Reconstruction of the velocity distribution function using the numerically integrated characteristics of the Vlasov equation

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I. Abstract

In this paper we investigate the spatial variation of the velocity distribution function (VDF) of electrons and protons injected into a non-uniform distribution of the electromagnetic field. The steady state magnetic field depends only on the x coordinate, with $B_x = 0$ everywhere. The magnetic induction, $\boldsymbol{B}$, may rotate with the shear angle α and the intensity varies as in the kinetic models of tangential discontinuities. Two stationary electric field distributions are studied: (A) an uniform electric field, as in steady-state models of magnetic reconnection; (B) a non-uniform electric field that varies such that $\boldsymbol{E}\times\boldsymbol{B}/\boldsymbol{B}^2$ is conserved everywhere. First we integrate numerically the orbits of the test-particles with a fifth-order Cash-Karp Runge-Kutta method with adaptive stepsize. Then we use the Liouville theorem to propagate along these trajectories the initial VDF, specified at the left handside of the simulation domain. We inject particles from sources aligned along the x-axis, with initial velocities distributed according to a displaced Maxwellian with $V_{0x}\neq 0$. In the case of an unidirectional nonuniform magnetic field increasing with the x-coordinate the VDF expands in the space of the perpendicular velocities. When the E-field is uniform and the B-field is unidirectional but takes a null value in $x = 0$ where it changes sign (i.e. it has an antiparallel distribution) the cloud is captured in the $\boldsymbol{B} = 0$ region. The velocity distribution functions of the protons and electrons are highly anisotropic in the phase space. The reconstructed VDF for different regions in physical space show the features imprinted in the particles distribution by the acceleration mechanism. When the E-field is non-uniform (case B) and the B-field has a sheared distribution the particles cloud penetrates the discontinuity. By reconstructing VDF for different regions in space we evidenced a process of "velocity picking".

II. Numerical method

Equation to solve: the Vlasov equation for the velocity distribution function (VDF) of each component species:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{r}} + \frac{q}{m}(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})\cdot\frac{\partial f}{\partial \boldsymbol{v}} = 0$$

Method to solve: numerical integration of the characteristics (Speiser et al., 1981; Lyons and Speiser, 1982; Williams and Speiser, 1984; Curran et al., 1987; Curran and Goertz, 1989).

Approximation: rarefied and non-diamagnetic plasma cloud. (The self-consistent perturbation of the TD magnetic field due to the incoming VDF, is neglected. Electric and magnetic fields are prescribed) ; test-particles.

Numerical algorithm: fifth-order Cash-Karp Runge-Kutta method with adaptive stepsize.

Code: C language, using MPI. The simulations were done on a small linux cluster of PCs.

III. Simulation setup

Electric and magnetic field distribution

Magnetic field distribution:

$$\boldsymbol{B}(x) = \frac{\boldsymbol{B}_1}{2}\,\mathrm{erfc}\left(\frac{x}{L}\right) + \frac{\boldsymbol{B}_2}{2}\left[2-\mathrm{erfc}\left(\frac{x}{L}\right)\right]$$

where $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ represent respectively the magnetic field at $x = -\infty$ and $x = +\infty$; L is the scale length of the TD; erfc is the complementary error function.

Electric field distribution:

Two stationary E-field distributions were tested:

⇨ **Case A**: a uniform electric field given by $\boldsymbol{E}=\boldsymbol{B}_1\times\boldsymbol{V}_0 =$ const.

⇨ **Case B**: a non-uniform electric field conserving the zero order drift, $\boldsymbol{U}_E$=const., given by $\boldsymbol{E}(x)=\boldsymbol{B}(x)\times\boldsymbol{U}_E$.

