Attitude Determination for Magion-5 Satellite Using Magnetometer Data Only

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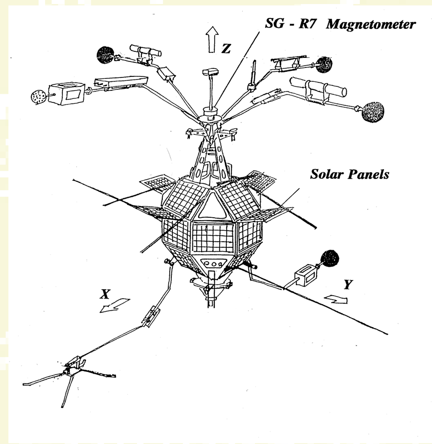
Summary

The attitude of a spin-stabilized satellite has been restituted from dynamical calculations using only three axes measurements of the Earth magnetic field vector. Magion-5 as a small subsatellite with 65 degrees inclination orbit and a perigee of 20000 Km, was intended for carrying out simultaneously measurements with the Auroral Probe main satellite, in the framework of Interball project.

In this work we have developed a batch numerical filter to search for the best-fit state parameters and to solve for the solutions of the dynamical and kinematic equations of motion for the satellite in it's own reference system.

An overall estimation for the effects of solar pressure, gravity, and magnetic disturbance torques as well as the internal passive stabilization torque has been done. These disturbance effects have been related to the torque-free attitude solutions of a nutating spacecraft.



Coordinate Systems

- **Non inertial local frame** (Σ_{loc}) is a coordinate system with one axis solidar to the geomagnetic field line while the satellite is moving on the orbit.
  origin: center of mass of the satellite
  oz-axis: directed parallel to the geomagnetic field line
  ox-axis: parallel to solar ecliptic plane at an initial epoch
  oy-axis: zx$x$
- **fixed frame** (Σ_{fix}) is the local frame at the initial epoch.
  origin: center of Earth
  oz-axis: parallel the the geomagnetic field line at the initial epoch
  ox-axis: parallel to the solar ecliptic plane
  oy-axis: zx$x$
- The **satellite frame**(Σ_{sat}) is a coordinate system fixed to the satellite body axes. The satellite frame is aligned with the principal axes of inertia.

Parameterization of Attitude

Indirect

- 2 successive transformations; first one starts from the fixed frame (or any other inertial coordinate, e.g. GSE system) to the local coordinate and the second one rotates the system to the satellite frame.
- geometric significance
- allow the prediction of the initial state parameters

$$A_{\Sigma_{loc}\to\Sigma_{sat}}(\theta,\psi,\phi/\mathbf{q})A_{\Sigma_{fix}\to\Sigma_{loc}}(\mathbf{q^I})$$

- Euler angle parameterization
  $A_{\Sigma_{loc}\to\Sigma_{sat}} = A_{313}(\theta,\psi,\phi) =$

$$\begin{pmatrix} \cos\psi\cos\phi-\cos\theta\sin\psi\sin\phi & \cos\psi\sin\phi+\cos\theta\sin\psi\cos\phi & \sin\theta\sin\psi \\ -\sin\psi\cos\phi-\cos\theta\cos\psi\sin\phi & -\sin\psi\sin\phi+\cos\theta\cos\psi\cos\phi & \sin\theta\cos\psi \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{pmatrix}$$

$\vec{B}^{obs} = (B_x, B_y, B_z)^{obs}$ is the measured magnetic field, $\theta = acos(B_z^{obs}/B^{obs})$, $\psi = atan(B_x^{obs}/B_y^{obs})$, $\vec{B}^{obs} = (B_x, B_y, B_z)^{obs}$, ϕ is the unobserved angle representing the rotated angle along the magnetic field line of the Σ_{loc} axis crossing from the ecliptic plane to the equatorial plane of the satellite. $\mathbf{q^I}$ are determined from the motion of the magnetic field vector in the solar ecliptic reference using IGRF 95 model.

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Direct

- 1 transformation from the fixed reference to the satellite frame via local system only at the initial epoch.
  $A_{\Sigma_{fix}\to\Sigma_{sat}}(\alpha,\beta,\gamma/Q_1,Q_2,Q_3,Q_4)$
- Quaternion representation

$$A_{\Sigma_{loc}\to\Sigma_{sat}}(\mathbf{Q}) = \begin{pmatrix} Q_1^2-Q_2^2-Q_3^2+Q_4^2 & 2(Q_1Q_2+Q_3Q_4) & 2(Q_1Q_3-Q_2Q_4) \\ 2(Q_1Q_2-Q_3Q_4) & -Q_1^2+Q_2^2-Q_3^2+Q_4^2 & 2(Q_2Q_3+Q_1Q_4) \\ 2(Q_1Q_3+Q_2Q_4) & 2(Q_2Q_3-Q_1Q_4) & -Q_1^2-Q_2^2+Q_3^2+Q_4^2 \end{pmatrix}$$

The Attitude Dynamics and Numerical Filter

Kinematic equations

- indirect

$$\dot{\mathbf{q}} = \left(\frac{1}{2}\Omega(\vec{\Omega}) - O^{-1}(\mathbf{q}^I)O(\dot{\mathbf{q}}^I)\right)\mathbf{q}$$

