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THE P- WAVE THRESHOLD EFFECT AND QUASIRESONANT SCATTERING

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The p-wave threshold effect is described in terms of the Reduced Scattering Matrix. The relationship of this approach to previous theoretical threshold models is established. We prove that this phenomenon is related to the reaction- mechanism of Quasiresonant Scattering: a Single Particle Neutron Threshold State and Direct Interaction Coupling to open observed channels. Spectroscopic aspects of the threshold effect, both with respect to the magnitude and microstructure, are discussed in terms of the Neutron Strength Function.

Keywords: Threshold phenomena; direct nuclear reactions; neutron single particle resonance.

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1. Introduction

The threshold effects in a multichannel reaction system originate in the conservation of flux. The opening of a new (threshold) reaction channel results into a redistribution of flux in complementary open channels, i.e. changes of their reaction cross-sections. The modification of the reaction cross-sections of open channels, due to opening of a threshold channel, is called threshold effect.

In a hydrodynamical description, the threshold effect's magnitude should be proportional to the amount of the flux absorbed in the threshold channel. If the threshold channel has no barriers, i.e. for a s- wave neutron, a violent transfer of the flux is produced, resulting in a specific threshold effect, called Threshold Cusp. Because the flux in the s-wave neutron threshold channel is proportional to the channel wave number, (i.e. $\sim \sqrt{E}$), one obtains that the Threshold Cusp has an infinite energy derivative at zero (threshold) energy; from this behaviour results the Cusp denomination. The Threshold Cusp is considered as an universal phenomenon, appearing whenever a new *s*-wave neutron channel opens. For higher partial waves, the centrifugal barrier inhibits the flux transfer between the neutron threshold channel and the observed open ones and, accordingly, will result in a smaller threshold effect.

The Wigner-Breit-Baz Cusp Theory[1,2,3] predicts nuclear threshold effects for the *s*-wave neutron opening channel. *S*-wave threshold effects were observed in some Reactions on Light Nuclei; an extensive review on threshold effects in proton scattering on 1-*p* Shell nuclei was presented in Ref. 4. As a matter of fact, the corresponding experimental data were not interpreted as evidence for a genuine Threshold Cusp, but rather analyzed in terms of a multiparametric method developed by same authors. On the other hand, *p*-wave threshold effects have been observed in some Low Energy Nuclear Reactions[5,6], becoming subject of many experimental and theoretical investigations, (see Ref. 7 and references therein). Recently, the spectroscopical aspects of *p*-wave threshold effects (relation to Neutron Strength Function) have been discussed[8].

The Threshold Physics field appears to be more rich in phenomena than in schematic descriptions. Understanding of the physical aspects of the *p*-wave threshold effect should be of interest not only in Low Energy Nuclear Physics[4], but also in the other Quantum Scattering fields as Atomic Collisions (*e.g.* the dipole induced threshold effects in electron-molecule collisions, see Ref. 9) and High Energy Physics (*e.g.* the *p*-wave near-threshold state in $p\bar{p}- > \Lambda\bar{\Lambda}$ reaction, see Ref. 10).

The aim of the present contribution is to develop a formal treatment, covering, in an unitary view, previous descriptions and conclusions. Relations of the *p*-wave threshold effect to Nuclear Reaction Mechanisms (Quasi-Resonant Scattering) as well as to Nuclear Structure Physics (Spectroscopy of Neutron Threshold State) are obtained.

2. Physical Aspects of the P-Wave Threshold Effect

The first experimental evidence of a *p*-wave threshold effect in Low Energy Nuclear Physics comes from deuteron stripping reactions on A $\sim$ 90 mass target nuclei. A sharp anomaly was found in the excitation functions of (d, p_0) reactions near the threshold of neutron analogue channel $(d, \bar{n})$, see Ref. 5. The threshold anomaly was also observed in deuteron polarization studies[6], resulting in well-defined experimental characteristics.

The anomaly occurs mainly as a dip in cross-section data and as resonant or S-shape form in analyzing power excitation functions; its width between half-way points is approximative 0.7 MeV. The anomaly's centre is not always at $(d, \bar{n})$ threshold but rather in a region of 0.1 - 0.2 MeV. The anomaly's magnitude is largest for A $\sim$ 90 mass nuclei, becoming smaller both for larger or smaller mass nuclei, as for example A $\sim$ 80. An illustrative example for the deuteron stripping

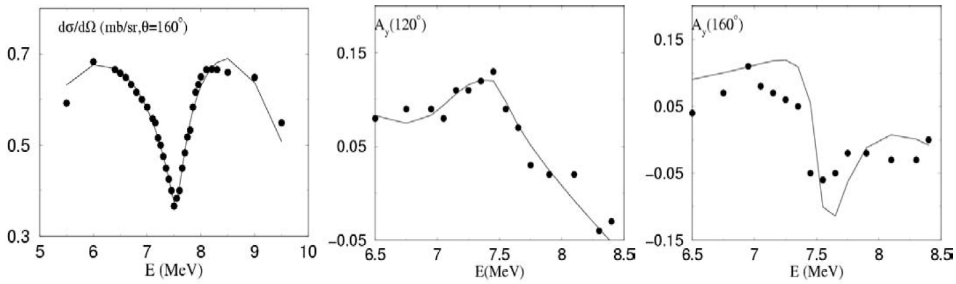

Fig. 1. The deuteron stripping threshold anomaly in the cross-section and analyzing power data. The effect is described (solid line) as the interference between the Direct Reaction Mechanism and the p-wave Neutron Threshold Single Particle Resonance.

threshold anomaly, (experimental data on $^{88}Sr(d,\vec{p})^{89}Sr$ reaction, from Ref. 7), is presented in the Fig. 1.

