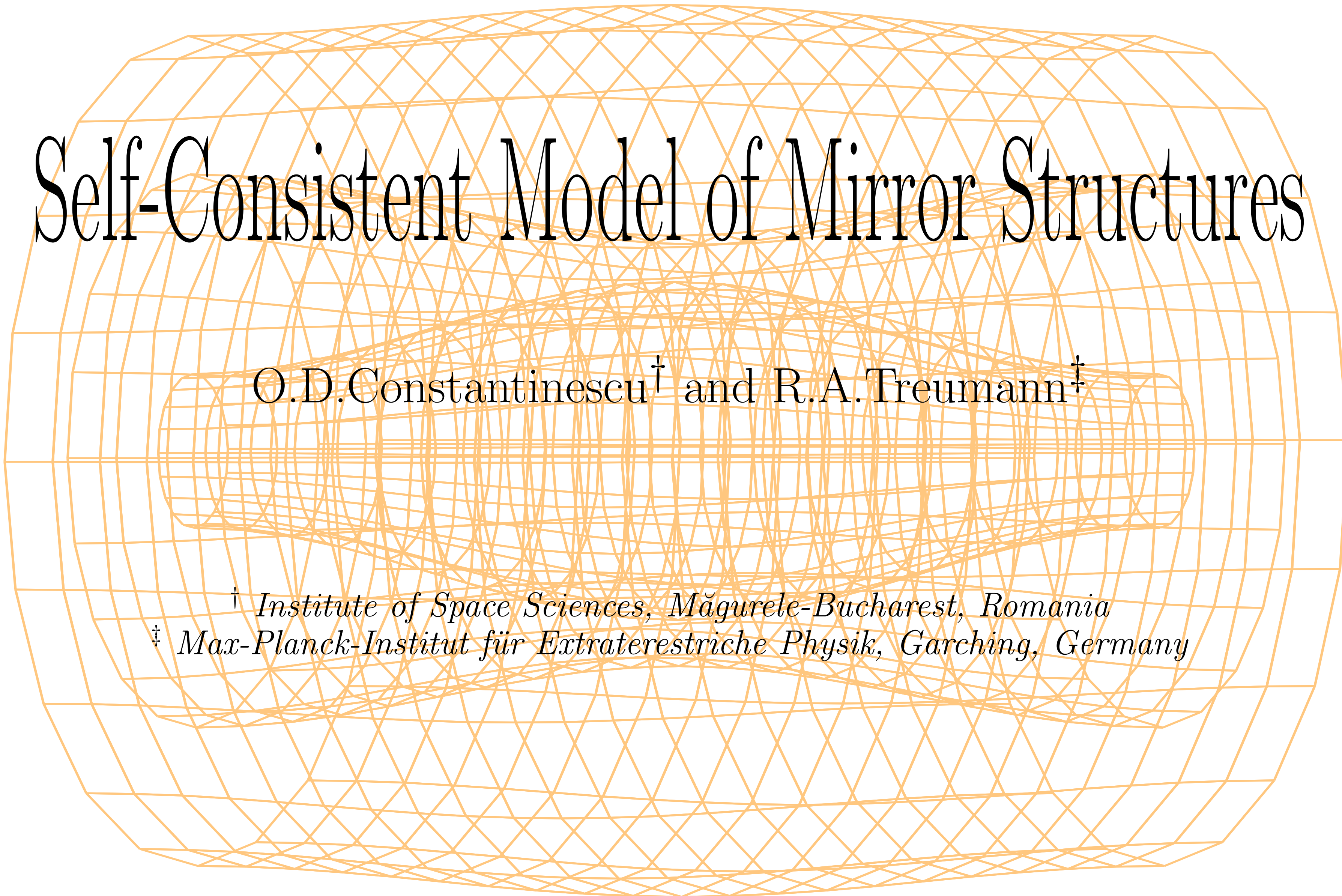
Self-Consistent Model of Mirror Structures

O.D.Constantinescu[†] and R.A.Treumann[‡]

[†] *Institute of Space Sciences, Măgurele-Bucharest, Romania*
[‡] *Max-Planck-Institut für Extraterestriche Physik, Garching, Germany*



Magnetic mirror structures have been observed in the day side of the terrestrial magnetosphere, between bow-shock and magnetopause and also in the distant magnetotail. The purpose of this work is to give a self-consistent model of the mirror structures using a perturbative magnetohidrostatic approach.

