution: High (22 vec/sec)
- Location: Dusk side magnetosheath
- Plasma flow: 815 km/s , C1 -> C3
- Magnetic field almost:
  - *aligned with Z_{GSE} axis*
  - *orthogonal to plasma flow*



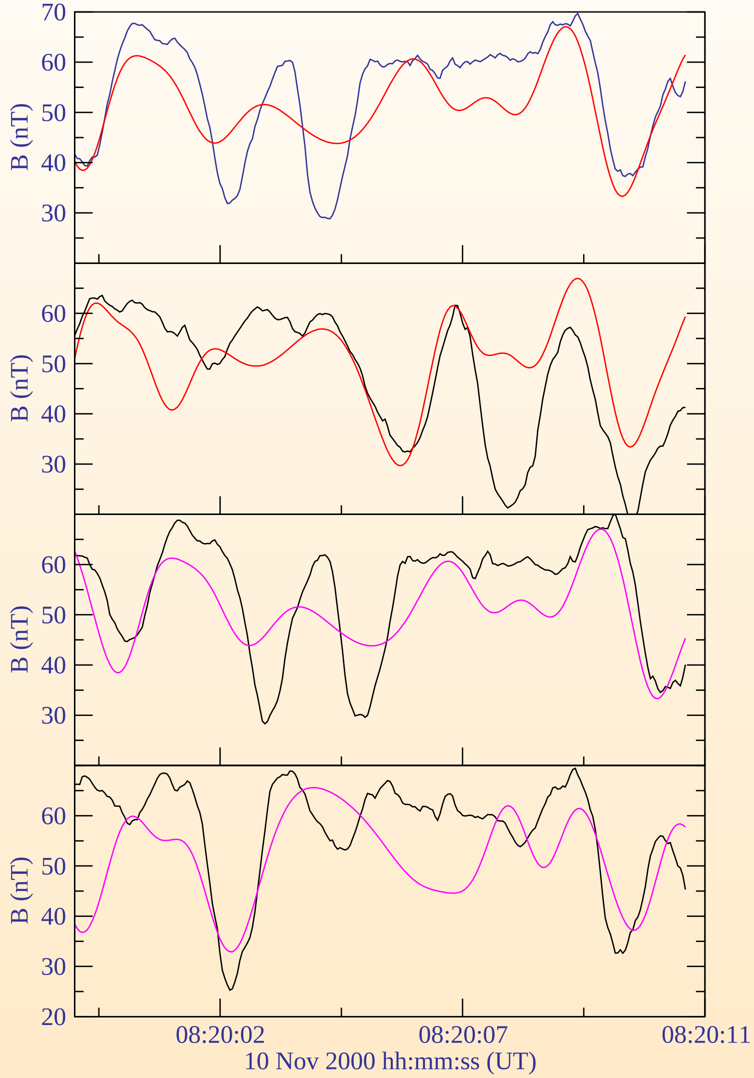


Fit parameters

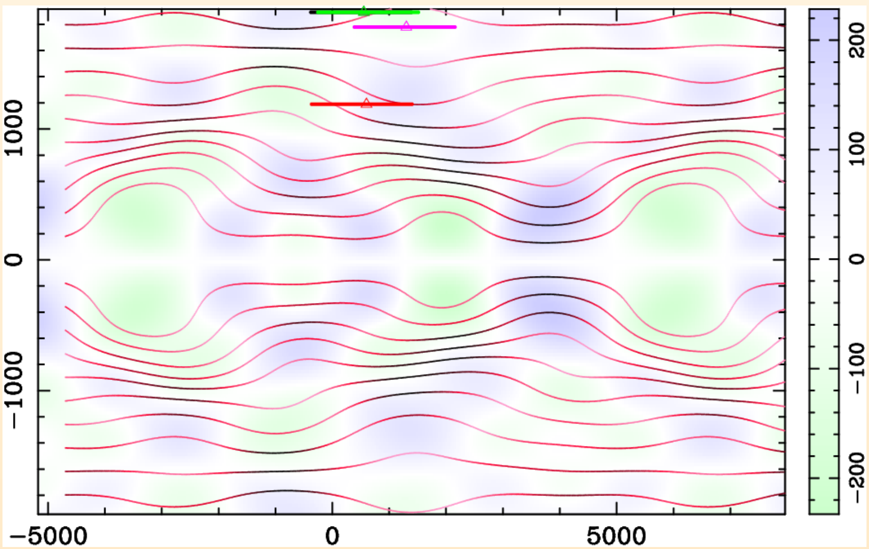
- (1-3) trajectory coordinates (h, d, γ)
- (4) initial position of the spacecraft on its path (s_0)
- (5) the length of the MM (L)
- (6) the unperturbed magnetic field intensity (B_0)
- (7) the α plasma parameter
- (8-n) the Fourier coefficients a_j and b_j



