THE DEUTERON STRIPPING ANOMALY AT THRESHOLD OF ANALOGUE CHANNEL AND THE $3-p$ WAVE NEUTRON STRENGTH FUNCTION

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We prove in this letter that the magnitude of anomaly in deuteron stripping reactions on $A \approx 80$ - 110 mass target nuclei, at threshold of analogue channel, is proportional to the $3-p$ wave neutron strength function. The result is obtained from analysis, within an empirical approach and computational frameworks, of the cross-section and analyzing power experimental data related to this (d,p) anomaly.

1. Introduction

The problems of the threshold effects[1] and of the neutron strength function[2] were formulated in the early days of the low-energy nuclear physics; nevertheless they both are still topical basic subjects of research in contemporary physics of nuclear reactions,[3,4] although apparently as non-related ones.

The purpose of this work is to establish a relation between the $3-p$ wave neutron strength function and the anomal effect observed at the threshold of the neutron analogue channel, in deuteron stripping reactions on $A \approx 80-110$ mass number. By analyzing the existing (cross section and analyzing power) experimental data on this (d,p) anomaly, one proves that the magnitude of the threshold effect is proportional to $3-p$ wave neutron strength function, in their dependence on mass number.

The present work connection between threshold effects and neutron strength function proves that the p-wave threshold effects are directly related to spectroscopy of ancestral zero-energy neutron single particle resonance. The threshold effects depend not only on penetration factors of opening neutron channel, as in Cusp Theory, but also on spectroscopic amplitude of neutron threshold state.

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2. On Threshold Effects in Low Energy Nuclear Reactions

The threshold effects originate in conservation of the flux, (Unitarity of the Scattering Matrix). If a new channel opens, a redistribution of the flux in old open channels appears, i.e. a modification of their reaction cross-sections; such changes of cross-sections of open channels, due to opening of a new one, are called threshold effects. The magnitude of a threshold effect depends on amount of flux absorbed by the threshold channel. If the threshold channel has no (coulombian and centrifugal) barriers, i.e. it is a s-wave neutron one, then absorption of flux happens suddenly and this results in a specific threshold effect named Threshold Wigner Cusp [1].

The dynamics of the flux to and from threshold channel is controlled by the reaction mechanism. The factor governing the leakage of the flux is the penetration factor, as e.g. in Cusp Theory.

The primary factor governing the leakage of the flux from internal region into neutron channel is neutron reduced width[5]. This could be the physical explanation why the threshold effects observed with light nuclei are related to a resonance coincident with neutron threshold and having a large neutron reduced width, see e.g. corresponding threshold data in tables of Ref. 3.

For medium-heavy nuclei, having a high density of actual states, the corresponding physical quantity is the neutron strength function; it is defined as ratio of the averaged neutron reduced width to the mean spacing of adjacent levels and it is a measure of how much the single particle states are mixed with actual states of the nucleus. Both the reduced width, for isolated resonances, and the strength function for giant resonances are spectroscopic quantities which, in addition to genuine reaction mechanisms, could play a role in dynamics of the flux in neutron threshold channel which at its turn is responsible in producing threshold effects.

Extensive experimental studies on threshold effects in nuclear reactions were done, along many decades, since formulating the problem; in spite of these searches only relatively few threshold effects were observed [3]. This proves that the threshold effects are related to some restrictive conditions; the aim of this letter is to elucidate, based on existing data, the spectroscopical determination of (d,p) threshold anomaly.

3. Deuteron Stripping Threshold Anomaly: Isospin Coupling and Lane Model

The deuteron stripping anomaly is the only one threshold effect reported in low energy nuclear reactions on medium-heavy nuclei; it was observed in deuteron stripping reactions on $A \approx 80 - 110$ mass nuclei both in cross section[6] and analyzing power[7] experiments. The anomaly appears like a dip in the (d,p) excitation functions. The center of the dip is located at neutron analogue threshold in limit of 100-200 KeV, while the typical half-width is 700 KeV. The most pronounced anomalies were observed with $^{88}Sr,^{90}Zr$, $^{92}Zr,^{94}Zr,^{92}Mo,^{94}Mo$ and other target nuclei[10]. In polarization measurements the anomaly occurs at more forward angles, having either a resonance or a S -shape form.