In the absence of the perturbation the magnetic field will be considered uniform and parallel with the z axis. The magnetic field perturbation will be chosen to be symmetrical in respect to the z axis in the meridian plane. In order to describe a mirror structure the perturbation must be periodical along the z axis $(B(z+2L) = B(z))$. The plasma will be a mixture of electrons and protons at equilibrium: $n^{(e)} = n^{(p)} = 1/2n; T^{(e)} = T^{(p)} = T$.

The basic relations we use are:

$$\nabla\left(p_\perp + \frac{B^2}{2\mu_0}\right) + \nabla\left[\left(p_\parallel - p_\perp - \frac{B^2}{\mu_0}\right)\frac{\underline{BB}}{B^2}\right] = 0$$

$$A(\rho, z) = \left[1 - \left(1 - \frac{1}{A_0}\right)\frac{B_0}{B(\rho, z)}\right]^{-1}$$

where $\underline{BB}$ is the tensor with elements $(\underline{BB})_{ij} = B_i B_j$, $T_\parallel, T_\perp$ are the parallel and the perpendicular temperatures and $A_0 = T_\perp / T_\parallel$ is the unperturbed anisotropy.

The previous equations lead us to the magnetic field perturbation:

$$\delta B_z(\rho, z) = \sum_{n=-\infty}^{\infty} C_n J_0\left(\frac{n\alpha\rho}{L}\right) e^{-in\frac{\pi z}{L}}$$

$$\delta B_\rho(\rho, z) = \sum_{n=-\infty}^{\infty} \frac{i\pi}{\alpha} C_n J_1\left(\frac{n\alpha\rho}{L}\right) e^{-in\frac{\pi z}{L}}$$

where J_n is the cylindrical Bessel function and by α we have denoted the subsequent adimensional quantity:

$$\alpha = \pi\sqrt{\frac{\frac{1}{2}\left(1-\frac{1}{A_0}\right)+\frac{1}{\beta_{0\perp}}}{A_0-1-\frac{1}{\beta_{0\perp}}}}$$

$\beta_{0\perp}$ being the plasma parameter, i.e. the ratio between the orthogonal plasma pressure, $p_{0\perp}$ and the magnetic pressure, $\frac{B_0^2}{2\mu_0}$.

The solution for the magnetic field perturbation is non-divergent only if the argument of the Bessel function is a real number. Hence, $\alpha^2 > 0$ is a condition for the existence of the mirror structures. This condition is equivalent with:

$$A_0 > 1 + \frac{1}{\beta_{0\perp}}$$
$$or$$
$$A_0 < \frac{\beta_{0\perp}}{\beta_{0\perp}+2}$$

The first inequality is the same as the mirror instability condition derived from the kinetic theory. On the other hand, the second shows us that the mirror structures could exist even for anisotropies less then one.

Because of the gradient and the curvature of the magnetic field, inside the magnetic bottle an electric current density is rising. We can write this current density in terms of the current density derived from the Ampere's law:

$$\mathbf{j}_d = \frac{1}{4}\frac{3\beta_{0\perp} + 2}{2A_0 + 1}\mathbf{j}_B$$

At equilibrium $\mathbf{j}_d = \mathbf{j}_B$. In this case the magnetic field perturbation produces a drift current which in turn sustains the perturbation. Depending on A_0 and $\beta_{0\perp}$ the drift current might be grater or smaller then the current $\mathbf{j}_B$ required to sustain the magnetic field perturbation. If $\mathbf{j}_d > \mathbf{j}_B$ then the perturbation induced by the drift current will be grater then the original perturbation, consequently the drift current intensity will increase. In this situation the mirror structure is unstable. Similarly, if $\mathbf{j}_d < \mathbf{j}_B$ the magnetic field perturbation will decrease. Therefore the instability condition for the magnetic mirrors is:

$$A_0 < \frac{1}{4}\left(\frac{3}{2}\beta_{0\perp} - 1\right)$$

The parameters used for the numerical calculations are: Gaussian perturbation on the axis: $\delta B_z(0, z) = \delta B_z(0, 0)\exp(-z^2/a^2)$, $a = L/3$, the initial anisotropy: $A_0 = 0.4$, the unperturbed magnetic field: $B_0 = 20nT$, the perturbation in the middle of the bottle: $\delta B_z(0, 0) = -5nT$, the unperturbed number density: $n_0 = 50cm^{-3}$, the length of the bottle: $2L = 20Km$, the initial orthogonal temperature: $T_{0\perp} = 10^6K$. With these parameters the radius of the main structure is $R = 15.42km$ and $\beta_{0\perp} = 4.33$.

