Numerical simulations of an electron and a proton cloud injected into a magnetic field distribution of a 1D tangential discontinuity

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I. Abstract

This paper examines the velocity distribution function (VDF) of a hydrogen plasma injected into a steady state magnetic field distribution of a 1D tangential discontinuity (TD). In our model, the magnetic field changes his orientation and magnitude from $\mathbf{B}_1$ at the "left" hand side of TD to $\mathbf{B}_2$ at the "right" hand side. We used two prescribed electric field distributions: (i) a uniform electric field equal to $\mathbf{E}=\mathbf{B}_1\times\mathbf{V}_0$, where $\mathbf{V}_0$ is the initial bulk velocity of the two species and (ii) a non-uniform electric field that is everywhere perpendicular to $\mathbf{B}$ and has a magnitude that satisfies the conservation of zero order drift: $\mathbf{E}\times\mathbf{B}/\mathbf{B}^2$=const. The electromagnetic field is unidimensional. The initial VDF of the two populations is described by a displaced Maxwellian. The particles are injected from sources aligned along x-axis. The solution of the Vlasov equation is constructed using characteristics method. We reconstructed VDF for protons and electrons inside/outside the TD and we evidenced the effects introduced by the non-uniformities of the electromagnetic field. In the case of an uniform electric field perpendicular to a unidirectional magnetic field, a broadening of the VDF in the perpendicular direction is illustrated when the particles cloud moves in an increasing magnetic field. In the case of a non-uniform electric field conserving the zero order drift, the VDF remains a displaced Maxwellian all along the TD.

II. Numerical method

Equation to solve: the Vlasov equation for the velocity distribution function (VDF) of each component species:

$$\frac{\partial f_1}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f_1}{\partial \boldsymbol{r}} + \frac{q}{m}\left(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}\right)\cdot\frac{\partial f_1}{\partial \boldsymbol{v}} = 0$$

Method to solve: numerical integration of the characteristics (Speiser et al., 1981; Lyons and Speiser, 1982; Williams and Speiser, 1984; Curran et al., 1987; Curran and Goertz, 1989).

Approximation: rarefied and non-diamagnetic plasma cloud. (The self-consistent perturbation of the TD magnetic field due to the incoming VDF, is neglected. Electric and magnetic fields are prescribed) .

Numerical algorithm: fifth order adaptive Runge-Kutta-Cash-Karp algorithm.

Code: C language, using MPI. The simulations were done on a small linux cluster of PCs.

III. Simulation setup

Electric and magnetic field distribution

Magnetic field distribution:

$$\boldsymbol{B}(x) = \frac{\boldsymbol{B}_1}{2} erfc\left(\frac{x}{L}\right) + \frac{\boldsymbol{B}_2}{2}\left[2 - erfc\left(\frac{x}{L}\right)\right]$$

where $\mathbf{B}_1$, $\mathbf{B}_2$ represent respectively the magnetic field at $x=-\infty$ and $x=+\infty$; L is the scale length of the TD; *erfc* is the complementary error function.

Electric field distribution:

Two stationary **E**-field distributions were tested:

⇨ **Case A**: a uniform electric field given by $\mathbf{E}=\mathbf{B}_1\times\mathbf{V}_0$=const.

⇨ **Case B**: a non-uniform electric field conserving the zero order drift, $\mathbf{U}_E$=const., given by $\mathbf{E}=\mathbf{B}\times\mathbf{U}_E$.

Initial and reconstructed VDF

The particles were injected from sources aligned along x axis, velocity initialization being done according to a displaced Maxwellian distribution:

$$f_{\alpha 1}(v_x, v_y, v_z) = N_{\alpha 1}\left(\frac{m_\alpha}{2\pi K T_{\alpha 1}}\right)^{\frac{3}{2}} e^{-\frac{m_\alpha\left[(v_x - V_0)^2 + v_y^2 + v_z^2\right]}{2KT_{\alpha 1}}}$$

The VDF is reconstructed at different moments of time, inside/outside the TD by applying the Liouville theorem that says that along each particle's orbit:

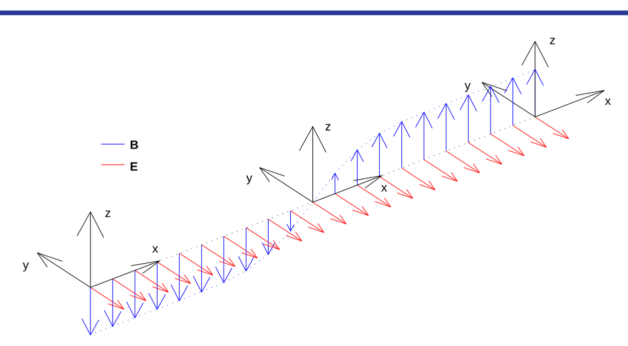
$$df_{\alpha 1}/d\tau = 0$$

Thus in any moment later to the injection, when the particle 'a' has the velocity $\mathbf{v}_a=(v_{ax}, v_{ay}, v_{az})$, the probability density $P_{va}(v_{ax}, v_{ay}, v_{az})$ to have the velocity in $(v_{ax}+dv_x, v_{ay}+dv_y, v_{az}+dv_z)$ is equal to the initial probability density, P_{va0}, that corresponds to the initial velocity of the particle, $\mathbf{v}_0$.

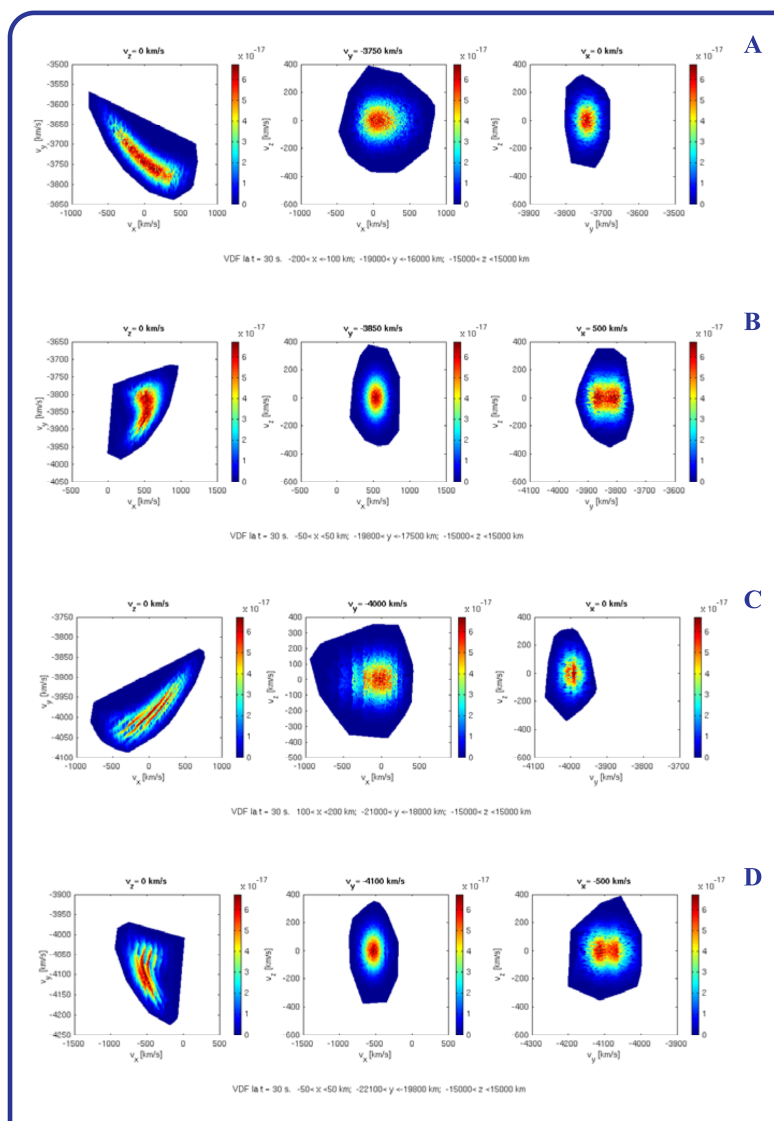
$$P_{va0}(x_0,y_0,z_0,v_{0x},v_{0y},v_{0z}) = P_{va}(x,y,z,v_x,v_y,v_z)$$