The anomaly does not appear for lowest neutron threshold but rather it is related to opening of neutron analogue channel; this observation was considered as an experimental evidence for isospin coupling of proton and analogous neutron channels of deuteron stripping reactions. The isospin coupling of analogue channels is physical basis for a Coupled Channel Born Approximation (CCBA) model[8] which was developed in order to explain this threshold anomaly. Numerical experiments[9] with CCBA model did emphasize the role of the $3-p$ wave neutron single particle state at zero energy, for producing a threshold effect. Lane[10] has proposed a phenomenological model based on the $3-p$ wave neutron single particle resonance which is energy coincident with threshold in the $A \approx 90$ mass region. The isospin interaction couples it to the p-wave of the observed proton channel, resulting into a p-wave resonant-like term in Scattering Matrix element of the (d,p) reaction

$$\begin{aligned} S_{dp} &= S^0_{dp} + S^\pi_{dp} = \\ & S^0_{dp} + i \sum_{j_p=1/2,3/2} \frac{\alpha_{j_p}(\hbar^2/ma^2)}{E_{j_p} - E - (S_1 + iP_1 - b)\gamma^2_n - \frac{i}{2}\Gamma'} \end{aligned} \tag{1}$$

with S^0_{dp} as a (d,p) reaction DWBA background S-Matrix element; α_{j_p} - phenomenological coupling parameters; E_{j_p} - energy eigenvalues corresponding to boundary condition b at neutron channel radius a; γ^2_n is the neutron reduced width; S_1 and P_1 are p-wave (l=1) neutron shift and penetration factors, Γ' is sum of the proton decay width and of the broadening width. The strong energy dependence of the resonance denominator (S_1 and P_1) results into a change of the energy scale ("compression factor"). The Lane model did reproduce the anomaly both in cross section and analyzing power;[11] a rather extensive experimental and computational study was presented in Ref. 12.

A further insight in the Lane's model resulted in the conclusion that the phenomenological coupling parameter α is proportional to the $3-p$ wave neutron strength function.[13] One could expect, on this basis, to correlate the observed characteristics of the anomaly with behaviour of the neutron strength function. For example, the structure observed in energy dependence of the anomaly could be interpreted as originating in the intermediate structure of the neutron strength function; a similar conclusion was obtained by Lane[14] from a different perspective. One expects also that the mass dependence of the anomaly strength α has to follow mass dependence of the $3-p$ wave neutron strength function.

4. On Analysis of Threshold Anomaly

An empirical procedure (used as an experimental evidence) for direct extraction from experimental data of anomaly's magnitude was devised for analysing threshold anomalies observed in deuteron stripping on the following target nuclei: ^{80}Se,[15]

^{86}Kr ,[16] ^{88}Sr,[11,17] ^{90}Zr,[7,12,18] ^{92}Zr,[19] ^{92}Mo,[12,19] ^{94}Zr,[19] ^{94}Mo [19] and ^{106}Cd.[12,20] One evaluates maximal deviations of the cross-section with respect to median (values) at half-widths points of the anomaly dip. The magnitude of the anomaly is normalized with respect to (d,p) background cross-section including corresponding spectroscopic factor. This procedure results into a global parameter for anomaly's magnitude with no explicit reference to different deuteron channels contributing to same p-wave proton channel. The 'empirical' anomaly strengths and their errors (determined by a standard error analysis), are represented by triangles in Fig. 1(a). The recent experimental data on $3-p$ neutron strength function for the investigated nuclei, i.e. ^{80}Se,[21] ^{86}Kr,[22] ^{88}Sr,[23] ^{90}Zr,[24] ^{92}Zr,[24] ^{92}Mo,[24] ^{94}Zr,[25] ^{94}Mo [25] and ^{106}Cd,[26] are displayed in the figure by filled circles; for previous data see e.g. Ref. 27. The values of the anomaly strengths are given in relative units, as they have been obtained from the empirical analysis.

The procedure is not a very accurate approximation. It neglects the type of deuteron channel partitions, the spin orbit coupling in exit proton channel and corrective factors as energy dependence of α parameters. Also it is not appropriate for evaluating the double-dip type of anomalies, and in evaluation of analyzing power data. However, in spite of its limitations, it exhibits the relation between anomaly's magnitude and neutron strength function.

To take into account all the above inaccuracies, we performed numerical analyses by using a standard DWBA method for background's description, two alternative subroutines for S-Matrix threshold term and a minimization procedure[28] for best fits of experimental data.

The purpose of numerical analysis is quantitative extraction, from experimental data, of strengths of α phenomenological parameters and, subsequently, of their spectroscopic component.