- direct

$$\dot{\mathbf{Q}} = \frac{1}{2}\Omega(\vec{\Omega})$$

$$\mathbf{\Omega}(\vec{\Omega}) = \begin{pmatrix} 0 & \Omega_3 & -\Omega_2 & \Omega_1 \\ -\Omega_3 & 0 & \Omega_1 & \Omega_2 \\ \Omega_2 & -\Omega_1 & 0 & \Omega_3 \\ -\Omega_1 & -\Omega_2 & -\Omega_3 & 0 \end{pmatrix} \qquad O(\mathbf{q}) = \begin{pmatrix} -q_4 & q_3 & -q_2 & q_1 \\ -q_3 & -q_4 & q_1 & q_2 \\ q_2 & -q_1 & -q_4 & q_3 \\ q_1 & -q_2 & -q_3 & -q_4 \end{pmatrix}$$

Dynamic equations

$$\begin{aligned} I_1\dot{\Omega}_1 + \dot{h}_1 &= -(I_3-I_2)\Omega_2\Omega_3 + \Omega_3 h_y + M_{control1} + M_{E1} \\ I_2\dot{\Omega}_2 + \dot{h}_y &= -(I_1-I_3)\Omega_3\Omega_1 - \Omega_3 h_1 + M_{control2} + M_{E2} \\ I_3\dot{\Omega}_3 &= -(I_2-I_1)\Omega_1\Omega_2 - \Omega_1 h_2 + \Omega_2 h_1 + M_{control3} + M_{E3} \end{aligned}$$

$$\begin{aligned} \dot{h}_1 &= I_0\dot{\Omega}_1 - c_n^{(1)} \\ \dot{h}_2 &= I_0\Omega_2 - c_n^{(2)} \end{aligned}$$

- $\vec{\Omega}$ - angular rate vector in satellite frame, $\hat{I}$ - momenta of inertia, $\vec{M}_{control}$, $\vec{M}_E$ - control and environmental torques, $\vec{h}, m_0, I_0, c_n$ - internal angular momentum, fluid mass, moment of inertia and friction coefficient of the viscous damper, respectively.
- Batch numerical filter

State Vector (12 state elements)	4 quaternions, 3 angular velocities, 4 moments of Inertia, 1 coefficient of viscosity
Measurement	3 magnetometer readings
Attitude Integrator	4/5-th Order Runge Kutta Method
Earth Magnetic Field Model	IGRF 1995 (up to 10-th Harmonic Order)
Environmental Torque	gravity-gradient torque solar-radiation torque magnetic disturbance torque

- Initial conditions

$$\begin{aligned} &\theta_0, \psi_0, \phi_0 \\ \Omega_1^{(0)} &= \frac{(\omega_0-\dot{\psi}_0)}{\cos\theta_0}\sin\theta_0\sin\psi_0 + \dot{\theta}_0\cos\psi_0 \\ \Omega_2^{(0)} &= \frac{(\omega_0-\dot{\psi}_0)}{\cos\theta_0}\sin\theta_0\cos\psi_0 - \dot{\theta}_0\sin\psi_0 \\ \Omega_3^{(0)} &= \omega_0 \end{aligned}$$

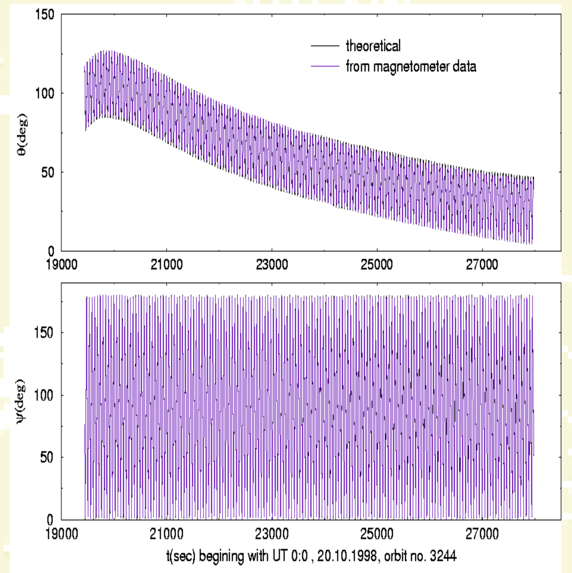
- ϕ_0, ω_0 fit parameters
- Loss Function is computed from:

$$\vec{B}_{model} = \hat{\mathbf{A}}^{-1}\left(\mathbf{S}\vec{B}^{obs} + \vec{\mathbf{b}}_{bias}\right)$$