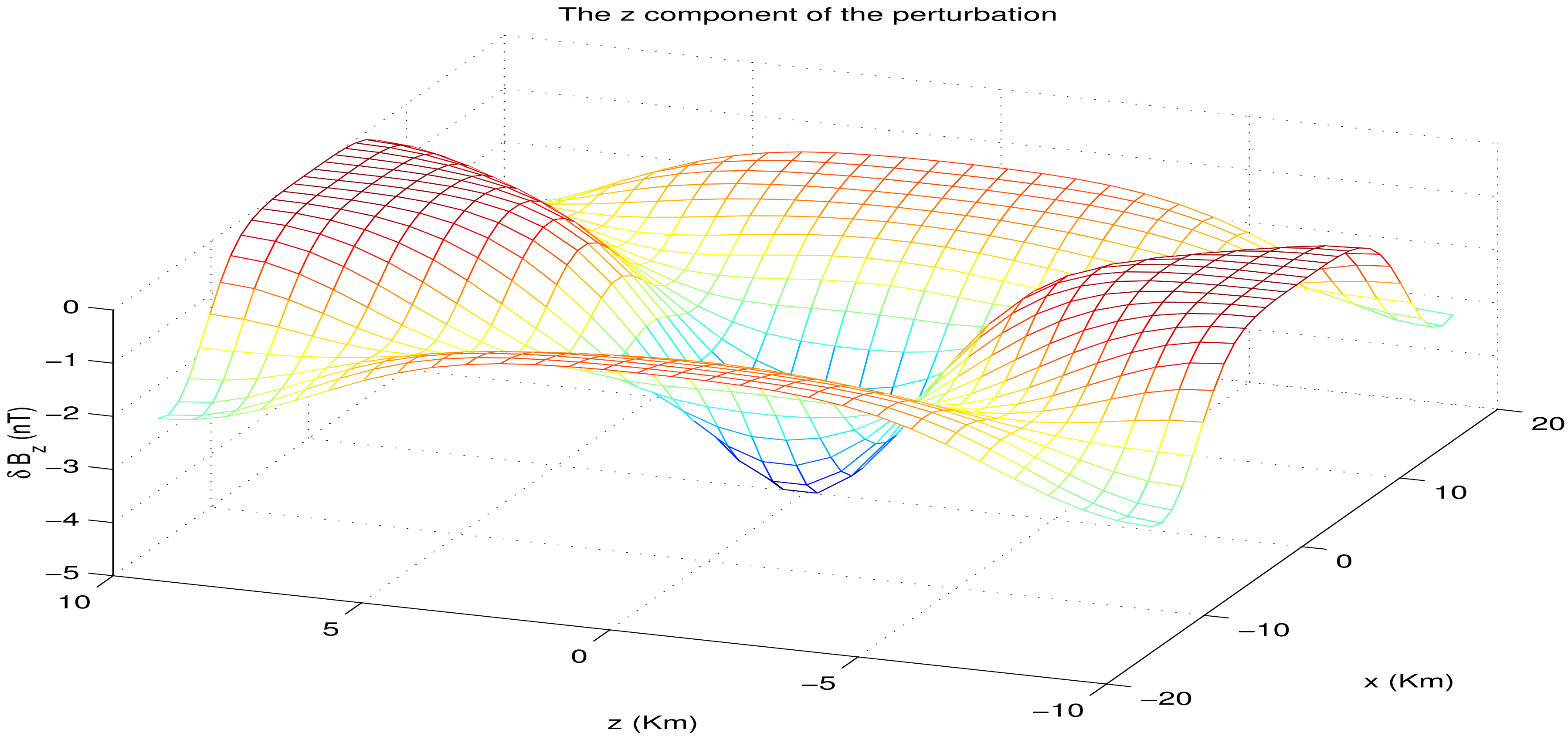
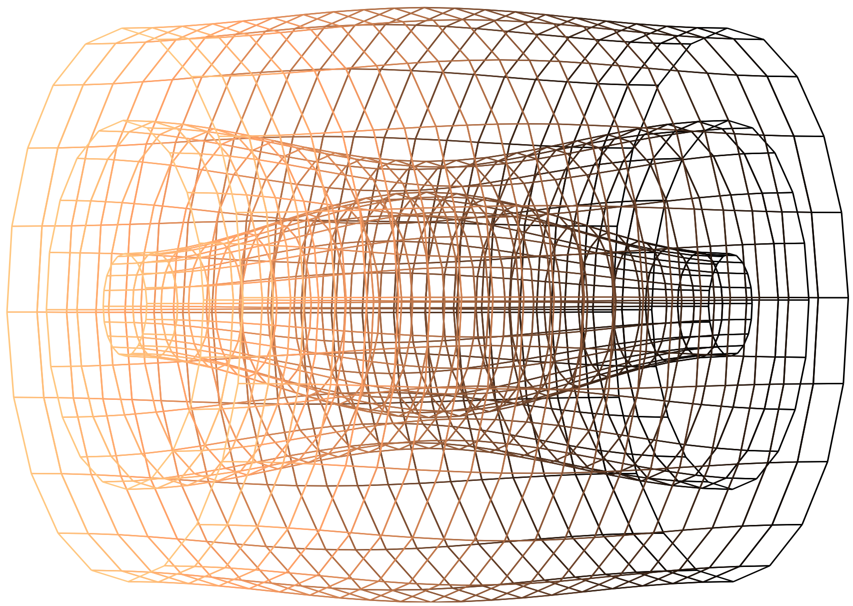