In the theoretical analysis of the threshold anomaly the α parameter could be considered not as a genuine free one but rather subject of some general physical constraints. The parameter α is dependent on the coupling strength of the proton and neutron analogue channels; it is also dependent on the deuteron input channel data (angular momenta $l_d j_d$) compatible with those $(l_p j_p)$ of p-wave single particle resonance. If the resonance would be shifted far away from neutron threshold, the resonant-like term should behave as proton p-wave background. In that limit one deduces that α-parameter's dependence on deuteron energy should be similar to that of the background terms of the Scattering Matrix which describe transitions to the p-wave proton channel; in other words α-dependence on deuteron and proton energies is contained in (d,p) DWBA radial integrals corresponding to $l_p = 1$ proton partial wave. Formally this can be deduced from the relationship existing between the total angular momentum - (specific to a resonance) and transferred angular momentum - (specific to a direct process) parameterizations of the transition amplitude for a direct reaction.[29]

The transition amplitude describing direct - and anomal - reaction mechanisms in (d,p) reactions consists, according to Lane formula, from a DWBA - and a res-

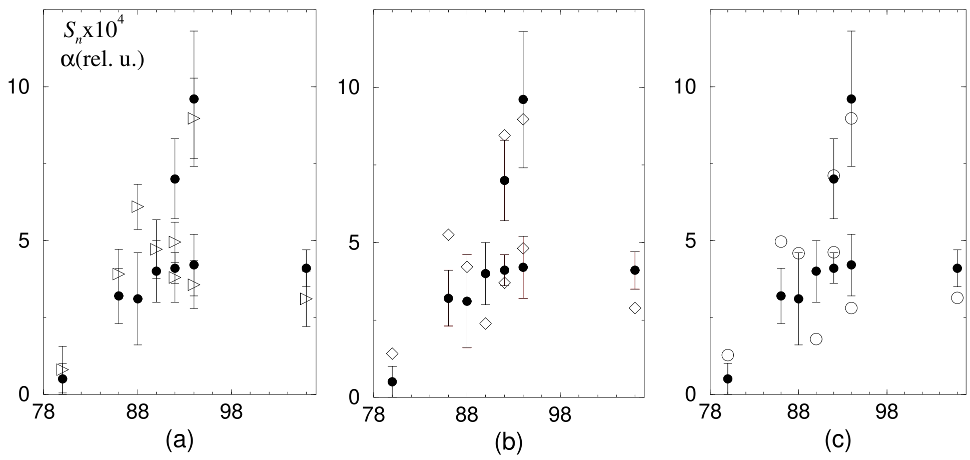

Fig. 1. The mass dependence of the experimental $3-p$ wave neutron strength function (•) and of the: (a) 'empirical' (▷); (b), (c), 'computational' (◇ and ∘) strengths of threshold anomaly.

onant - term; these terms are constructed, respectively, in transferred angular momentum coupling scheme,[29] and in the total angular momentum coupling scheme,[30] and they are added up in the code DWUCK.[31] The DWBA description of the background is realized in terms of deuteron[32] and proton[33] optical global (averaged) parameters and by a suitable choice of spectroscopic factors[34,35,36,37,38,39]. The anomal threshold term is approached[10] in terms of R-Matrix description,[30] for $3-p$ wave neutron single particle resonance.

A physical constraint in analysis is to obtain same spectroscopic component of anomaly, from all possible transitions (corresponding to $\alpha_{l_d j_d; l_p j_p}$ parameters) populating p wave proton channel from different (l_d, j_d) deuteron channels; apart kinematical and penetration factors, the different transition strengths $\alpha_{l_d j_d; l_p j_p}$ should provide same (reduced) spectroscopic quantity, i.e. neutron strength function.