Example: Two spacecraft fit



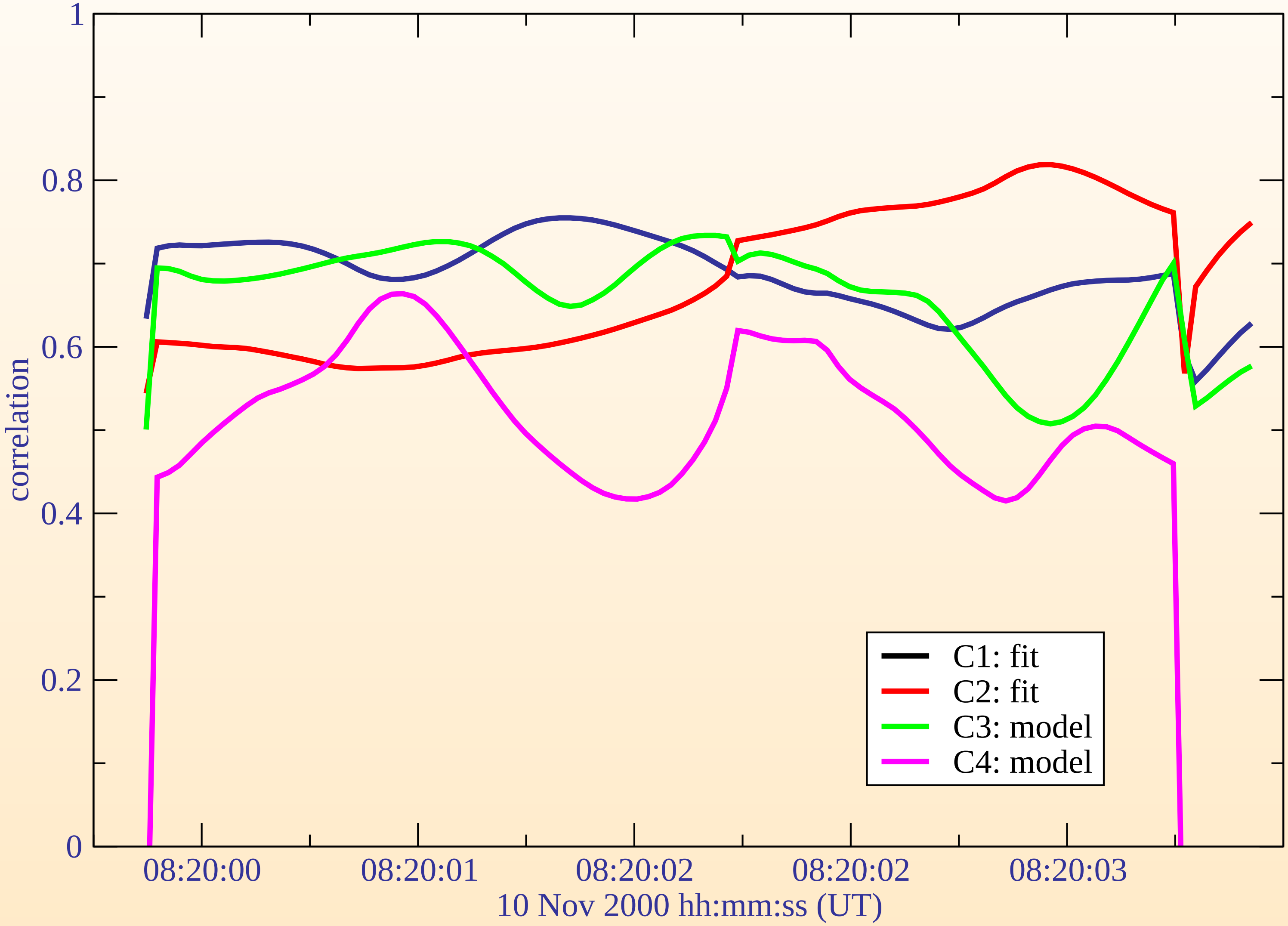
- Parameters:
  - ▷ $h = 552$ *km*
  - ▷ $d = 1892$ *km*
  - ▷ $\gamma = 75^o$
  - ▷ $L = 4690$ *km*
  - ▷ $B = 51.74$ *nT*
  - ▷ $\alpha = 10$
  - ▷ $\Rightarrow R = 1797$ *km*



Scan procedure

- Simultaneous fit using $n \leq 4$ spacecraft for data interval $[i, i + k]$
- Compute model field for the other spacecraft using parameters found from fit
- Compare model field with data
- Decide starting guess parameters for the next interval
- Return to the first step for interval $[i + 1, i + k + 1]$

