Collisionless transport equations in the solar wind with Kappa distribution

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Exospheric model of solar wind

The solar wind is the continuous outflow of completely-ionised gas from the outermost region of the solar atmosphere - solar corona. It consists of protons and electrons, with an admixture of a few percent of heavier ions. Number density is estimated at a few particles per cm^3. The temperatures exceed one million degrees Kelvin. The hot coronal plasma never reaches equilibrium, is continually being accelerated up to supersonic velocities and flows outward into interplanetary space.

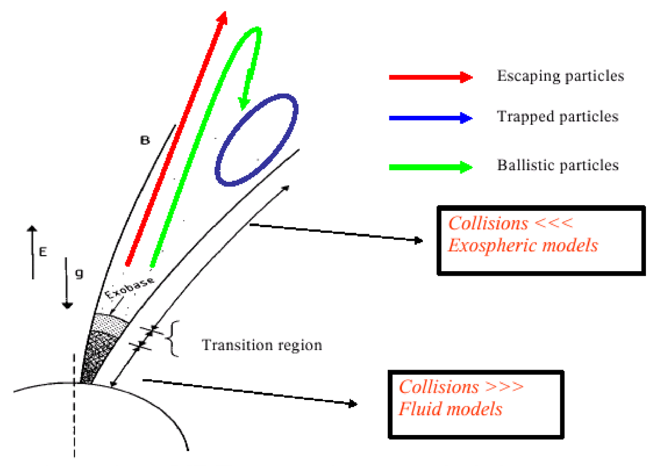
The solar wind exospheric model assumes existence of a sharp boundary, called exobase, which separates a collision dominated region (where a fluid model would apply) and a completely collisionless region, named exosphere. This boundary is defined as the distance r_0 from the Sun where the Coulomb mean free path of the particles becomes equal to the local density scale height. Above the exobase the dynamic of charged particles (electrons and protons are considered in our studies) is determined by the gravitational potential, electrostatic potential and magnetic field distribution. Owing to the lack of collisions in the exosphere, the Boltzmann equation describing evolution of the Velocity Distribution Function (VDF) of the particles reduces to the Vlasov equation. With Liouvilles theorem and with conservation of the total energy:

$$E = \frac{mv^2}{2} + m\Phi_g + Ze\Phi_e = const$$

and magnetic moment:

$$\mu = \frac{mv_\perp^2}{2B} = const,$$

a solution of Vlasov equation is obtained.



Once a VDF is assumed for particles at the exobase level r_0, their VDF at any larger radial distance $r$in the exosphere is uniquely determined by Liouvilles theorem. Furthermore, macroscopic quantities for different species (like density, bulk velocity, temperature and heat flux) are recived by by integrating moments of VDF.

The key-point of exospheric kinetic model of the solar wind is the correct determination of interplanetary electrostatics potential is the key point of exospheric model. To avoid charge separations and currents on large scales in the exosphere, the electrostatic potential gives rise to a force which attracts the electrons towards the Sun and repels the protons.

General assumptions:

- only protons and electrons are considered
- the velocity distribution function (VDF) of electrons at the exobase is given by a Lorentzian (κ) distribution, VDF of protons is Maxwellian
- model is one dimensionall - radial dependence on r only is assumed, time stationary
- strictly collisionless
- rotation of the Sun is neglected

Input parameters:

- radial distance of the exobase r_0
- temperature of electrons T_e and protons T_p at r_0
- value of κ index
- maximum radial distance
- domain of r_{max}

Exospheric model of the solar wind

Numerical model

Output parameters:

- the electrostatic potential Φ_E
- the total normalized potential of the protons
- the number density, the flux, bulk velocity, temperature, heat flux

Exospheric solutions

- suprathermal electrons (κ function) in atractive potential

$$f_\kappa(v) \propto (1 + \frac{m_e v^2}{\kappa V_\kappa^2})^{-(\kappa+1)}$$

- quasi-neutrality

$$n_e(\Phi_e, \Phi_g, B) = n_p(\Phi_e, \Phi_g, B)$$

- zero current condition (F_e, F_p - field-alligned fluxes of particles):

$$F_e(\Phi_e, \Phi_g, B) = F_p(\Phi_e, \Phi_g, B)$$

Key point of exospheric model: Determination of interplanetary electrostatic potential $\Phi_E(r)$. It is found by iterating potential difference between infinity and the exobase, until the fluxes of electrons and protons are equal.