IV. Case A: Uniform electric field. Antiparallel magnetic field distribution

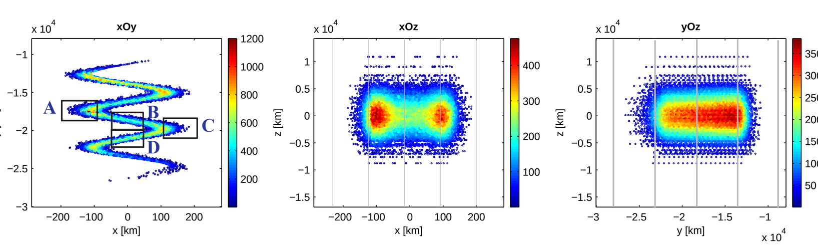
Proton case - Input parameters
$\mathbf{B}_1$= (0,0,-30) nT, $\mathbf{B}_2$= (0,0,30) nT, L= 500 km **E**= (0,-4.2,0) mV/m
20 sources, 27000 protons/source x_0= -3380 km, dx_0= 20 km
temperature 100 eV bulk velocity 140 km/s
time of simulation 30 s



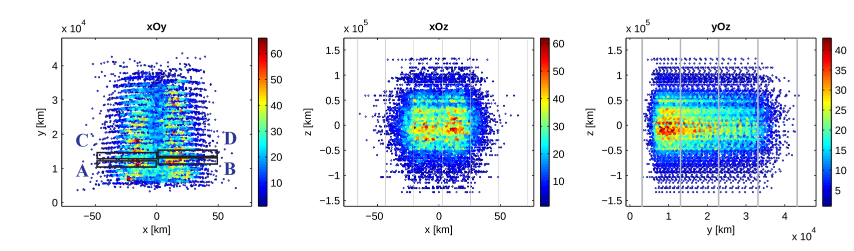
Electron case - Input parameters
$\mathbf{B}_1$= (0,0,-30) nT, $\mathbf{B}_2$= (0,0,30) nT, L= 500 km **E**= (0,-4.2,0) mV/m
21 sources, 1500 electrons/source x_0= -3100 km, dx_0= 5 km
temperature 500 eV bulk velocity 140 km/s
time of simulation 20 s



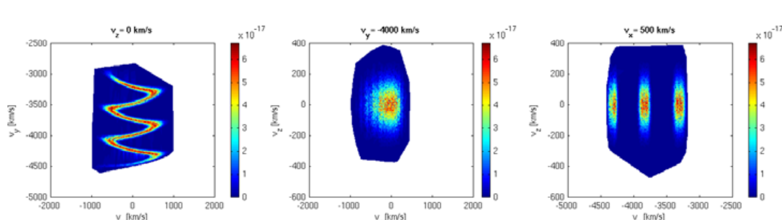
Panels show the final VDF for protons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space. The VDF is reconstructed for the particles localized in the rectangles defined in (x,y) plane identified by the corresponding letters (see figures in the middle).



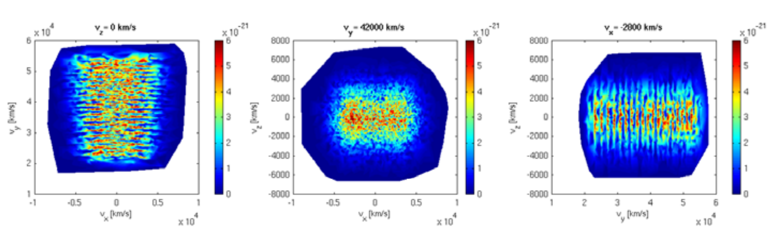
Projections in the planes (x,y), (x,z) and (y,z) of the final positions of all the protons in the jet. The density is color coded. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.