Two spacecraft fit: Correlations



Two spacecraft fit: Parameters

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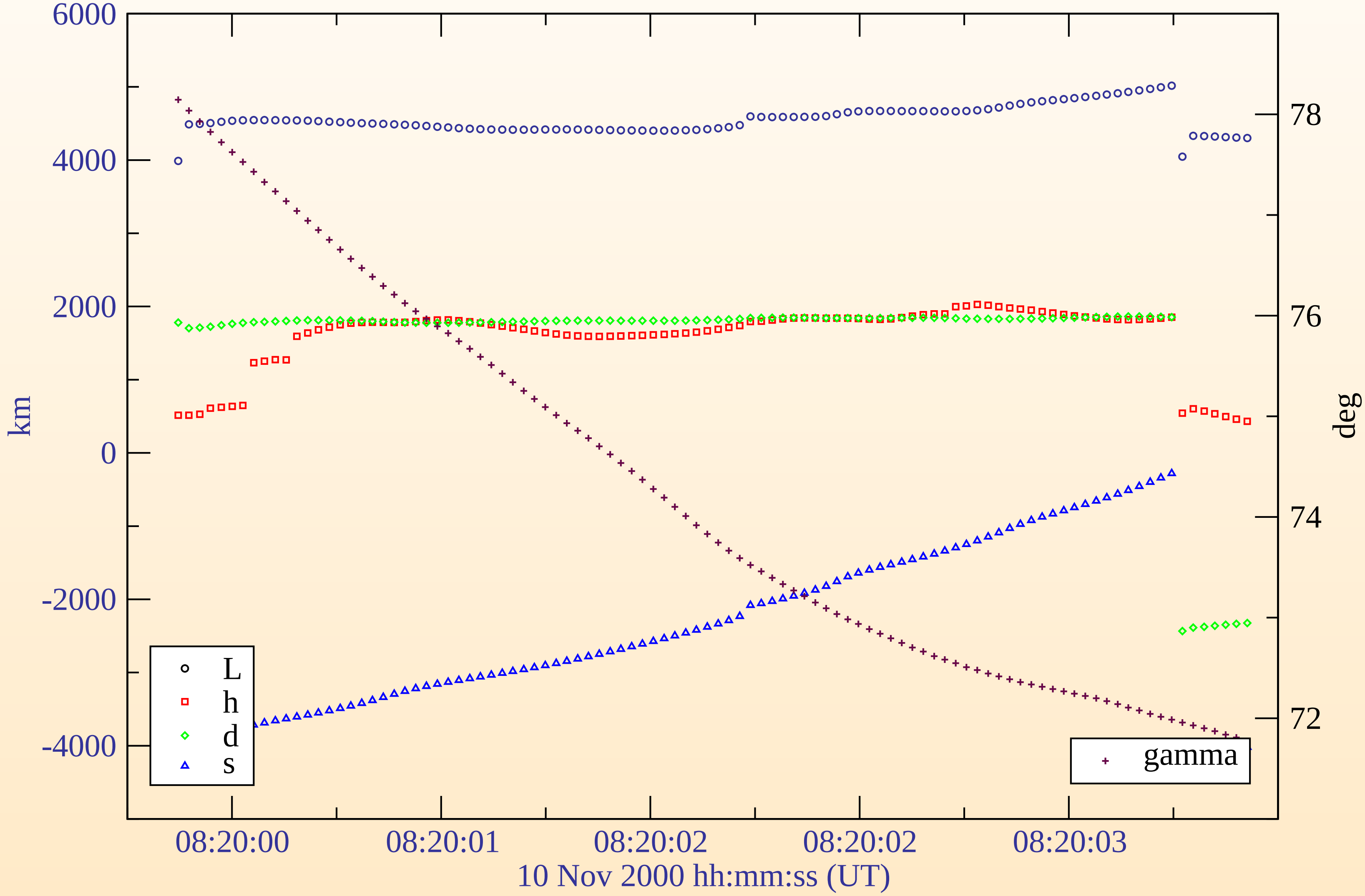
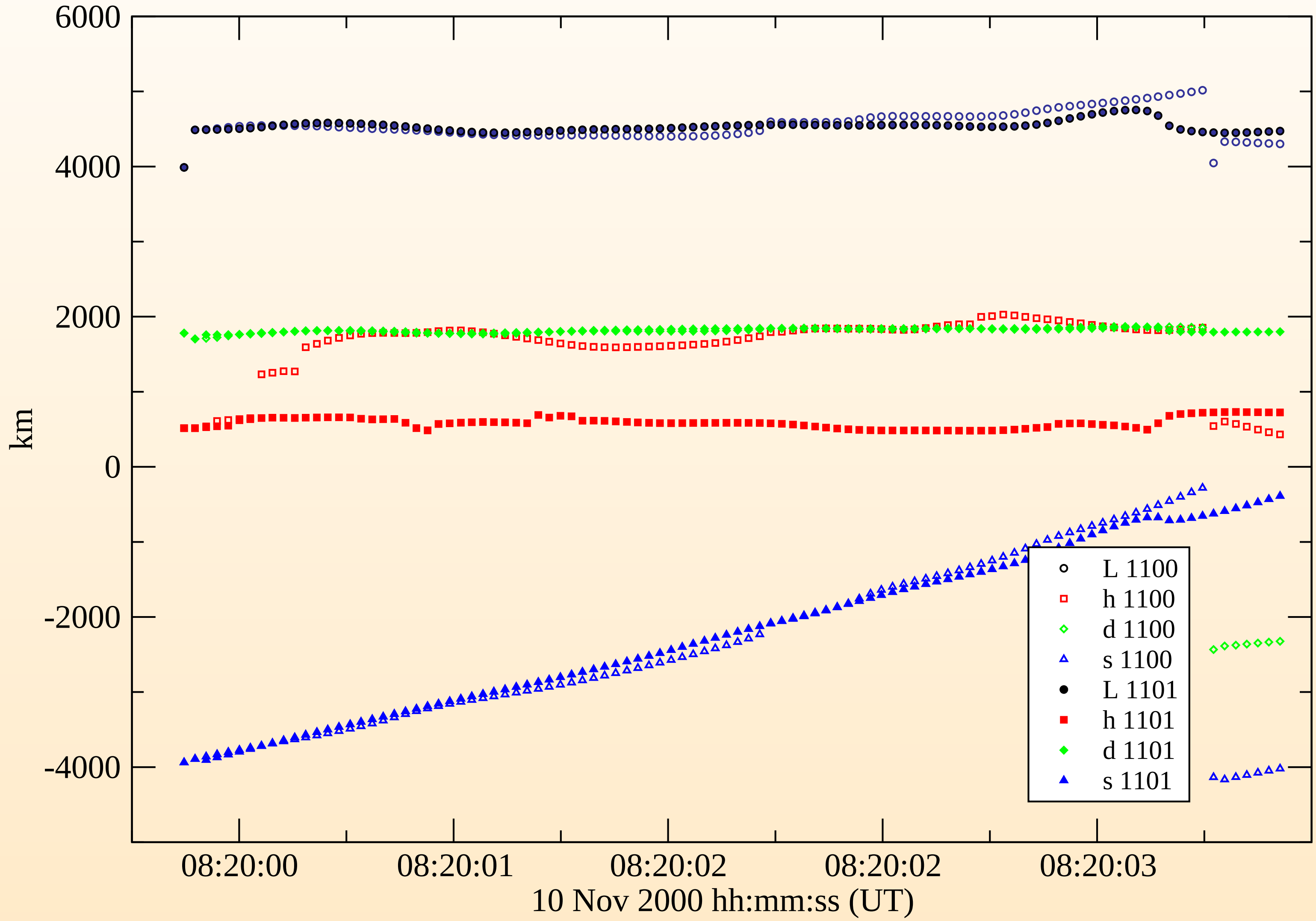