Initial and reconstructed VDF

The particles were injected from sources aligned along x axis. Initial velocities are alocated according to a displaced Maxwellian distribution:

$$f(v_x, v_y, v_z) = N_0\left(\frac{m}{2\pi kT}\right)^{3/2} \mathrm{e}^{-\frac{m[(v_x-V_0)^2+v_y^2+v_z^2]}{2kT}}$$

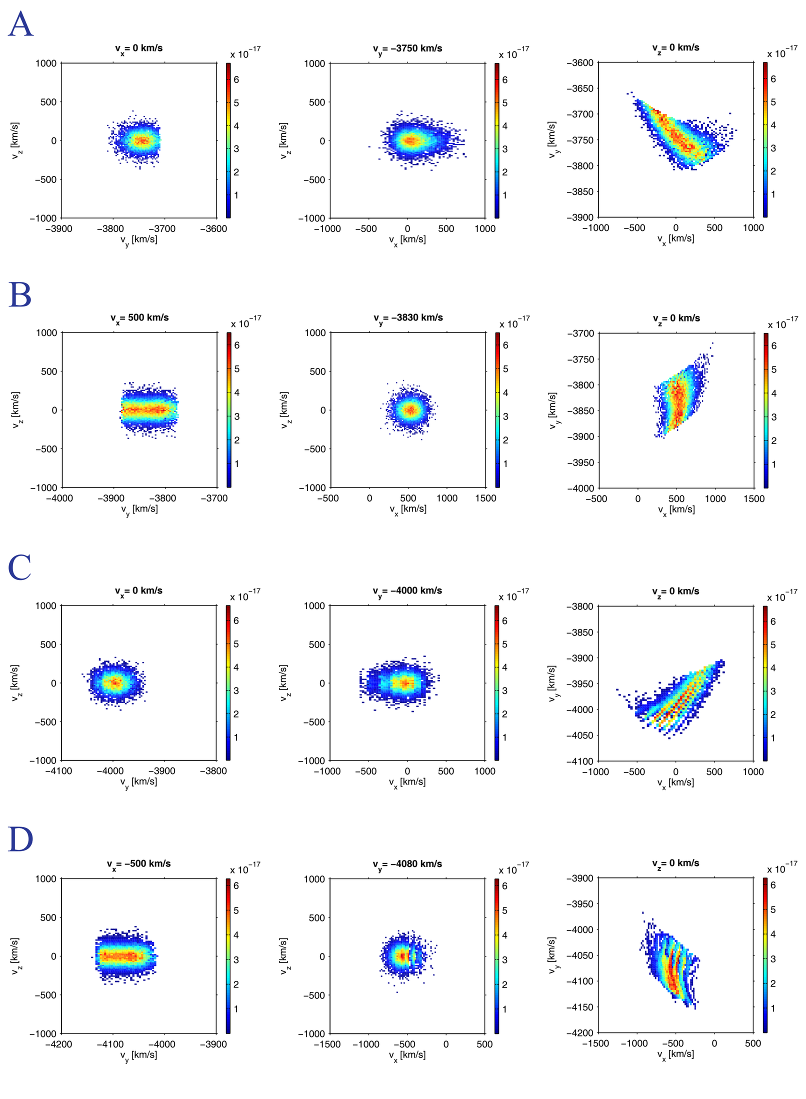
The VDF is reconstructed at different moments of time, inside/outside the TD by applying the Liouville theorem:

$$\frac{df}{dt} = 0$$

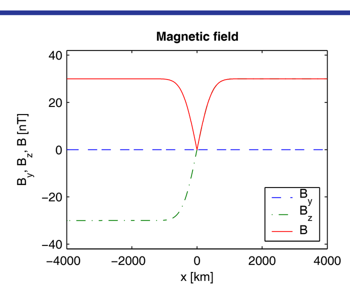
Thus in any moment later to the injection, when the particle 'a' has the position $\boldsymbol{r}_a = (x, y, z)$ and velocity $\boldsymbol{v}_a = (v_x, v_y, v_z)$, the VDF, f_a, is equal to the initial VDF, f_{a0}, that corresponds to the initial position $\boldsymbol{r}_{a0}$ and velocity $\boldsymbol{v}_{a0}$ of the particle:

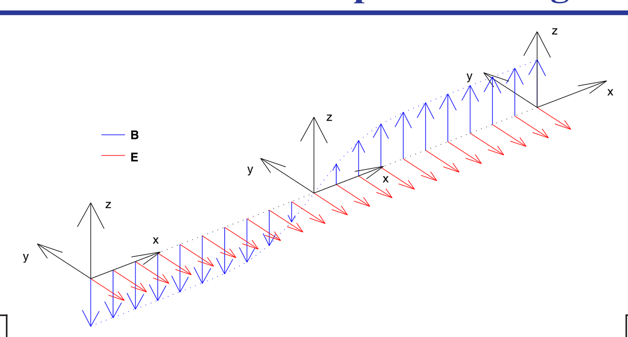
$$f_{a0}(x_0, y_0, z_0, v_{0x}, v_{0y}, v_{0z}) = f_a(x, y, z, v_x, v_y, v_z)$$

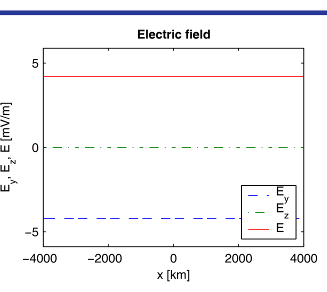
IV. Case A: Uniform electric field. Antiparallel magnetic field distribution



Panels show the VDF for protons in four different regions A, B, C, D at the end of simulation. The four rectangles A, B, C, D are defined in the figures showing the particle distributions at the end of simulation (see panels in the middle).

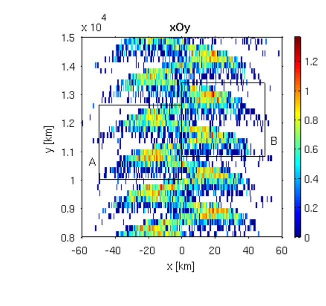




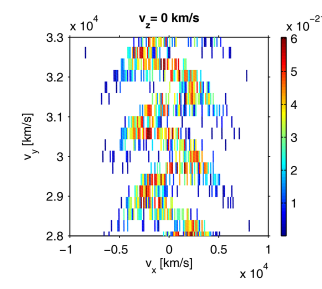


Proton case - Input parameters
20 sources, 27000 protons/source
x_0= -3380 km, dx_0= 20 km
temperature 100 eV bulk velocity 140 km/s
time of simulation 30 s

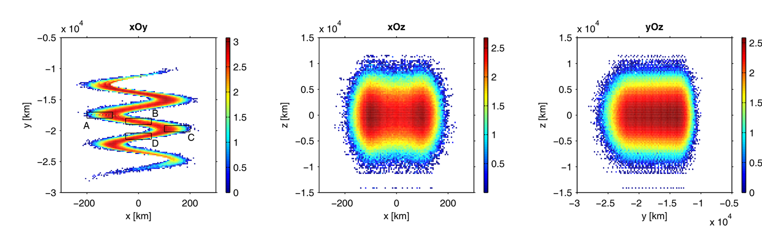
Electron case - Input parameters
21 sources, 1500 electrons/source
x_0= -3000 km, dx_0= 5 km
temperature 500 eV bulk velocity 140 km/s
time of simulation 20 s



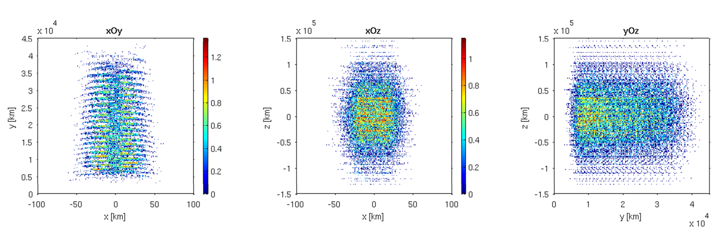
Zoom in the (x,y) plane. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.