Collisionless transport equations

Main goal of this project is to establish that the moments of the VDF fulfill transport equations that give a macroscopic description of solar wind plasma. Under the assumption of steady state conditions and radial symmetry, collisionless transport equations are (*Lemaire and Scherer*, 1973):

mass continuity equation:

$$nVr^2 = const. \quad (1)$$

momentum conservation equation for each species (electrons, protons):

$$\underbrace{nmV\frac{dV}{dr}}_{\text{inertial term } (T_1)} + \underbrace{\frac{d}{dr}(nk_BT_\parallel)}_{\text{pressure gradient } (T_2)} + \underbrace{\frac{2nk_B}{r}(T_\parallel - T_\perp}_{\text{magnetic mirror force } (T_3)}) = - \underbrace{nm\frac{d\Phi_g}{dr}}_{\text{gravitational term } (T_4)} - \underbrace{Zen\frac{d\Phi_E}{dr}}_{\text{electrostatic term } (T_5)} \quad (2)$$

energy conservation equation:

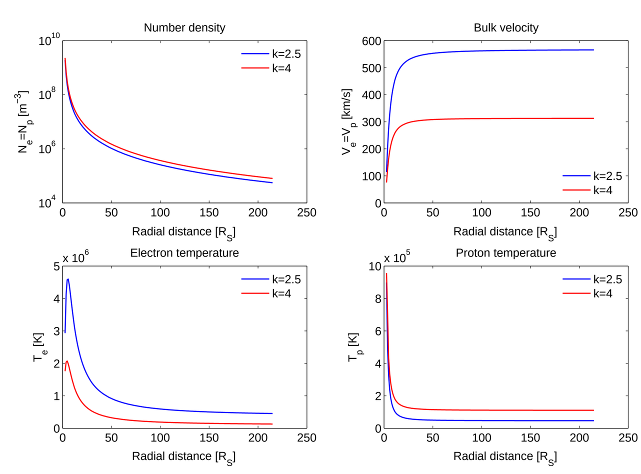
$$\underbrace{r^2q}_{\text{heat flux}} + C\left[\underbrace{\frac{mV^2}{2}}_{\text{kinetic energy}} + \underbrace{\frac{k_B(3T_\parallel + 2T_\perp)}{2}}_{\text{enthalpy}} + \underbrace{m\Phi_g}_{\text{gravitational energy}} + \underbrace{Ze\Phi_E}_{\text{electrostatic energy}}\right] = E_\infty \quad (3)$$

where: n - density, V - bulk velocity, r - radial distance, C - constant, $T_\perp$ - perpendicular temperature, $T_\parallel$ - parallel temperature, Φ_g - gravitational potential, Φ_E - electrostatic potential, q - heat flux.

Numerical results

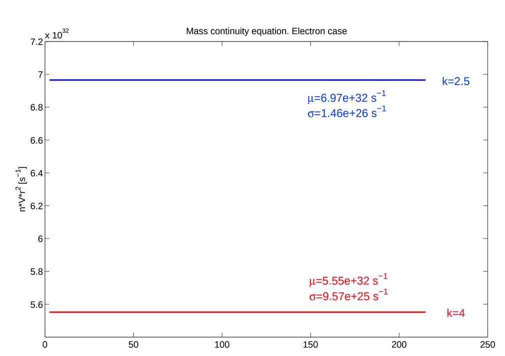
The moments of the VDF for both protons and electrons are introduced into the mass continuity, momentum and energy conservation equations. The analysis is carried for the following parameters: $r_0 = 1.5\, R_\odot$, $T_e = 10^6$ K, $T_p = 2 \cdot 10^6$ K, $\kappa_1 = 2.5$, $\kappa_1 = 4.0$.

Moments of VDF function:



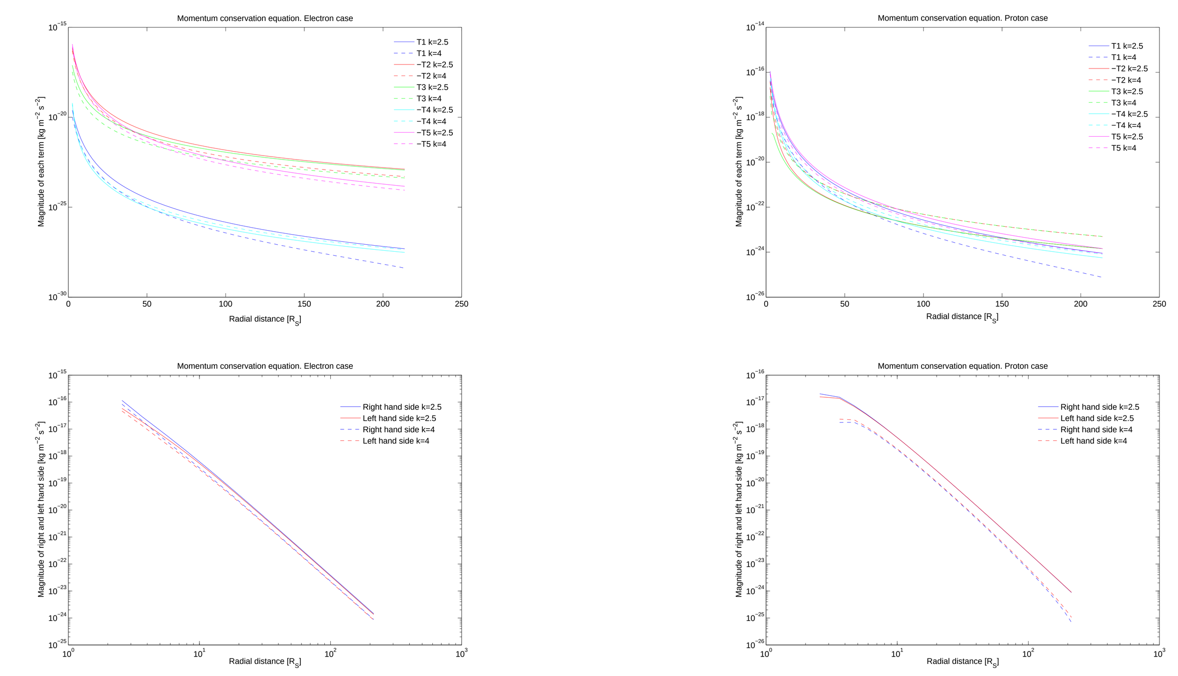
For lower values of κ (for faster solar wind), the bulk velocity is higher. For $\kappa = 2.5$ we obtain the velocity at the value of approximately 600 km/s and for $\kappa = 4.0$ the solar wind velocity is 300 km/s. For lower κ index electron temperature is higher, this fact emphasises the importance of suprathermal electrons on acceleration of the solar wind.