The anomaly occurs only at neutron analogue thresholds; it is an experimental proof of isospin coupling of the analogue channels. An Isospin Coupled Channels Born Approximation Model was devised in order to describe this phenomenon[11]. It was found numerically, with Optical Model calculations[12], that the 3-p wave neutron wave function has a strong energy dependence near threshold, for A $\sim$ 90 mass nuclei; there exists a p-wave neutron single particle state at zero energy. This neutron state moves far away from threshold (zero energy) for other mass nuclei. The Coupled Channel Born Approximation Model (CCBA) does reproduce the threshold anomaly only in cross-section data.

The Cusp Theory, both in its original or its energy-averaged extension, cannot account for this p-wave threshold effect, both for experimental and theoretical reasons, see Refs. 13, 14, 15.

Lane has proposed[16] a very physical model for understanding of the p-wave threshold anomaly. This model is based on two experimental facts: (1) there exists, for A $\sim$ 90 mass nuclei, a p-wave neutron single particle state at zero-energy, and, (2) isospin coupling of the exit neutron and proton analogue channels. The near-threshold p- wave neutron-single particle resonance, ($l = 1, j = 3/2, 1/2; E_j \sim 0; L_1 = S_1 + iP_1$ - p- wave neutron channel logarithmic derivative; b - boundary condition at channel radius a; γ_n^2 - neutron reduced width; Γ - resonance's total width), is reflected, by isospin coupling as a resonant term in the proton channel S-Matrix elements:

$$S_{dp} = B_{dp} + \Sigma_{j=3/2,1/2}\alpha_j(\hbar^2/ma^2)/[E_j - E - (S_1 + iP_1 - b)\gamma_n^2 - i\Gamma/2]$$

The background S- Matrix elements, B_{dp} are generated by DWBA, $(\hbar^2/ma^2)$ is Wigner unit of the reduced width; the coupling constants α_j , related to isospin coupling strengths, are free parameters of this model. Although, formal equivalent to a single level formula, it is specific for the threshold phenomena due to the strong energy dependence of the neutron channel logarithmic derivative L_n near zero-energy. This energy dependence results into a distorsion of the resonance shape, esp. for s- and p- waves. The distorsion of the resonance's shape can be viewed as an asymmetric compression of the energy scale in the threshold range. The compression factor[17], $\beta_l = 1/[1 + \gamma_n^2 dS_l/dE]$, which is subunitary, results into a shift to the threshold of the resonance's position $E_j \to \bar{E}_j = \beta_l[E_j - E - (S_1(0) - b)\gamma_n^2]$ as well as into a width compression ,$\Gamma \to \bar{\Gamma} = \beta_l[\Gamma + 2P_l\gamma_n^2]$. For $\beta \to 0$, the resonance is shifted just to zero (threshold) energy. A large reduced width is essential in obtaining small values of β. For a compression factor which can reproduce the anomaly's width ~ 0.7 MeV, a reduced width γ_n^2 exceeding several Wigner units is necessary. (Such a large value of the reduced width can be obtained both from the Shell Model and Optical Model calculations or from an empirical formula, relating the width's increase to the nucleus surface's diffuseness) . By using the Lane's model, the p-wave threshold anomaly, both in cross-section and analyzing power, was reproduced[7].

Another aspect of the Lane Model is an empirical Q- classification of the 3-p wave threshold anomalies. It was proved[18], that the Q- classification is a kinematical one, according to transfered linear momentum and transfered angular momentum in the (d, p_0) reaction. The threshold anomaly results from interference of the (d, p_0) background and the threshold- resonance terms: different forms of the background result into different shapes of the anomaly.

Another classification of the anomalies, related to their microstructure, was proposed too[18]: single dip and fluctuating-type anomalies. All fluctuating -type anomalies do correspond to the residual nuclei in 0^+ ground states, while the spin of residual nucleus' state is non-zero for the single dip anomalies. Therefore the nature of the residual state could be another criterion for the anomalies' classification. It was proved[18], that the anomaly is related to the Neutron Strength Function. The single particle state can be distributed in many ways since the system (neutron + a non-zero spin state) can be coupled to various angular momenta. In this way the configuration can be fragmented by residual interactions into many components; if they are uniform distributed, one obtains a Breit-Wigner form. If the corresponding components are not uniformly spread into each other, then there occur fluctuations of the Breit-Wigner line shape. This can result in fluctuating - type' anomalies if the residual state (and its analogue) has a relatively simple structure, e.g. even-even nuclei in their ground state. G.E. Brown[19] has predicted fluctuations of the Neutron Strength Function, depending on the nature of the involved nuclear states; these "Intermediate Structures" are displayed as microstructures of the Neutron Strength Function. A similar interpretation was developed by Lane[20]. The analysis[18,20,21] of the p-wave threshold anomaly does sustain the "Intermediate Structure" interpretation for the fluctuations observed in the excitation functions of this threshold effect.