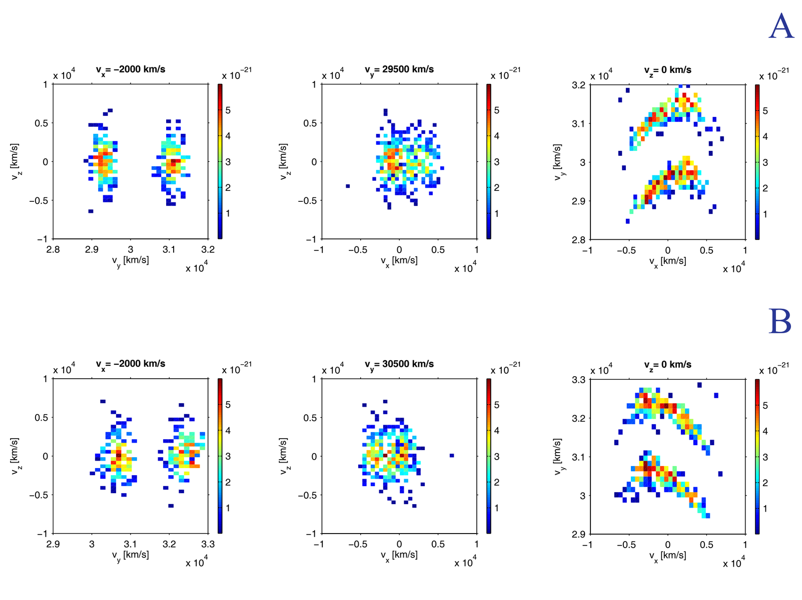
Zoom in the (v_x,v_y) plane of the velocity space.

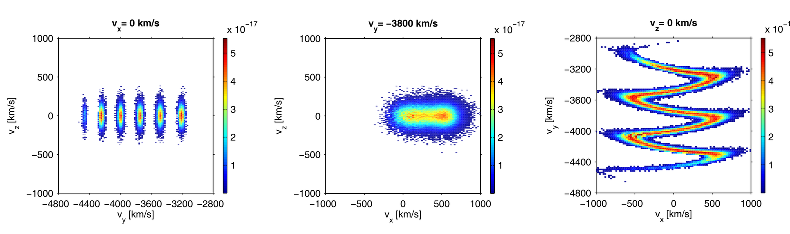


Particle distributions of all the protons at the end of simulation. The density is color coded. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.

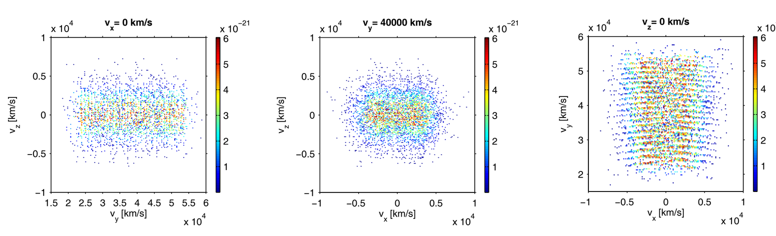


Particle distributions of all the electrons at the end of simulation. The density is color coded.