Comparison between 2 and 3 spacecraft fit



Conclusions

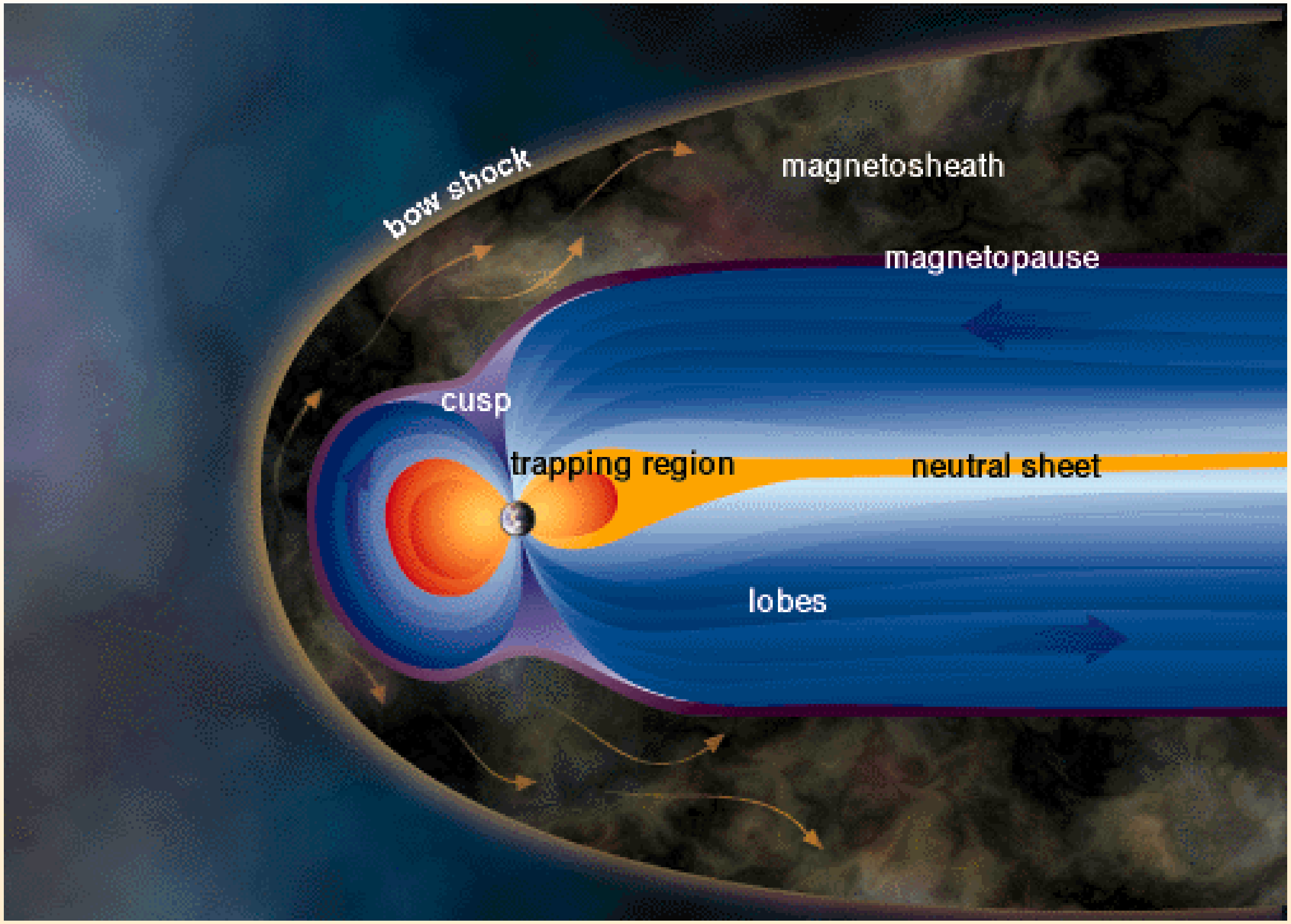
- We have developed an analytical model which describes the properties and geometry of MM

- We have demonstrated the possibility of deriving the shape and parameters of the MM using multi spacecraft data

- Further investigation is necessary

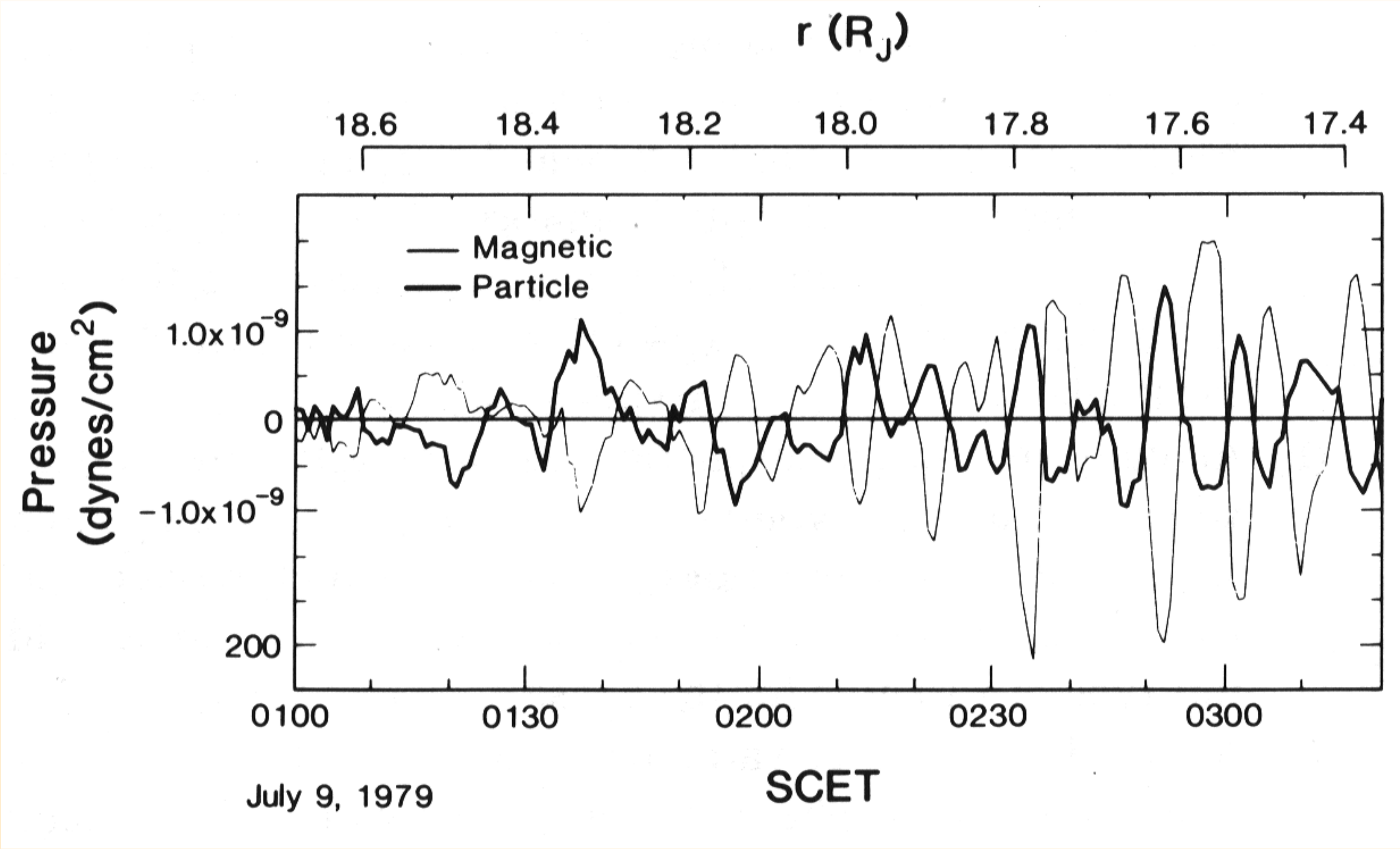
  ▷ *include particle data*
  ▷ *perform particle simulations for model magnetic field configurations*
  ▷ *look at the distribution function*
  ▷ *improve or develop a nonlinear model*
  ▷ *...*