Mass continuity equation for electrons

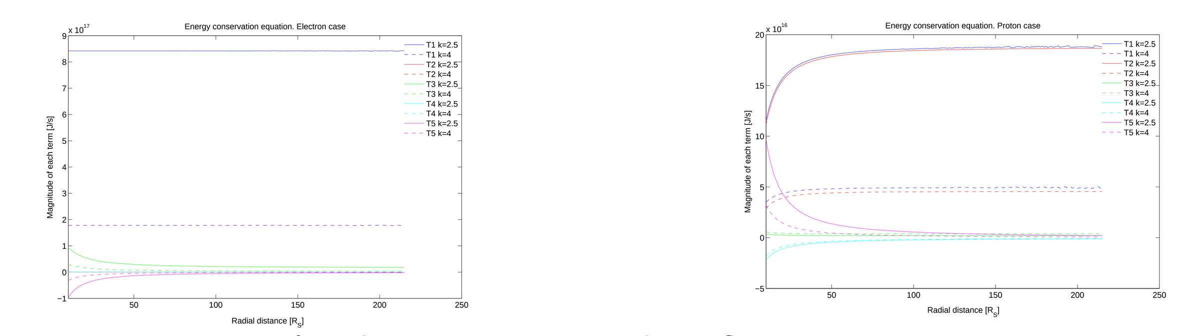


For both considered values of κ, function remains constant, so we conclude that mass continuity equation is conserved. Identical results were obtained for protons.

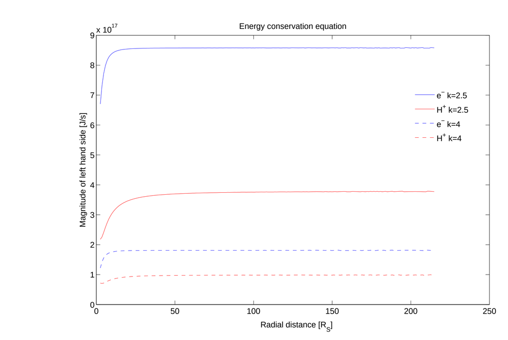
Momentum conservation equation for electrons and protons Magnitude of each term of momentum equation (2) is presented as the function of solar radii. In case of electrons, due to their very law mass, inertial and gravitational terms are negligible at all radial distances. Comparison of left hand side and right hand side of equation (2), respectively for protons and electrons, assures that momentum equation is conserved.



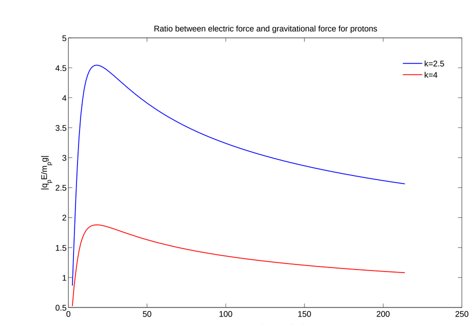
Energy conservation equation for electrons and protons



In a same manner as for the momentum equation, each term of energy equation (3) is presented. Analysis lead to conclusion that energy is conserved. With the increase of value of *kappa* parameter, total energy of particles decrease. For electrons the greatest part of total energy comes from heat flux. Since the protons are considered, main contribution to total energy is associated with convection of kinetic energy and heat flux.



Relation between electric and gravitational force



For protons, we obtained that at large radial distance outward electric force becomes larger than the inward gravitational force. Closer to the Sun, the gravitational potential dominates the elctrostatic potential.

Conclusion

- It is shown that the moments derived for a Kappa VDF fulfill the transport equations and give an accurate macroscopic description of plasma. Mass continuity is satisfied by the kinetic exospheric solution.
- Energy conservation equation is also satisfied by the moments of the kinetic exospheric model.
- We have identified the radial distance where the outward electric force becomes larger than inward gravitational force.
- Faster solar wind is produced when the flux of suprathermal electrons increases.
- We have been able to show that close to the acceleration region the pressure gradient is equal to polarization electric field.

Refrences

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