Figure 1: *The z component of the perturbation. We kept the first five Fourier components. For the chosen parameters the radius is $R = 15.42km$. At this radius the field lines are strait and parallel with z-axis*



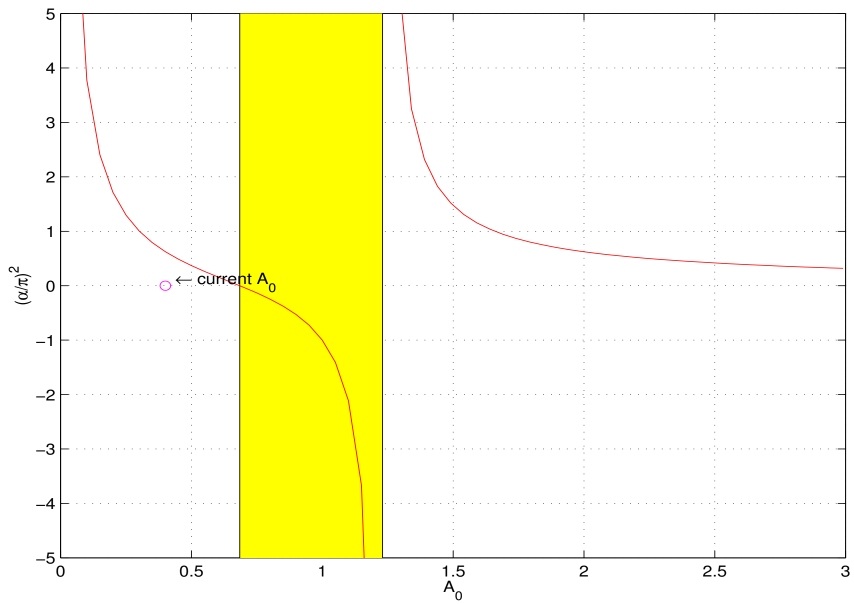


Figure 2: *The surfaces defined by the field lines for the first Fourier component. The main structure (closest to the symmetry axis) has the well known bottle shape. As we move away from the axis, we encounter other structures with similar symmetry, wrapping up each other.*

Figure 3: *The* $(\alpha/\pi)^2$ *parameter versus the unperturbed anisotropy for* $\beta_{0\perp} = 4.33$. *In the highlighted region (*$\alpha^2 < 0$*) the magnetic mirrors can not exist.*