A1. The Earth magnetosphere



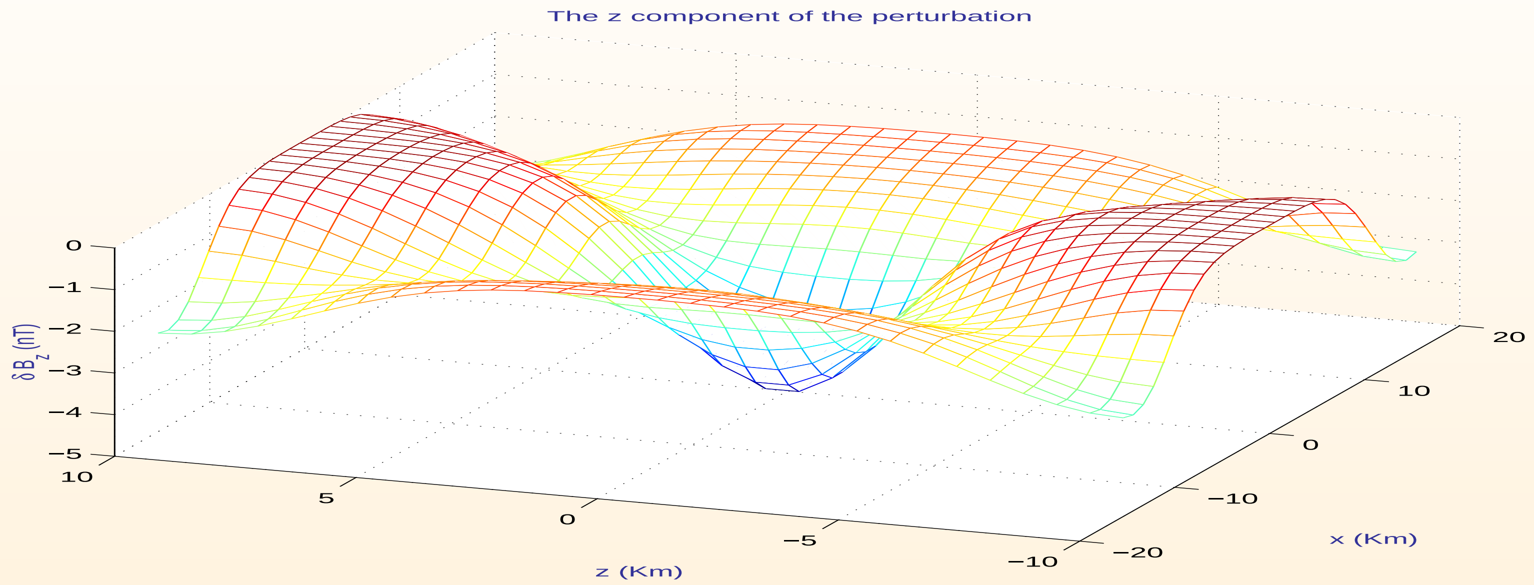
Courtesy of Windows to the Universe, http://www.windows.ucar.edu