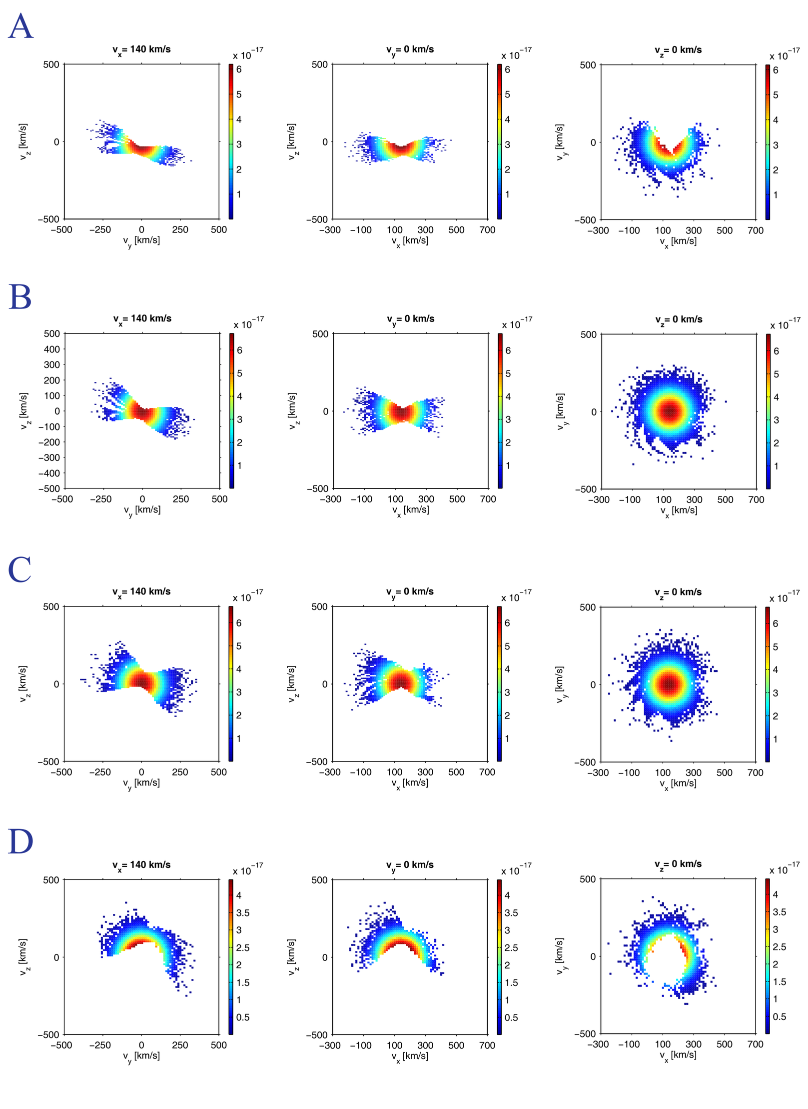
VDF for protons at the end of simulation. The VDF is reconstructed using the particles within the entire spatial domain.



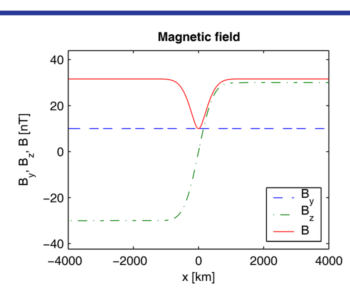
VDF for electrons at the end of simulation. The VDF is reconstructed using the particles within the entire spatial domain.

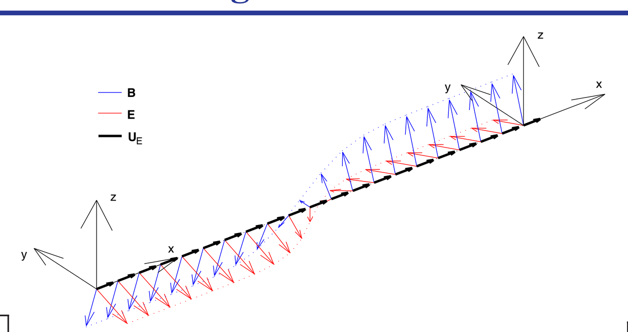
Panels show the VDF for electrons in two different regions A, B at the end of simulation. The two rectangles A, B are defined in the figures showing the particle distributions at the end of simulation (see panels above).

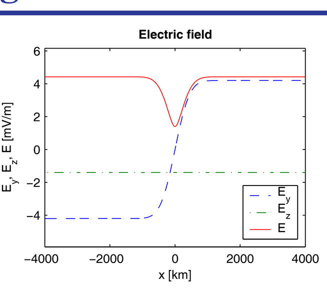
V. Case B: Non-uniform electric field conserving the zero order drift. Sheared magnetic field distribution



Panels show the VDF for protons in four different regions A, B, C, D at the end of simulation. The four rectangles A, B, C, D are defined in the figures showing the particle distributions at the end of simulation (see panels in the middle).