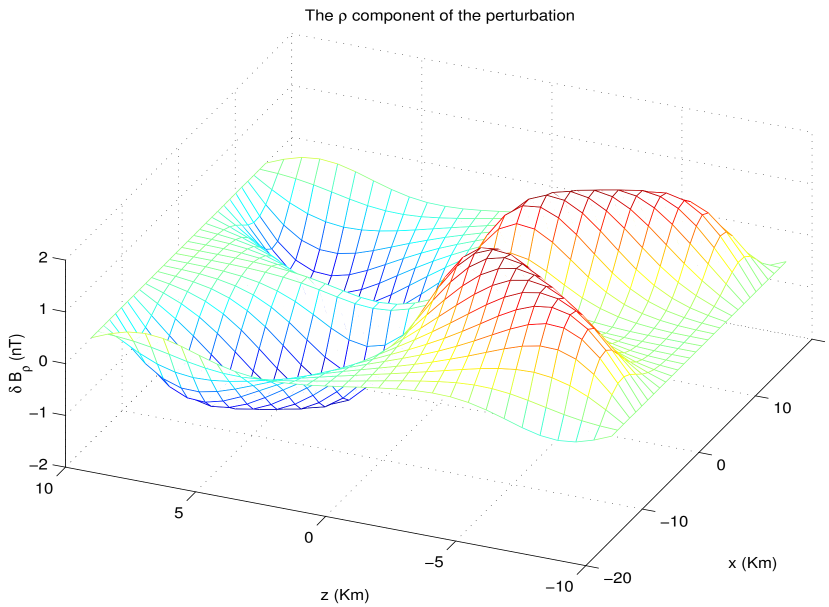


Figure 4: *The ρ component of the magnetic field perturbation. The radius of the bottle is found imposing the vanishing of the ρ component.*

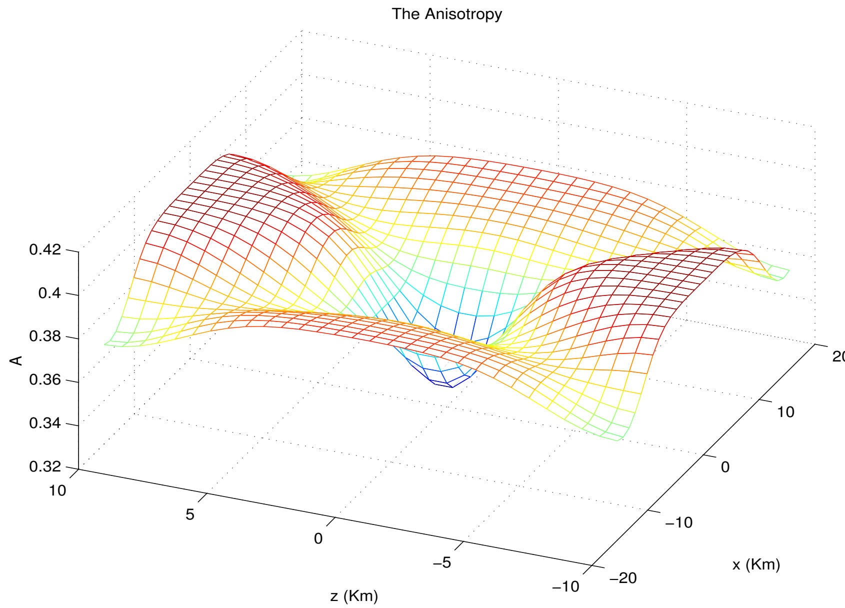


Figure 5: *The anisotropy. The unperturbed anisotropy is $A_0 = 0.4$. For this case ($A_0 < 1$) the variation of the anisotropy is in phase with the variation of the magnetic field magnitude. For $A_0 > 1$ the variations are in opposite phase.*