The Lane's Model predicts even a more pronounced threshold effects for the s-waves, which were not observed experimentally, (A $\sim$ 50, 3-s; A $\sim$ 140, 4-s). This absence was explained[22], in terms of the Neutron Strength Function, too. The S-wave Neutron Strength Function, defined according to the S-Matrix, is depressed

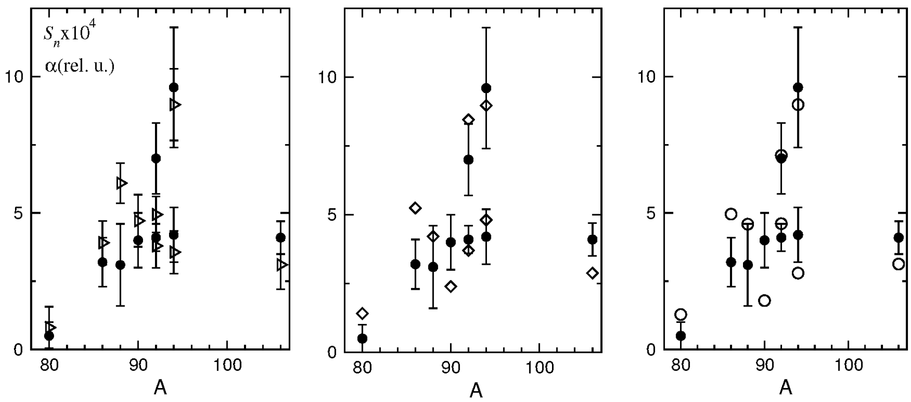

Fig. 2. Spectroscopic strengths (α) of the p-wave threshold effects obtained from: an empirical ($\triangleright$) and different computational procedures ($\diamond$ and $\circ$). The threshold effects follow the mass dependence (A) of the S_n Neutron Strength Function data ($\bullet$).

by an order of magnitude as compared to R-Matrix Strength Function and has no resonance dependence[23]. Due to peculiar behaviour of the s-wave neutron virtual states, perhaps they cannot induce appreciable s-wave threshold effects.

A quantitative analysis of the relation between the p-wave threshold effect and the p-wave Neutron Strength Function has been done[8]. The analysis took into account all experimental data on threshold effects in A = 80 - 110 mass-region, both cross-section and analyzing power. According to this analysis, the magnitude of the p-wave threshold effect is proportional to the Neutron Strength Function, in their mass dependence, see Fig. 2. This analysis does evince the spectroscopic properties of the threshold effects; the strength of the threshold effect (in open channel) is proportional to the spectroscopic strength of the Neutron Threshold State (from opening channel). It is a proof that the threshold effects depend not only on the penetration factors, as in the Cusp Theory, but also on the spectroscopic factor of the ancestral neutron state in opening channel.

3. Reduced S-Matrix Approach to Threshold Effects

The Nuclear Physics of seventies was confronted with several problems related to the threshold phenomena: (1) the threshold effects are rare, appearing under restrictive conditions, (2) the threshold effects are apparently peculiar, differing each from other in many aspects, (3) special reaction models were devised to describe different threshold effects or even same threshold anomaly. For example, the p-wave threshold effect under study was approached in terms of Cusp Theory, Coupled Channel Born Approximation and Lane Model.

A theory of threshold effects has to fulfill corresponding requirements: (1) to understand necessary conditions for experimental observation of a threshold effect, (2) to provide an unitary description of different types of threshold effects, (3) to include previous theoretical threshold models as limit cases.

Formally, the problem of the Threshold Effects can be viewed as a Scattering Problem in the truncated space of open (observed, retained) channels; one has to take into account the coupling of open channels to the threshold (invisible, eliminated) channel. The usual approach to multichannel scattering problems in truncated space of channels is either Reduced R-(K-) Matrix[24], or Effective Hamiltonian (Projector Method)[25]. Since the Scattering Matrix is primary object of the Scattering Theory, the concept of "reduced" or "effective" operator should be extended to the S- Matrix.

The S-Matrix contains the complete Reaction Dynamics and it is the primary object of the Scattering Theory; it is directly related to experimental observables as cross-section or polarizations. Developing the Reduced S- Matrix approach to the threshold phenomena[26], one expects to cover, in an unitary view, some previous descriptions and conclusions. The Reduced S-Matrix approach aims to account for the known physical aspects of the p-wave threshold effect.