The fit of the $3-p$ threshold anomalies has been done for every allowed values of the angular momentum corresponding to the total angular momentum coupling scheme and related to $l_p = 1$ exit channel. Two fit parameters were monitored in these calculations: the magnitude of the the α coefficients and the relative phases between the direct and resonant terms of the transition amplitude. The 'computational' anomaly strengths, obtained in transferred angular momentum coupling scheme and in the total angular momentum coupling scheme, are plotted respectively in Fig. 1(b) and Fig. 1(c) by unfilled diamonds and circles. The trend of the estimated data follows the experimental strength function irrespective the used analysis method or the set of quantum numbers of the involved deuteron channel. One can observe a good agreement between the 'empirical' (Fig. 1(a)) and 'computational' (Fig. 1(b) and Fig. 1(c)) anomaly strengths and the values of the $3-p$

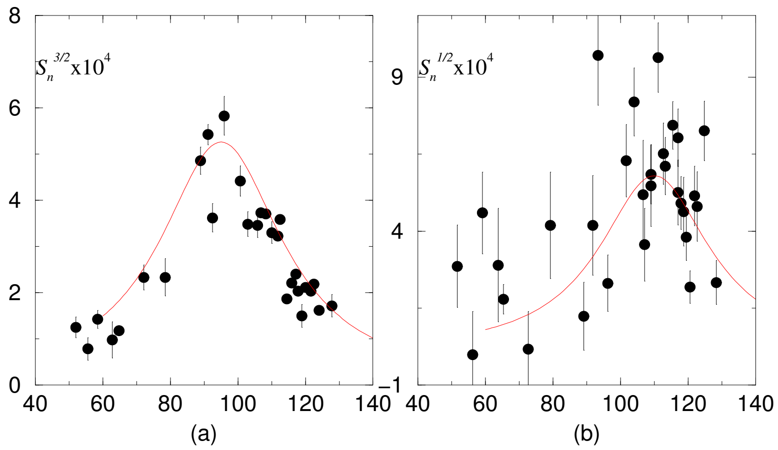

Fig. 2. The mass dependence of the experimental $3-p$ wave neutron strength function (•) and of the α's anomaly strength derived within a Lorentzian distribution (-) for the: (a) $p3/2$ spin orbit component; (b) $p1/2$ spin orbit component.

neutron strength function for all investigated nuclei, within the limits of the experimental errors. The errors of the numerical evaluated strengths have been extracted from the Error Matrix generated by the MINOS procedure (see Ref. 28). The evaluated spectroscopic strengths of anomaly, considering a 90 percents confidence level, are in agreement with the experimental values of the neutron strength function.

We also performed a 'global' fit for all the investigated cases by describing the anomaly strengths through a certain distribution function versus the nuclei masses. The Lorentzian distribution seems to be very appropriate for our purpose; it also has been used by the authors from Ref. 4 to estimate the spin orbit splitting of the $3-p$ neutron strength function. The 'global' fit does search for the best estimates nor of the each anomaly strength but rather for the parameters of the Lorentzian function (the width and position of the maximum). The difference between our Lorentzian function and that from Ref. 4 is contained in the deriving procedure; ours does fit for the deuteron anomaly only (for the existent investigated cases) and their's describes the experimental data of 3/2 and 1/2 neutron strength function. In Fig. 2(a) and Fig. 2(b) there are presented the most recent data of the spin orbit splitting of the neutron strength function[4] and α's estimated functions.

5. Conclusions

The conclusion of the works from seventies regarding the threshold anomaly is focused to the concept of isospin coupling of proton and neutron analogue channels and the existence of $3-p$ wave neutron single particle resonance at zero energy for $A \approx 90$ nuclei. Also Lane did emphasize the role of the 3-p wave resonance as reaction mechanism in producing threshold anomaly. In Lane's work attention is paid only to resonance denominator. The Lane's phenomenological parameter of anomalies, the alpha's, are related to isospin coupling only. The results from our work converge to idea that the alpha's are directly related to the neutron strength function, also.

The physical result from our analysis is that the magnitude of deuteron stripping threshold effect is determined not only by reaction mechanism (isospin coupling of neutron zero-energy resonance to observed proton channel), but also by the neutron strength function, i.e. the spectroscopic amplitude of neutron threshold state. A large spectroscopic amplitude of neutron threshold resonance does act as amplifier of flux transfer to and from neutron threshold channel, resulting in increase of threshold effect's magnitude.

In this paper it is first time when the deuteron threshold anomaly strengths and the $3-p$ wave neutron strength function are pictured together within their dependence of the mass number in the region $A \approx 90$. This proves the existence of a proportionality relationship between the above two physical quantities. According to this work, the threshold effects depend not only on penetration factors of opening channel, as in Cusp Theory, but also on spectroscopic amplitude of threshold state from neutron opening channel. This result is also a computational and empirical demonstration according to the ancestral source of anomaly is zero-energy neutron p-wave single particle resonance. The numerical analysis have been supported by actual developed numerical tools and large hardware computing capabilities, inaccessible those former times.

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