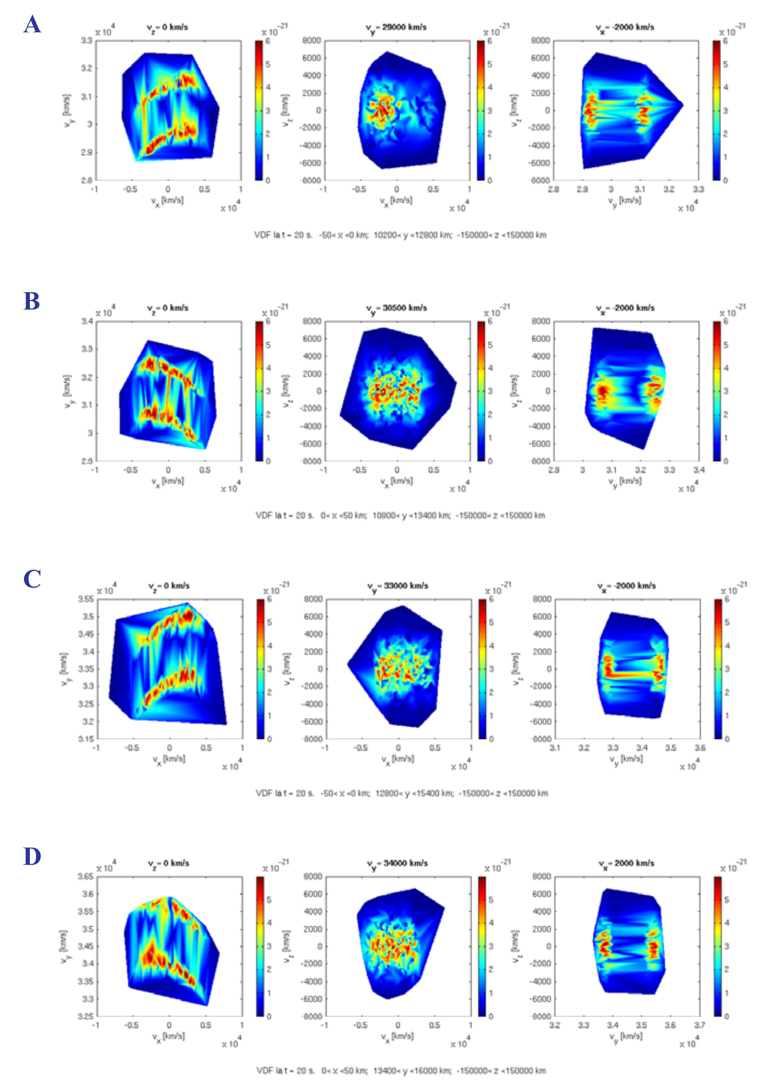
Projections in the planes (x,y), (x,z) and (y,z) of the final positions of all the electrons in the jet. The density is color coded. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.



The final VDF for protons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space is reconstructed using the particles within the entire spatial domain.



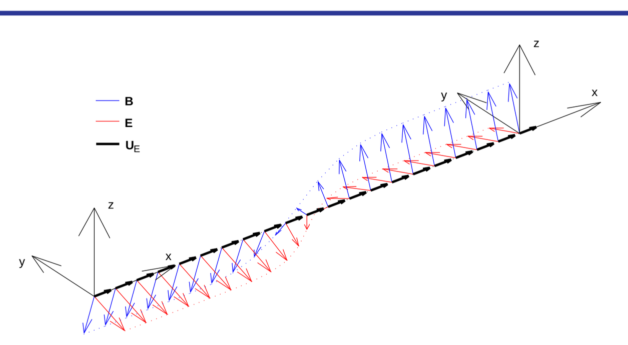
The final VDF for electrons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space is reconstructed using the particles within the entire spatial domain.