Consider the multichannel system of N open (retained) channels , decoupled from the threshold (unobserved, eliminated) channel n. The "bare" independent open channels are described by the unitary Scattering Matrix $S^0_N = ||S^0_{ab}||$. By coupling the threshold channel $n = N+1$, to N open ones, via S_{na} matrix elements, one obtains the Reduced Scattering Matrix $S_N = ||S_{ab}||$ for the retained channels[26]; it includes both bare S^0_N Scattering Matrix and the effect ΔS, of eliminated channel

$$S_{ab} = S^0_{ab} + \Delta S_{ab} = S^0_{ab} + S_{an}(1 + S_{nn})^{-1} S_{nb}$$

The Reduced Scattering Matrix does include as limit case the Cusp Theory[3], $S_{nn} \to 1$. The formal merits of this method are: it is valid both near and far away from threshold, it is valid both for potential and resonant scattering and it is valid even for threshold channel with barrier, as e.g. a p-wave one. The physical merit of the method is it does establish a relation between the threshold effects, ΔS, and the reaction mechanism in the threshold channel, via S_{nn} -matrix element. Different reaction mechanisms will result in different types of threshold anomalies.

The Threshold Cusp is related to the nonresonant Potential Scattering. In zero-energy limit, $S_{nn} \to 1$, the Reduced S-Matrix results in the Cusp formula, $\Delta S_{ab} = 1/2 S_{an} S_{nb}$. The flux transfer involved in a Cusp Effect is essentially determined by the penetration factors of the threshold channel S-Matrix elements.

A Compound Nucleus resonance (π), located in neutron threshold vicinity, $|E_\pi - E_n| < \Gamma_\pi$, and decaying preferentially in the neutron threshold channel, $\Gamma_\pi \sim \Gamma_{\pi n}$, induces a non-negligible threshold effect for s-wave only. Most of the threshold effects observed with light nuclei, (1-p Shell), do belong to this class[4]. The flux transfer to and from the neutron threshold channel is controlled not only by the penetration factors, but also by the spectroscopic neutron reduced width; the reduced width is primary factor governing the flux leakage from the compound nucleus to the channels[27]. However the threshold compound nucleus resonance cannot account for a p- wave threshold effect.

A p-wave threshold effect does require (1) Resonant energy dependence of the threshold channel related S- Matrix elements, S_{nn}, S_{an} and S_{nb}, and (2) Direct Interaction in open channels, S^0_{ab} - (monotone energy dependence). Otherwise, the effective term of the Reduced S- Matrix, ΔS, goes to zero in all threshold range. In the following we will approach the p-wave threshold effect problem in different formal ways, all converging to the same physical conclusion: a non-negligible p-wave threshold effect involves (1) a Neutron Threshold Single Particle Resonance and (2) its Direct Interaction Coupling to open channels[26].

The two formal conditions could be realized in terms of the Final-State Interaction[28]. The final fragments, neutron and corresponding residual nucleus, have an interaction producing a resonance at zero energy; the Jost function has, then, a zero in the complex k-wave plane, just below the real axis, near origin. The S-Matrix elements, having in denominator the Jost function, are strongly enhanced; this is Final (or Initial) -State Interaction. This refers in our case to the S-matrix elements S_{nn}, S_{an} related to the n-channel only. On the other hand, it is required that the potential responsible for transition between channels should be assumed weak, (Direct Interaction transitions). The forces producing reaction are weak, excepting interaction in the (threshold) channel where the Final State Interaction does produce a resonance. The Final State Interaction is effective mainly at low energies, where one-channel resonances are produced by centrifugal barrier effects[28].

In the R-Matrix theory, usually, one considers only compound system multichannel resonances described by poles of all R- Matrix elements. The multichannel resonances originating in single particle states are described, in this theory, by a perturbative approach developed by Bloch, see Ref. 17. By means of perturbative residual interactions, the single (-channel) -particle resonance of non-perturbated (independent particles) system is subject to transitions to actual states of compound system and to couplings to other reaction channels. The R- Matrix of reaction system becomes a series of resonant terms, collected, with statistical assumptions, in a Single Particle Resonance formula. Its total width comprises an additional spreading component related to the flux lose into actual states and the other reaction channels. We approach this aspect in more quantitative way by using the Bloch's procedure for describing the Single Particle Resonances in R- Matrix Theory[17]. Implementing this procedure into the Reduced S-Matrix one obtains, in second-order perturbation theory,

$$
\begin{aligned}
\Delta S_{ab} &= \mathcal{P}_a^{1/2}\gamma_{\pi n}\gamma_{\pi n}\mathcal{P}_b^{1/2}[E_\pi - \Delta + i(\Gamma + G)]^{-1} \\
\mathcal{P}_a^{1/2} &= \Sigma_b P_a^{1/2}(S_N^0 + 1)_{ab} R^0_{bb} V_{bn} \\
G &= (E_\pi - E)^2/(P_n \gamma^2_{\pi n}) \\
-\Delta + i\Gamma &= -\gamma^2_{\pi n}\Sigma_{ab}(P_a^{1/2} R^0_{aa} V_{an}, (S_N^0 + 1)_{ab} P_b^{1/2} R^0_{bb} V_{bn})
\end{aligned}
$$

with R^0_{aa} describing uncoupled (independent) open channel a, V_{an} - the Direct Interaction coupling between open channel a and threshold one n, R^0_{nn}–R-Matrix element of the Neutron Single Particle Resonance π, $R^0_{nn} = \gamma_{\pi n}\gamma_{\pi n}/(E_\pi - E)$, and $\gamma^2_{\pi n}$- its neutron reduced width. The width Γ is positive due to the unitarity of the

S_N^0 - Scattering Matrix. The additional width term G does increase the "Escape width", thus inforcing the doorway aspects of the Neutron Threshold State. (If applied to s-wave case, one obtains a very broad effect, mixing up with background, while Lane formula does predict a very strong s-wave threshold effect). One has to remark, however, the artificial aspect of this term, resulting from exact coincidence of the Neutron Single Particle State with threshold. Anyway, this description displays explicitly the role played by Direct Interaction Coupling of the Threshold channel to open observed ones.