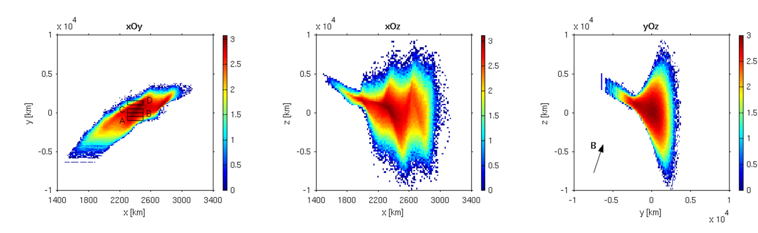




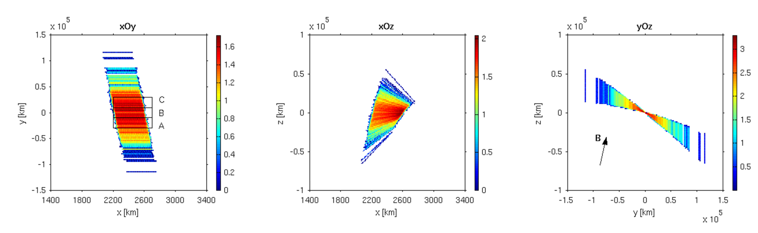


Proton case - Input parameters
20 sources, 27000 protons/source
x_0= -3380 km, dx_0= 20 km
temperature 100 eV bulk velocity 140 km/s
time of simulation 40 s

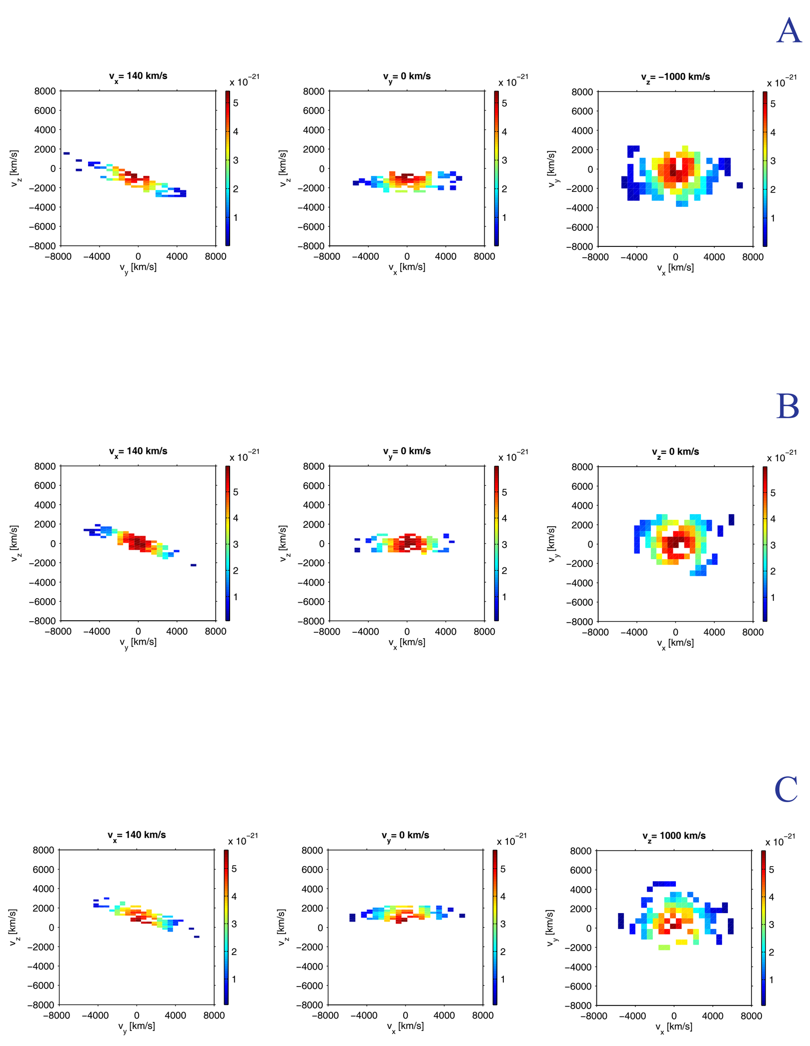
Electron case - Input parameters
77 sources, 1000 electrons/source
x_0= -3380 km, dx_0= 5 km
temperature 500 eV bulk velocity 140 km/s
time of simulation 40 s