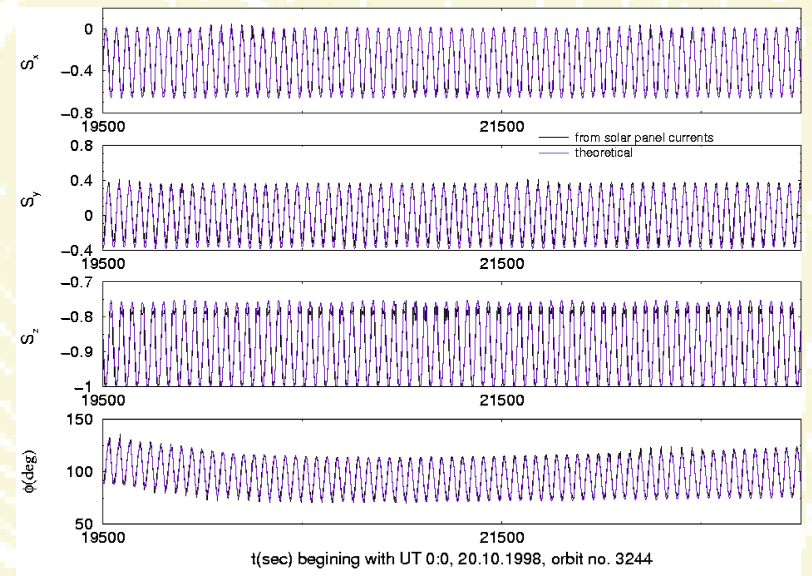
- **S** - scale factor + alignment matrix, 3x16 + bias axes coefficients, 3 axes general biases, 9 alignment (and channel mixing) coefficients

Numerical Results

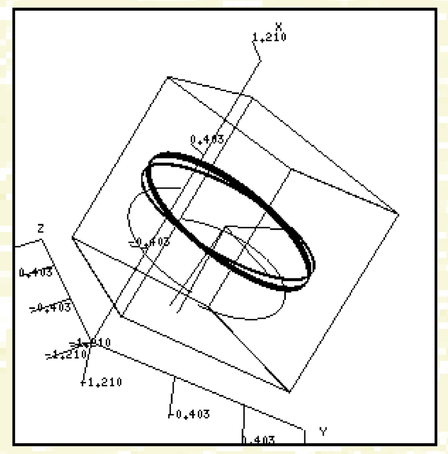
- Attitude calculation results for Magion-5, orbit 3244, from October 10, 1998



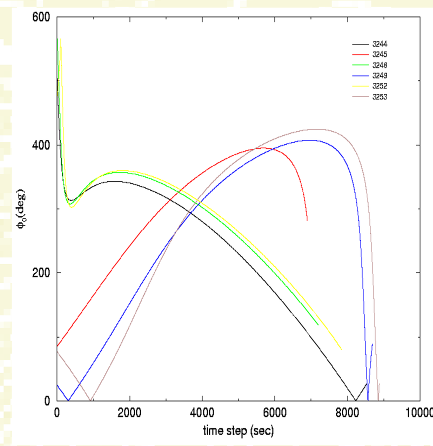
- Measured and estimated values of the θ and ψ angles for 3244 orbit



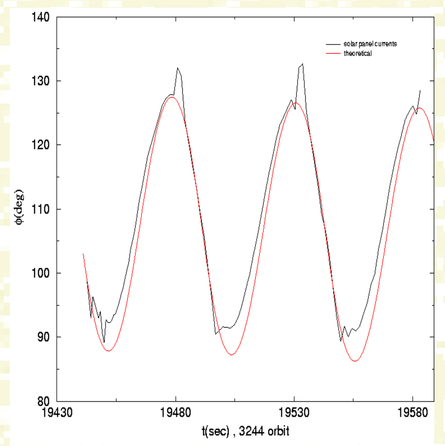
- Measured and estimated projections of the sun versor in the satellite frame, as well as the ϕ Euler angle behaviour. The measured values are computed by processing the solar panel currents and are subject of large errors.

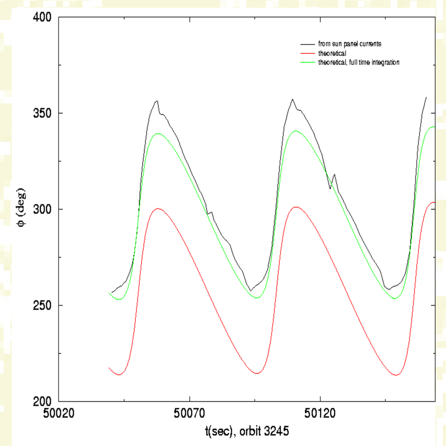


- 3-D motion of satellite's principal axes of inertia in GSE reference for 3244 and 3245 orbits



- ϕ_0 initial phase behaviour along the satellite trace





- numerical filter estimates trusted ϕ_0 initial phase values over a full orbit integration or for space regions with fast modification of the direction of magnetic field line.
- attitude estimated errors are less than 2 degrees (r.m.s. for θ and ψ angles)
- estimates for the third angle errors depends on quality of the data available for the sun vector restitution (r.m.s. values ≈ 5 degrees).

Conclusions

The restitution of the attitude matrix for a near earth satellite is possible to be achieved by processing 3 axis magnetometer data only. The method is based on a theoretical approach of the satellite motion as well as on the behaviour of the geomagnetic field along the orbital trace. The accuracy of the method depends on the correct estimates of external and internal torques acting on the satellite, on the exactness of the geomagnetic field model and on the accuracy of the onboard measurements.