Another possible description of the Single Particle Resonance from Neutron Threshold Channel could be in terms of the Neutron Channel Logarithmic Derivative. In order to deal explicitly with the Neutron Channel Logarithmic Derivative one has to work with the Collision Matrix U, parametrized in R-Matrix terms and Logarithmic Derivative $L = S + iP$, (S- Shift factor, P- Penetration factor), see Ref. 17,

$$U = 1 + 2iP^{1/2}(R^{-1} - L)^{-1}P^{1/2}$$

The Reduced Collision Matrix term is now dependent on n-threshold channel phase, ω_n, $e^{2i\omega_n} = -L_n^*/L_n$, namely

$$\Delta U_{ab} = U_{an}[e^{2i\omega_n} + U_{nn}]^{-1}U_{nb}$$

For s-wave, $L_n = iP_n$, or for specific boundary condition, $e^{2i\omega_n} = 1$, it is identical to the Reduced S-Matrix term ΔS. In the limit $U_n \rightarrow 1$, it reduces, up to a phase, to a Generalized Cusp formula[29],

$$\Delta U_{ab} = (1/2)U_{an}U_{nb}(L_n/iP_n)$$

(This formula was used in description of a threshold anomaly in a transfer reaction on a light target nucleus, provided L_n has a resonance behaviour.) The Kapur-Peierls Matrix, $(R^{-1} - L)^{-1}$ has to be rewritten in order to separate resonant L_n term, from the rest included in "background" (β). One obtains

$$\Delta U_{ab} = (1/2)U_{an}^\beta U_{nb}^\beta (L_n/iP_n)(1 - L_n R_{nn}^\beta)^{-1} = 2iP_a^{1/2}R_{an}^\beta R_{nb}^\beta P_b^{1/2}(1/L_n - R_{nn}^\beta)^{-1}$$

A resonant term in the logarithmic derivative, describing a single-channel resonance, $L_n = L_n^\beta/[E_n - E - i\Gamma_n], (L_n^\beta$- monotone), will result in a resonant term in the Reduced Collision Matrix, of the form, $L_n^\beta/[E_n - E - L_n^\beta R_{nn}^\beta - i\Gamma_n]$. The background Kapur-Peierls term, R_{nn}^β, will result into a resonance's shift as well as in the width increase. The condition $(1/L_n - R_{nn}^\beta)$ =0 has also another physical interpretation which will be discussed below, in a different framework.

A straightforward interpretation of the p-wave threshold effect is in terms of the Quasiresonance. The Quasiresonant Scattering[30], consists from a Single Channel Resonance preceded and/or followed by Direct Transitions to other reaction channels. Such phenomena are evinced as resonances in some reaction channels; other competing reaction channels show Direct Interaction monotone energy dependence.

A K- Matrix description is below outlined,

$$S = -1 + 2i[K + i \cdot 1]^{-1}$$
$$K = K^{\beta} + K^{\rho}$$
$$K^{\rho} = \Sigma_{\lambda}(\gamma_{\lambda} * \gamma_{\lambda})[E_{\lambda} - E]^{-1}$$

with labels β and ρ referring to "background" and "resonant", respectively. If this formalism is specialized to a Single Channel Resonance, (π), ($\gamma_{\pi n} \neq 0$; all other $\gamma_{\pi a} = 0$), then the Scattering Matrix becomes

$$S_{ab} = S^{\beta}_{ab} - 2iT^{\beta}_{an}\gamma^2_{\pi n}T^{\beta}_{nb}[E_{\pi} - E + Re\ T^{\beta}_{nn}\gamma^2_{\pi n} - i(1 - \Sigma_l|T^{\beta}_{ln}|^2)\gamma^2_{\pi n}]^{-1}$$

with the Transition Matrix defined by $S^{\beta} = 1 + 2iT^{\beta}$ and index l running over all channels either open (a) or threshold (n). The magnitude of the Quasiresonant Scattering process is proportional both to the single channel resonance reduced width $\gamma^2_{\pi n}$ and to the channel coupling strengths, $T^{\beta}_{an}T^{\beta}_{nb}$. An interesting property of the Quasiresonant Scattering is the "Direct Compression" of the width, due to channel couplings, $\gamma^2_{\pi n} \rightarrow (1 - \Sigma_l|T^{\beta}_{ln}|^2)\gamma^2_{\pi n}$. This result could be related to the "Channel Coupling Pole"[31], observed in numerical experiments for multichannel reactions; it does appear for strong channel coupling interaction. It is considered that the Channel Coupling Pole originates in distant poles, (located at infinity in complex energy or wave planes, when channel couplings tend to zero), which are driven to physical region when channel coupling becomes strong.