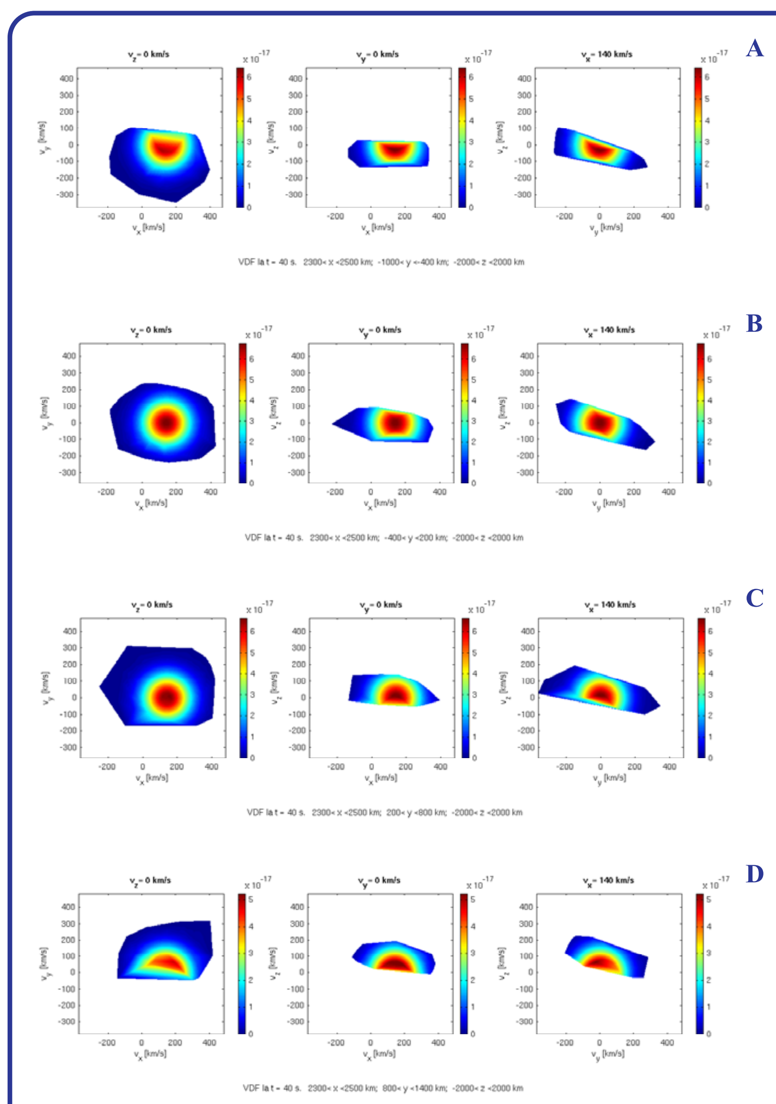
Panels show the final VDF for electrons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space. The VDF is reconstructed for the particles localized in the rectangles defined in (x,y) plane identified by the corresponding letters (see figures in the middle).

V. Case B: Non-uniform electric field conserving the zero order drift. Sheared magnetic field distribution

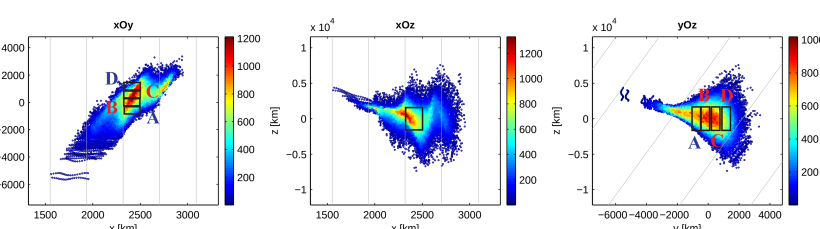
Proton case - Input parameters
$\mathbf{B}_1$=(0,10,-30) nT, $\mathbf{B}_2$=(0,10,30) nT, L=500km U_E= (140,0,0) km/s
20 sources, 27000 protons/source x_0= -3380 km, dx_0= 20 km
temperature 100 eV bulk velocity 140 km/s
time of simulation 40 s



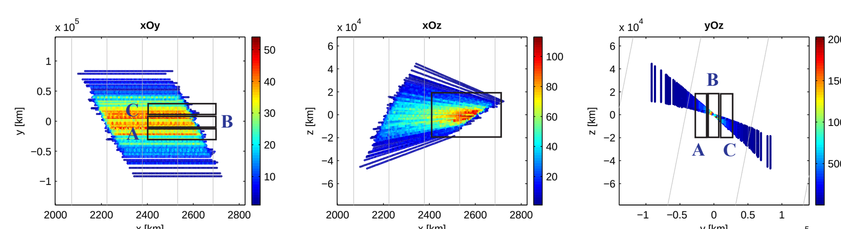
Electron case - Input parameters
$\mathbf{B}_1$=(0,10,-30) nT, $\mathbf{B}_2$=(0,10,30) nT, L=500km U_E= (140,0,0) km/s
77 sources, 1000 electrons/source x_0= -3380 km, dx_0= 5 km
temperature 500 eV bulk velocity 140 km/s
time of simulation 40 s