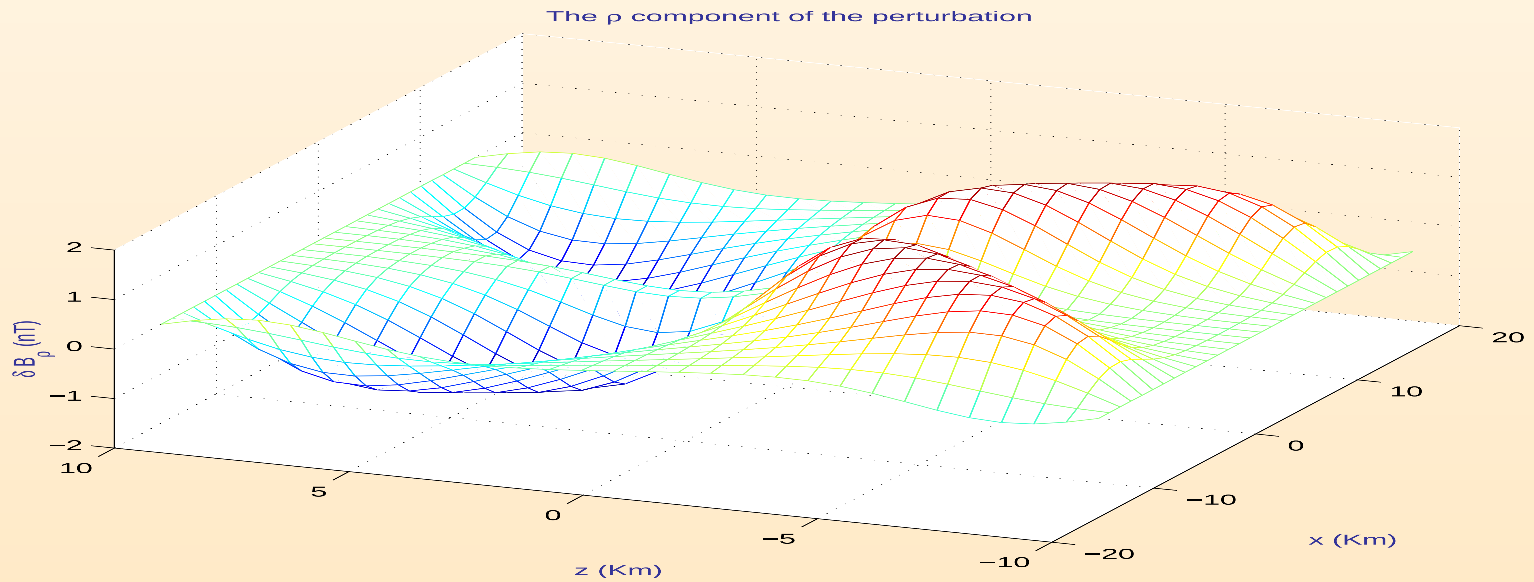
A2. Magnetic mirrors in spacecraft data



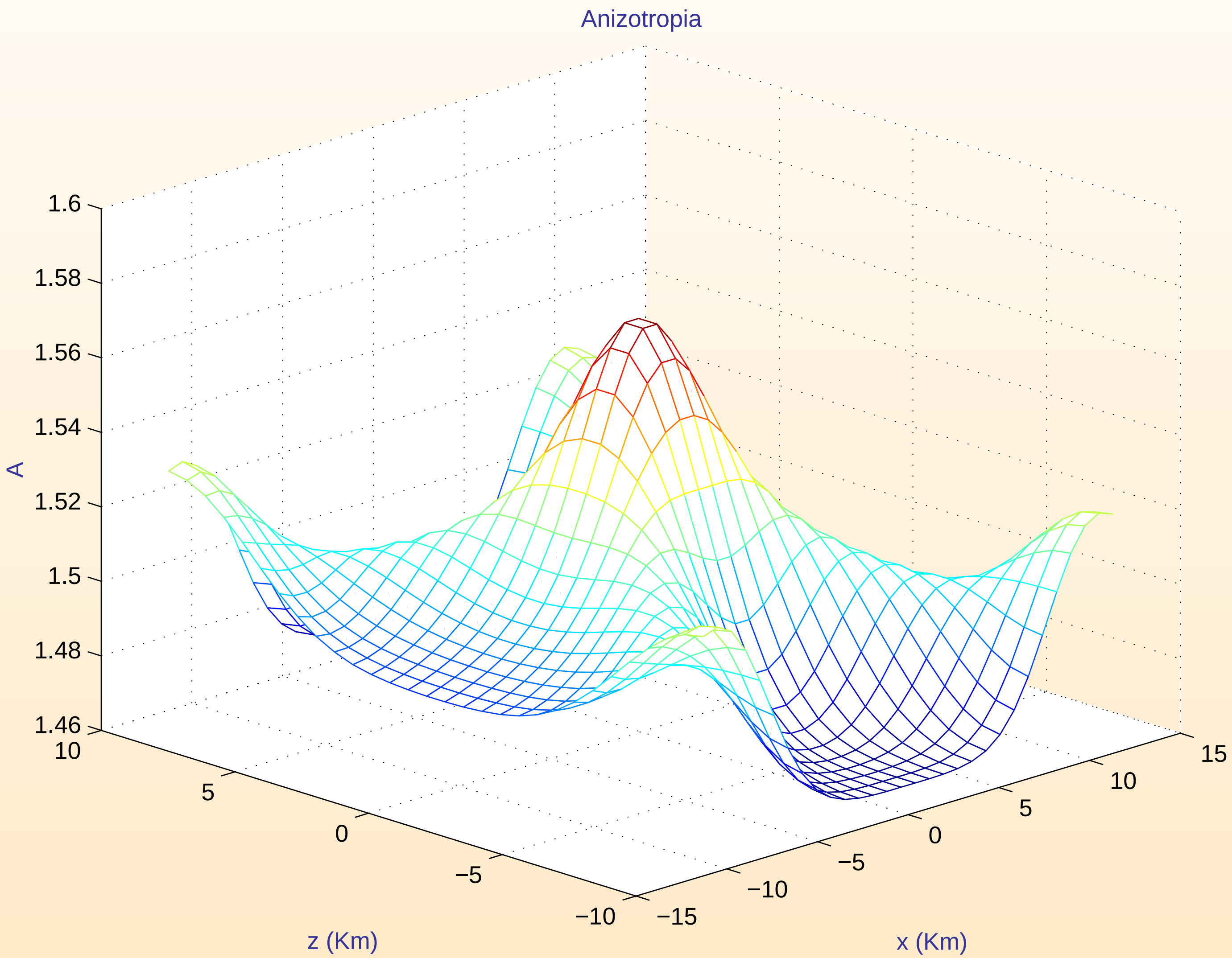
Khurana, K. K and M.G Kivelson 1989, Ultralow frequency MHD waves in Jupiter's middle magnetosphere, J. Geophys. Res. 94:5241