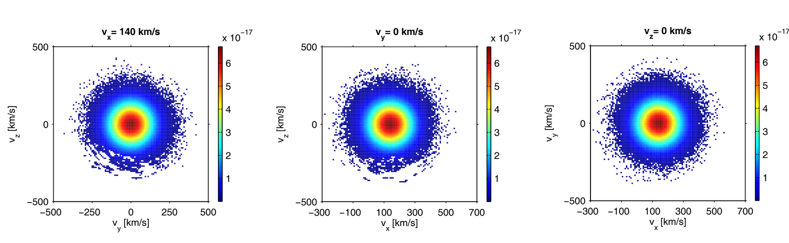


Particle distributions of all the protons at the end of simulation. The density is color coded. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.

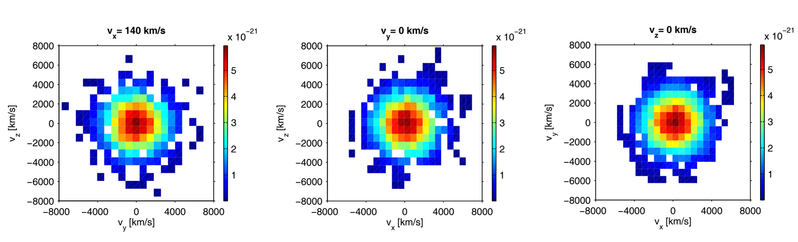


Particle distributions of all the electrons at the end of simulation. The density is color coded. Rectangles mark the space regions used to reconstruct the spatial variation of the VDF.





VDF for protons at the end of simulation. The VDF is reconstructed using the particles within the entire spatial domain.



VDF for electrons at the end of simulation. The VDF is reconstructed using the particles within the entire spatial domain.

Panels show the VDF for electrons in three different regions A, B, C at the end of simulation. The three rectangles A, B, C are defined in the figures showing the particle distributions at the end of simulation (see panels in the middle).

VI. Conclusions

The velocity distribution function (VDF) of the electrons and protons is reconstructed using the method of numerical integration of the characteristics of the stationary Vlasov equation. We provide assessment of the VDF in various space regions as a first step toward the goal of giving fully 6D distributions.

- **A**. The VDF of the protons and electrons of a jet injected into an anti-parallel magnetic field distribution and an uniform electric field is highly anisotropic in the velocity and position space. Note the iso-morphism between the distribution of the protons in space and the distribution of their velocities. Indeed, the reconstructed VDF for different regions from (x,y) plane correspond to different regions from the VDF reconstructed using all particles. That can be observed both for protons and electrons. It illustrates the process of the electrostatic acceleration in the region where **B**=0 and the electric field is constant. The structuring, both in physical and phase space has a much smaller scale for electrons than for protons. The VDF provided for the bins A,B, C and D show the features imprinted by the acceleration mechanism in the protons and electrons distribution . Our results provide possible tests for experimental data obtained for discontinuities in laboratory and space plasma.

- **B**. If **E** remains everywhere perpendicular to **B** and has a magnitude that satisfies the conservation of the zero order drift, the jet penetrates the TD and move across it into the right hand side. The electrons penetrate along the same distance as the protons. The shear angle of the magnetic field produces a deformation of the shape of the jet. By reconstructing VDF for different regions in space we evidenced a process of "velocity picking". That can be seen both in the case of protons and electrons. An interesting feature is the "transition" of the proton VDF from position A to position D. It reminds similar results ("velocity filtering") obtained with 2D kinetic models of plasma boundary layers (Echim and Lemaire, Phys. Rev E, 2005).

VII. References

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