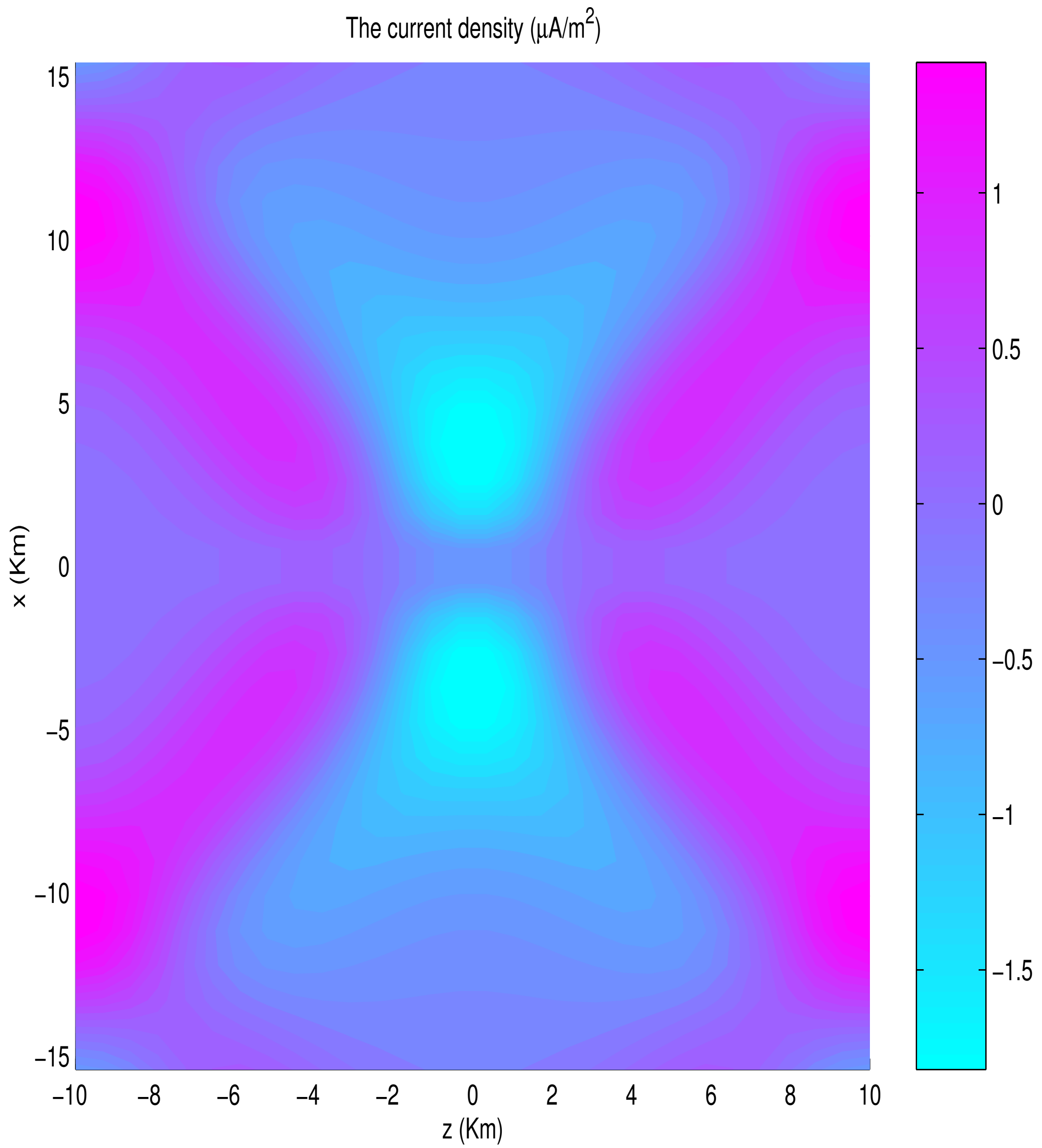


Figure 6: *The current density* $\mathbf{j}_B$ *calculated from Ampere's law. This current density is proportional to the gradient-curvature drift current density* $\mathbf{j}_d$*, actually being inside the magnetic mirror. It can be seen a central ring current bordered upon opposite sense ring currents. Comparing the Ampere current density with the gradient-drift current density we have established the instability regions in the* $(A_0, \beta_{0\perp})$*-plane.*

Figure 7: *The existence and stability domains for the mirror structures in the $(A_0, \beta_{0\perp})$-plane. In the yellow region the existence condition is not fulfilled, in the blue regions the mirror structures are unstable and in the green regions the perturbation will be dumped. For $A_0 > 1$ there is a minimum value for $\beta_{0\perp}$ ($\beta_{0\perp}^{>min} = 4$) corresponding to an anisotropy $A_0^{>} = 1.25$ for which both the instability an existence conditions are satisfied. For $A_0 < 1$ there is also a minimum value for $\beta_{0\perp}$ ($\beta_{0\perp}^{<min} = \frac{2}{3}$) for which the instability condition is satisfied.*