A3. The ρ and z perturbations

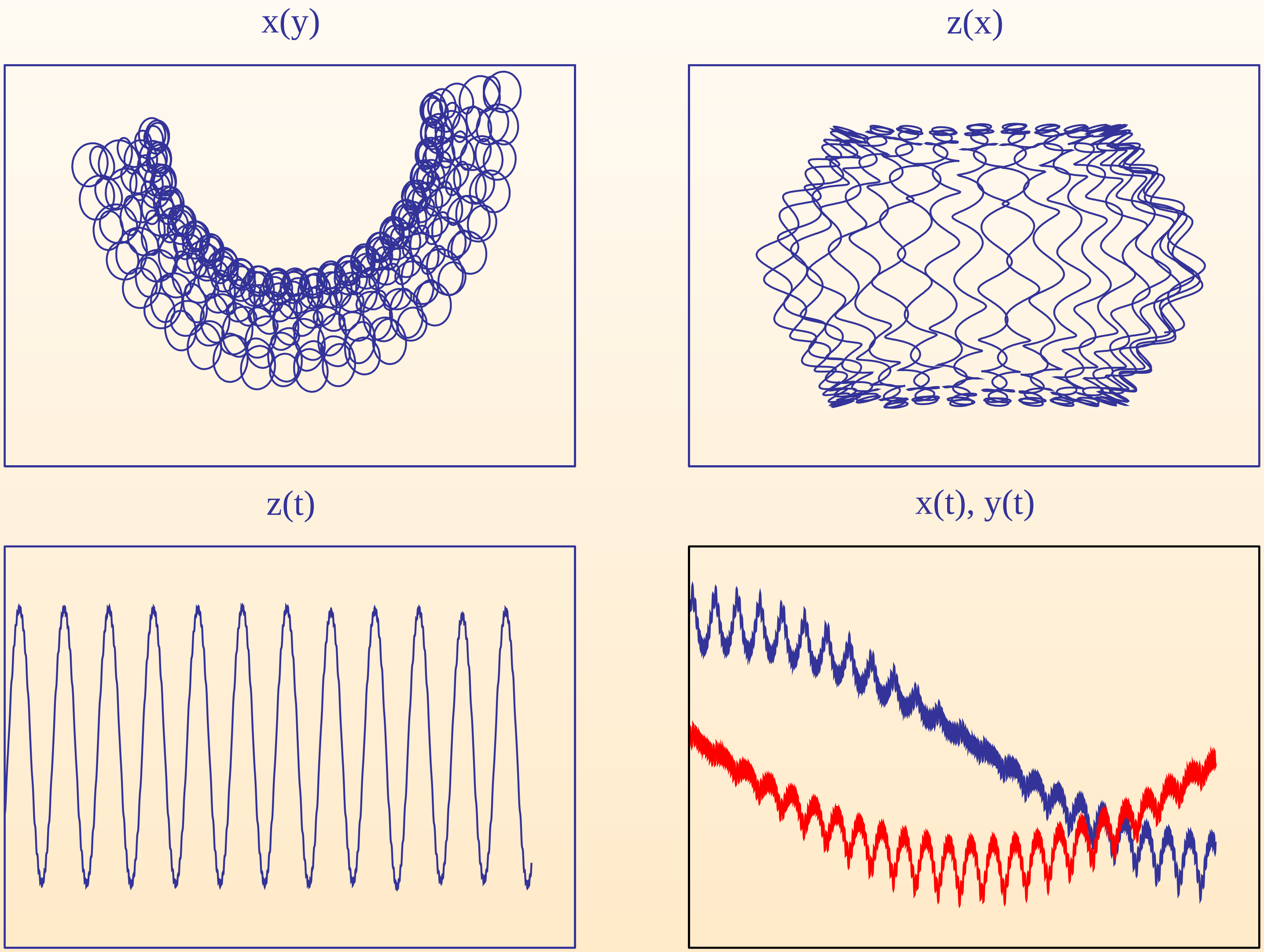




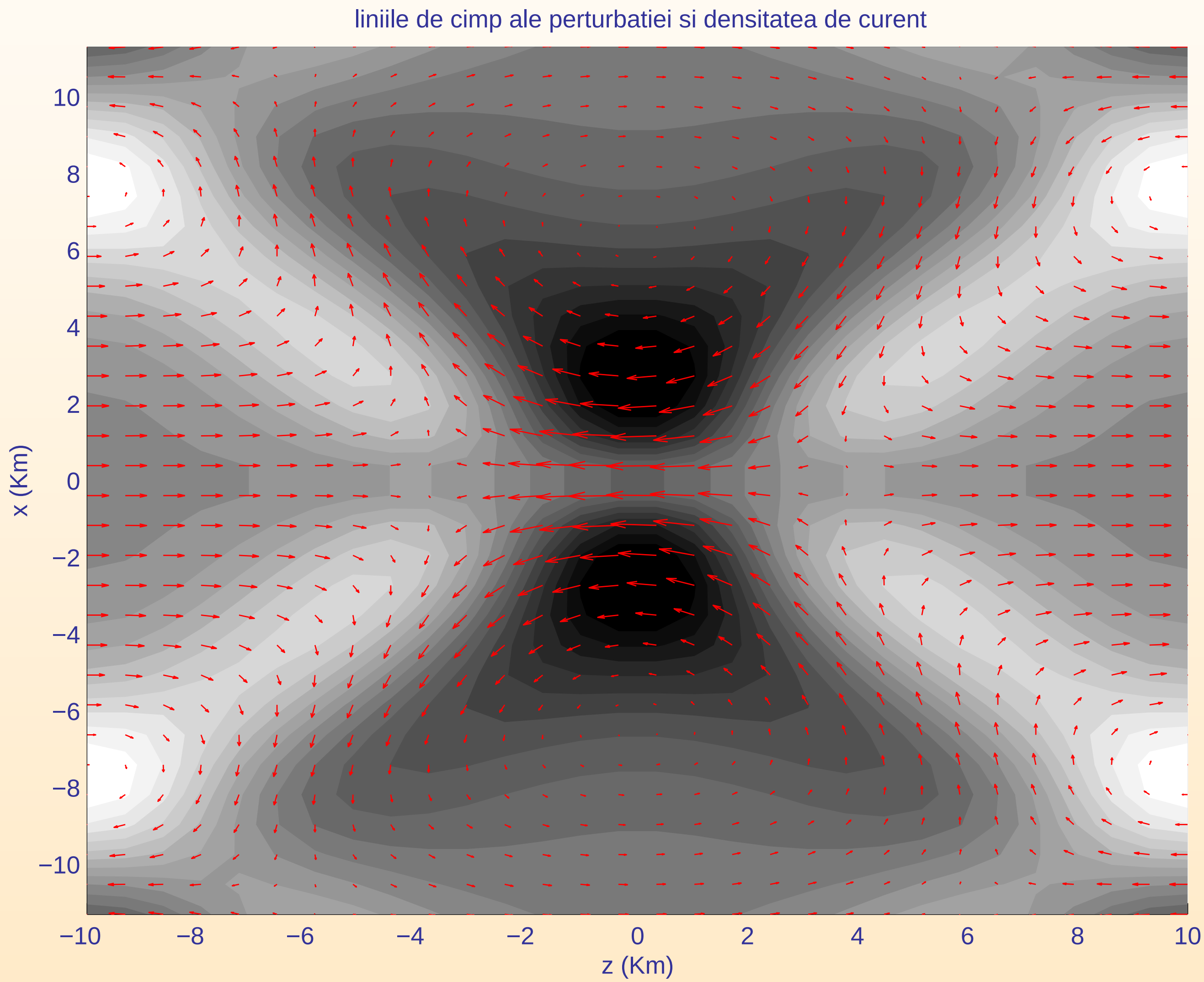
A4. The anisotropy



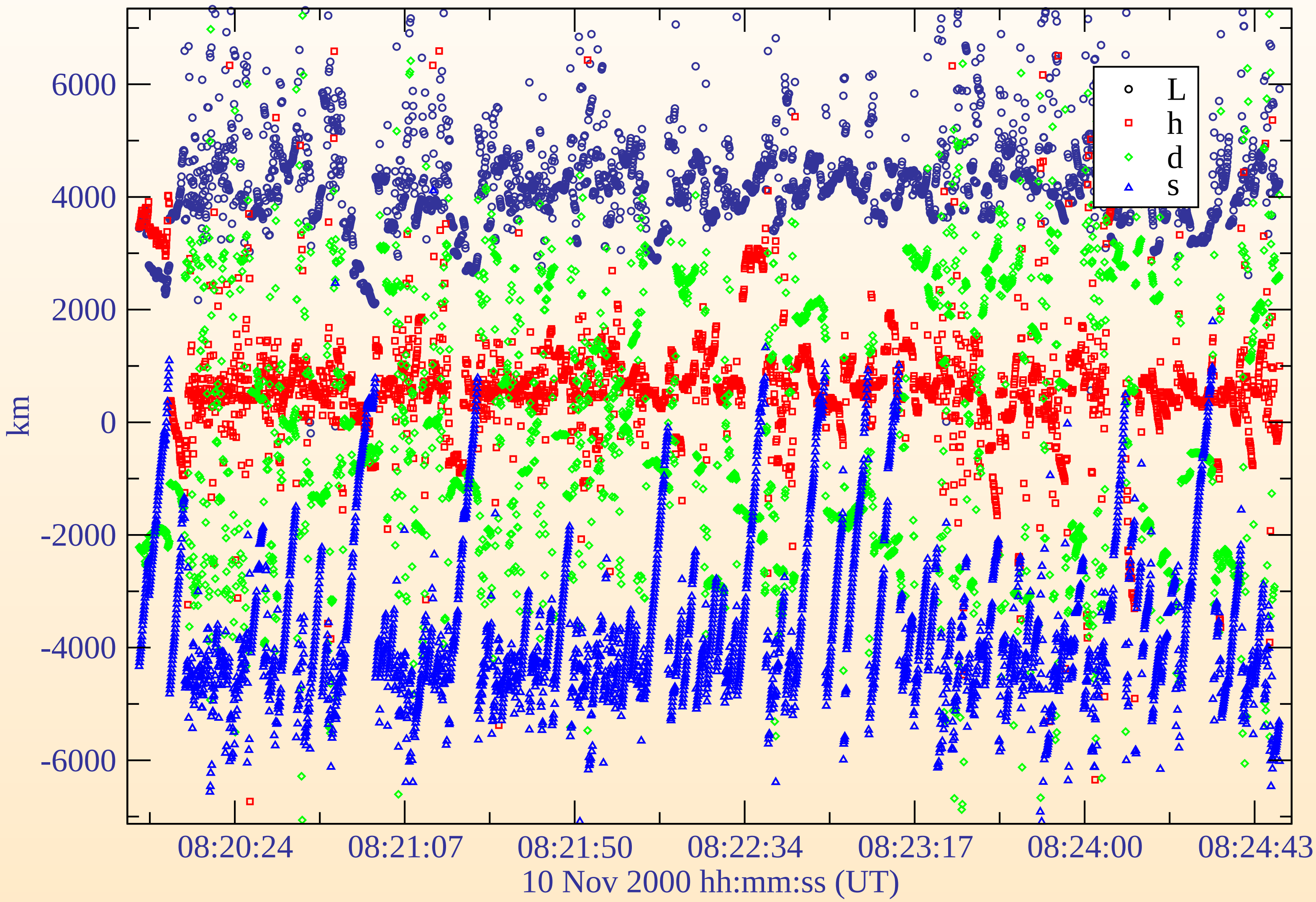
A5. Particle simulation



A6. Current density and magnetic field perturbation



A7. Full scan example



A8. Model examples parameters:

- Perturbation on the axis:
  - ▷ $\delta B_z(0,z) = \delta B_z(0,0)\exp(-z^2/a^2), \qquad a = L/3$
- Anisotropy:
  - ▷ $A_0 = 1.4$
- Unperturbed magnetic field:
  - ▷ $B_0 = 20nT$
- Perturbation in the middle:
  - ▷ $\delta B_z(0,0) = -5nT$
- Unperturbed number density:
  - ▷ $n_0 = 50cm^{-3}$
- Length of the bottle:
  - ▷ $2L = 20Km$
- Unperturbed orthogonal temperature:
  - ▷ $T_{0\perp} = 10^6 K$
- Plasma parameter:
  - ▷ $\beta_{0\perp} = 4.33$