The compression of the resonance width persists even in case of a single channel, being related to background scattering $S^{\beta}_{nn} = e^{2i\delta_n}$, ($\delta_n$ single channel background scattering phase shift). For the single channel scattering, the resonance's width is compressed to $\gamma^2_{\pi_{nn}} cos^2\delta_n$. For no-background scattering (δ_n=0) there is no compression. The 'strong' background scattering, (*e.g.* echo-descending phase shift δ_n multiple of $\pi/2$) results into resonance's extinction. This extinction effect has not yet been evinced.

In next step we discuss the relationships of the Quasiresonant Approach to p-wave threshold effect to Lane and CCBA Models. The two formulae (the Quasiresonance and Lane ones) are similar, however not identical. Firstly the anomaly's strength α has a definite physical meaning in the Quasiresonant Approach; the effect's magnitude is proportional to the transition strengths between the threshold and open observed channels. Secondly, the anomaly's strength is proportional to the neutron single particle reduced width; the effect is maximum in the single particle limit. The α parameters are not more free ones but rather subject of physical constraints[8] : the interchannel transition strengths play the role of generalized penetration factors and the flux leakage from resonance to n-channel is related to the neutron reduced width. The Resonance's Direct Compression is different from Lane's compression near threshold due to non-linearity of energy scale.

The CCBA formulae could be obtained from the Quasiresonance formula by assuming the exact Isospin Symmetry. In this limit, T^{β}_{np} transition matrix element between analogue neutron and proton channels is a constant ($\sqrt{2T}$; T- Multiplet's Isospin). Secondly, the resonance denominator could be absorbed in n-channel wave function from T^{β}_{dn} matrix element. Thus one obtains the CCBA formulae for (d,p) and $(d,\bar{n})$ analogue reactions. Ad-hoc, the analogue neutron threshold is fixed by physical considerations. However the first condition, conflicting the Isospin Symmetry breaking, does assume, in process of derivation, identical thresholds for both analogue channels, thus violating the Wigner threshold law for neutron channel.

All these approaches do emphasize the role of zero-energy Neutron Single Particle Resonance in producing p- wave threshold effects in open channels. There is another approach to Single Particle Resonances in Multichannel Reaction Systems, encountered both in the Nuclear Scattering and Atomic Collisions[32]. In order to have a deeper insight on Single Particle Resonances in Multichannel Reactions, we develop now the subject in terms of a formalism based on the Reduced Collision Matrix, both for quasistationary and bound single particle states in threshold channel.

4. On Single Particle Resonances in Multichannel Reactions

The resonances in multichannel scattering originate either in multiparticle excitations of an inner core or from excitation of far-away located states; they are called "inner resonances" and "channel resonances", respectively. In Lane's approach[33], both resonances are described in similar ways; the channel resonances are represented by a meromorphic term added to inner resonances of the genuine R-Matrix. The "inner" and "channel" resonances do correspond to "compound nucleus"- and to "single particle"-resonances, respectively.

In the R-Matrix Theory the inner multichannel resonances are described by standard R- Matrix techniques and the resonances originating in single particle states are approached by the Bloch's perturbative method. In the present approach the inner multichannel resonances are described by the R-Matrix while the channel resonances are related to the L_n logarithmic derivative, also to the n-channel Reduced R-Matrix, i.e. to the Reduced Collision Matrix. In the Reduced Collision-Matrix framework one has to separate the effects originating in the two groups of channels (retained and eliminated) in order to have a physical insight on phenomena developing in the n-closed channel.

In order to deal explicitly with the Neutron Channel Logarithmic Derivative one has to work with the Collision Matrix U, parametrized in R-Matrix terms and Logarithmic Derivative, $L = S + iP$, (S- Shift factor, P- Penetration factor), see Ref. 17,

$$U_N = 1 - 2iP_N^{1/2} L_N^{-1} P_N^{1/2} + 2iP_N^{1/2} L_N^{-1} (L^{-1} - R)_N^{-1} L_N^{-1} P_N^{1/2}$$

The Reduced Collision Matrix U_N refers to the retained (N) channels, but by taking into account the effect of the eliminated (n)-channel. It consists from the

Collision Matrix U_N^0 which describes the "bare" retained channels (N), uncoupled to eliminated (n) channel,

$$U_N^0 = 1 - 2iP_N^{1/2}L_N^{-1}P_N^{1/2} + 2iP_N^{1/2}L_N^{-1}(L_N^{-1} - R_N)^{-1}L_N^{-1}P_N^{1/2}$$

and from a term ΔU_N describing this coupling. The Reduced Collision Matrix evaluated above n-threshold is, see Ref. 34,

$$\begin{aligned} U_N^> &= U_N^0 + \Delta U_N^> \\ \Delta U_N^> &= U_{Nn}^>(-L_n^*/L_n + U_{nn}^>)^{-1}U_{nN}^> \end{aligned}$$