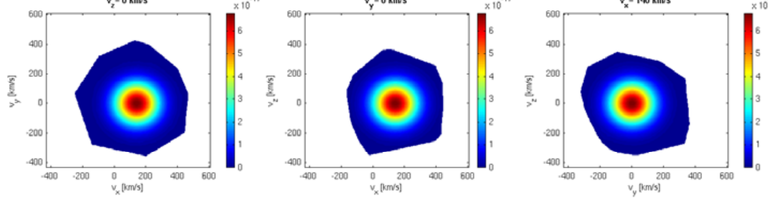
Panels show the final VDF for protons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space. The VDF is reconstructed for the particles localized in the regions identified by the corresponding letters (see figures in the middle).



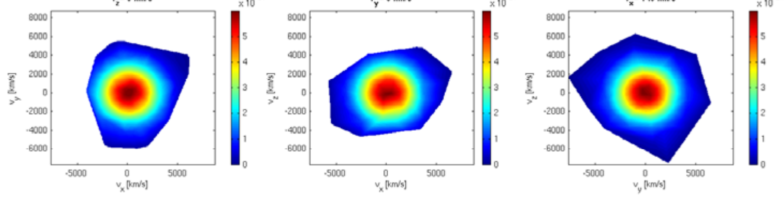
Projections in the planes (x,y), (x,z) and (y,z) of the final positions of all the protons in the jet. The density is color coded. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.



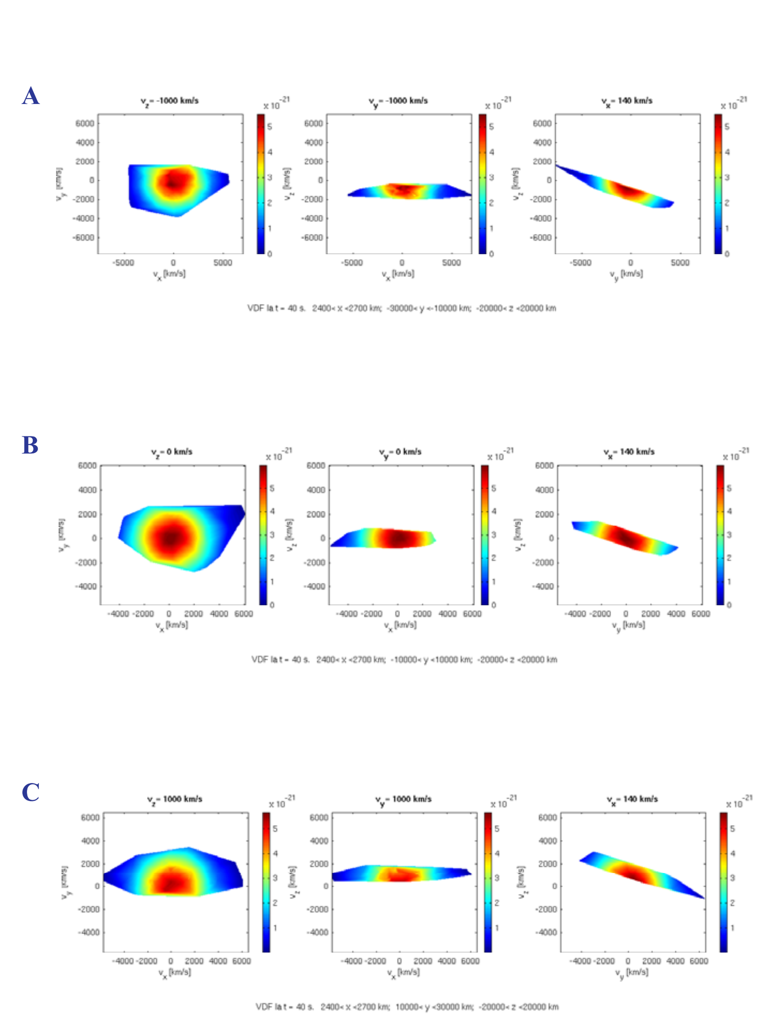
Projections in the planes (x,y), (x,z) and (y,z) of the final positions of all the electrons in the jet. The density is color coded. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.



The final VDF for protons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space is reconstructed using the particles within the entire spatial domain.



The final VDF for electrons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space is reconstructed using the particles within the entire spatial domain.



Panels show the final VDF for electrons projected in (v_x,v_y), (v_x,v_z), (v_y,v_z) planes of the velocity space. The VDF is reconstructed for the particles localized in the regions identified by the corresponding letters (see figures in the middle).

VI. Conclusions

The velocity distribution function (VDF) of the electrons and protons is reconstructed using the method of numerical integration of the characteristics of the stationary Vlasov equation. We provide assessment of the VDF in various space regions as a first step toward the goal of giving fully 6D distributions.

- **A**. The VDF of the protons and electrons of a jet injected into an anti-parallel magnetic field distribution and an uniform electric field is highly anisotropic in the velocity and position space. Note the iso-morphism between the distribution of proton in space and the distribution of their velocities in the velocity space. Indeed, the reconstructed VDF for different regions from (x,y) plane correspond to different regions from the VDF reconstructed using all particles. That can be observed both for protons and electrons. It illustrates the process of the electrostatic acceleration in the region where **B**=0 and the electric field is constant. The structuring, both in space and velocities has a much smaller scale for electrons than for protons. The VDF provided for the bins A,B, C and D show the features imprinted in the protons and electrons distribution by the acceleration mechanism. Our results provide possible tests for experimental data obtained for discontinuities in laboratory and space plasma.

- **B**. If **E** remains everywhere perpendicular to **B** and has a magnitude that satisfies the conservation of the zero order drift, the jet penetrates the TD and move across it into the right hand side. The electrons penetrate along the same distance as the protons. The shear angle of the magnetic field produces a deformation of the shape of the jet. By reconstructing VDF for different regions in space we evidenced a process of "velocity picking". That can be seen both in the case of protons and electrons. An interesting feature is the "transition" of the proton VDF from position A to position D. It reminds similar results ("velocity filtering") obtained with 2D kinetic models of plasma boundary layers (Echim and Lemaire, Phys. Rev E, 2005).

VII. References

[1] M. Echim and J. Lemaire, Phys. Rev E, 2005
[2] M. Echim, Cosmic Research, 40, 534, 2002
[3] M. Echim, J. Lemaire, "Earth's Low Latitude Boundary Layer" AGU Monograph 133, pp. 169-179, American Geophysical Union, 2003
[4] M. Echim, Ph. D. Thesis, Universite Catholique de Louvain, Belgium, 2004
[5] Speiser, T.W., D.J. Williams, H.A. Garcia, J. Geophys. Res., 86, 723, 1981
[6] Lyons, L.R., Speiser, T.W., J. Geophys. Res., 87, 2276, 1982
[7] Williams, D.J., Speiser, T.W., J. Geophys. Res., 89, 8877, 1984
[8] Curran, D.B., C.K. Goertz, T.A. Whelan Geophys. Res. Lett., 14, 99, 1987
[9] Curran, D.B., C.K. Goertz, J. Geophys. Res., 94, 272, 1989
[10] Demidenko, I.I., Lomino, N.S., Padalka, V.G., Rutkevitch, B.G. And Sinel'nikov, K.D., Soviet Physics-Technical Physics, 14, 16, 1969
[11] Demidenko, I.I., Lomino and N.S., Padalka, Soviet Physics-Technical Physics, 16, 1096, 1972
[12] Lemaire, J., Journal of Plasma Physics, 33, 425, 1985
[13] Adaptive Stepsize Control for Runge-Kutta, pp. 714-722, Numerical Recipes in C

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