For boundary conditions used by Lane[33] , the L_n^*/L_n modulus one quantity is expressed in terms of the Coulombian hard-sphere phase-shifts, $-L_n^*/L_n = \exp(2i\Phi_n)$, and one obtains the Reduced S- Matrix result[26],

$$S_N^> = S_N^0 + S_{Nn}^>(1 + S_{nn}^>)^{-1}S_{nN}^>$$

By the Reduced Collision Matrix procedures one can relate two reaction systems with same internal dynamics, differing only in interactions in eliminated channel, ($L^>$ and $L^<$).

$$\begin{aligned} U_N^< &= U_N^0 + \Delta U_N^< \\ \Delta U_N^< &= U_{Nn}^> \frac{1}{-(L_n^>)^*/(L_n^>) + U_{nn}^>} U_{nN}^> \frac{1/L_n^> - \mathcal{R}_{nn}}{1/L_n^< - \mathcal{R}_{nn}} \end{aligned}$$

For example, the Reduced Collision Matrix for negative energies (closed channel) is expressed in terms of positive energy quantities ($U^>$, $L^>$) and also of quantities specifying eliminated closed channel (logarithmic derivative $L_n^<$ and the Reduced R-Matrix element $\mathcal{R}_{nn}$)

$$\begin{aligned} \mathcal{R}_{nn} &= R_{nn} - R_{nN}(R_{NN} - L_N^{-1})^{-1}R_{Nn} \\ U_{nn}^> &= 1 - 2iP_nL_n^{-1} + 2iP_nL_n^{-2}(L_n^{-1} - \mathcal{R}_{nn})^{-1} \end{aligned}$$

The effective term ΔU_N of Reduced Collision Matrix, valid both below and above n-threshold, is

$$\begin{aligned} \Delta U_N = \frac{1}{2i}&(U_N^0 - L_N^* L_N^{-1})P_N^{-1/2}L_N \\ &R_{Nn}(L_n^{-1} - \mathcal{R}_{nn})^{-1}R_{nN} \\ &L_N P_N^{-1/2}(U_N^0 - L_N^* L_N^{-1}) \end{aligned}$$

where for the ΔU_N superscripts $>$ or $<$ one has to insert the corresponding logarithmic derivatives $L_n^>$ or $L_n^<$, respectively.

The K- Matrix form of the effective term in the Collision Matrix can be obtained via formal equivalence of K- Matrix and R- Matrix with natural boundary conditions[24], $B = S^>$, i.e. $K = P^{1/2}R_SP^{1/2}$, $L_N = iP_N$, $L_n^> = iP_n$, $L_n^< = S_n^< - B_n = S_n^< - S_n^> = -\Delta S_n$ and $(L_n^{-1} - \mathcal{R}_{nn})$ transforms into $(\tau + \mathcal{K}_{nn})$, with $\tau^> = i$ and $\tau^< = \tau_n$. In above derivation it is assumed that $L_{n<}$ is real and $\Delta L_n =$

$L_{n>} - L_{n<}$ is logarithmic derivative variation across threshold of the n-channel. The modulus one quantity $(\Delta L_n)^\star/(\Delta L_n)$ allows to define a " scattering phase shift" δ_n, and a corresponding "K-Matrix element" $\tau_n = \tan\delta_n = \mathcal{I}m\Delta L_n/\mathcal{R}e\Delta L_n$,

The K- Matrix form, corresponding to the Reduced S- Matrix, becomes now

$$\Delta S_N = \frac{1}{2i}(S_N^0 + 1)K_{Nn}(\tau_{nn} + \mathcal{K}_{nn})^{-1}K_{nN}(S_N^0 + 1)$$
$$\Delta S_N^< = \Delta S_N^> \frac{i + \mathcal{K}_{nn}}{\tau_n + \mathcal{K}_{nn}}$$

As a parenthesis we remark the last equation contains basic formulae of the Cusp Theory, both above and below n-threshold. For nuclear s-wave scattering, the logarithmic derivatives are $L_n^> = i\rho$ and $L_n^< = -\rho$, ($\rho = k_n \cdot a$; k_n - channel wave number, a - channel radius). It follows $\Delta U_N^< = \Delta U_N^>((L_n^>)^{-1} - \mathcal{R}_{nn})/((L_n^<)^{-1} - \mathcal{R}_{nn})$ which in zero energy limit, ($\rho \to 0$), reduces to the Cusp Theory result, $\Delta U_N^< = i\Delta U_N^>$. The same result is obtained with $\tau_n = 1$ for s-wave neutron threshold.

The n-channel effects on retained channels (N) are expressed by the product $R_{Nn}(L_n^{-1} - \mathcal{R}_{nn})^{-1}R_{nN}$, resembling to the additional term of $\mathcal{R}_N$ Reduced R- Matrix. However there is a difference, namely the "bare" R-Matrix element R_{nn} of eliminated n-channel is here replaced by its effective counterpart $\mathcal{R}_{nn}$; the Reduced $\mathcal{R}_{nn}$ - Matrix element does include also rescattering effects from complementary open channels. In next paragraphs we will discuss single particle resonances of multichannel scattering in terms of the Reduced Collision Matrix, which describes the two groups of channels by well-separated terms.

Below threshold, a pole in the $U_N^<$ Collision Matrix elements could be obtained from condition $\mathcal{R}_{nn}^{-1} = L_n^< = S_n^{(-)}$, ($S_n^{(-)}$ - shift function). In non-coupling limit, $\mathcal{R}_{nn}$ reduces to single channel R-Matrix element R_{nn}. Or this is just the bound state condition of the R-Matrix Theory[17]; a bound state appears at that energy at which the internal (R_{nn}^{-1}) and external $S_n^{(-)}$ logarithmic derivatives do match. This result is a R-Matrix proof that the single particle state from a closed channel does induce resonance in competing open channels of the multichannel system.

For positive energy eliminated channels the corresponding states should be quasistationary ones. A pole in U_N is now obtained by a condition which is analog to the bound state one, $\mathcal{R}_{nn}^{-1} = L_n^>$; the logarithmic derivative $L_n^>$ is the corresponding, at positive energy, of the shift function $S_n^{(-)}$ defined for negative energy. According to R-Matrix theory, the quasistationary (Siegert) state is defined by condition $|1 - R(H_\lambda)L| = 0$, (see Ref. 17, p. 297). A quasistationary state originating in an eliminated channel induces a quasiresonant structure in other open competing channels. Apparently, (see, for example, Ref. 35), this situation (multichannel resonance originating in a quasistationary single particle state from an unobserved channel) was not reported until now. As discussed before, in the literature[31] one reports on the "channel coupling pole" observed in numerical experiments for multichannel scattering; a single channel pole may be driven to physical region of the complex energy plane when the channel coupling becomes effective.

One can go a step further along lines developed in the Nuclear Physics, but with price of some assumptions. There one studies the dissolution of the (bound or quasistationary) single particle state amongst the actual states of nucleus, by energy averaging over last ones[36]. By using energy averaging procedures one has to avoid the threshold branch point; one can consider only energy averaging intervals which could be very near threshold but avoiding its overlap, (see Ref. 37, p.146). Another physical assumption, mostly used, is the R-Matrix elements are factorable $R_{Nn}R_{nN} \sim R_{NN}R_{nn}$, see Ref. 36. One proves[37], that the energy averaging is equivalent to replace the real energy E by a complex quantity $\mathcal{E}$, $\overline{R_{nn}(E)} = R_{nn}(\mathcal{E})$, and further this is related to the Reduced R-Matrix element, $R_{nn}(\mathcal{E}) = \mathcal{R}_{nn}(E)$, see Ref. 17. Within these assumptions one obtains the result according to the n-channel related term in averaged Collision Matrix is

$$\overline{\Delta U_N} \sim \mathcal{R}_{nn}(E)L_n/[1 - L_n\mathcal{R}_{nn}(E)]$$

which, at low energies for s-waves is proportional to the single- particle Strength Function $< \gamma_n^2 > /D$. The Strength Function is ratio of averaged width $< \gamma_n^2 >$ to the mean spacing D between adjacent levels; it does measure the mixing of the single particle nucleon state with nuclear actual states and will display maxima whenever the single particle states are present.

The physical interpretation of last formula is related to the spectroscopical aspects of multichannel resonance originating in the single particle state from an invisible channel. The magnitude of the effect in the open observed channel is proportional to the spectroscopic amplitude of the single particle state from closed channel. Assuming the single particle state is located at channel- threshold then, by strong coupling, a threshold effect has to be observed in open channels. The analysis of experimental data on threshold effects from Low Energy Nuclear Reactions, related to the p wave neutron single particle state at zero energy, do corroborate the relation between the resonance's strength and the spectroscopic amplitude of the ancestral single particle states[8]. In physical terms, the threshold effect magnitude depends on amount of the flux transfer between threshold channel and open observed one. A threshold single particle quasistationary state does act as an amplifier for the flux transfer to and from threshold channel, because state's overlap with this channel (reduced width) is very large.

5. Conclusions

A Reduced Scattering Matrix Approach to the p-wave threshold effect, embodying its physical characteristics, was developed. Its relationship to other p-wave threshold models, was established.

The p-wave threshold effect is related to the Quasiresonant Scattering: a Neutron Single-Channel Resonance coupled, by Direct Transitions, to the observed open channels. The p-wave threshold effect magnitude is proportional both to the channel coupling strengths and to the spectroscopical amplitude of the ancestral zero-energy neutron single particle state. In physical terms the flux leakage into threshold channel is determined not only by genuine Reaction Mechanisms (Penetration Factors, Coupling Strengths) but also by the Spectroscopy of the Neutron Threshold State (Neutron Reduced Width).

We proved that all significant spectroscopical characteristics of a multichannel threshold effect, originating from a Neutron Single Particle State, are embodied in the Neutron Strength Function. The Neutron Strength Function dependence on the mass number A is reflected in a mass-dependence of the p-wave threshold effect. The energy dependence of the Neutron Strength Function, displayed via its microstructures, is reflected in an "Intermediate Structure" of the p-wave threshold effects (Micro-Giant structures of the p-wave threshold effects).

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