NON stationary
NON linear
NON gaussian
NON sense ?

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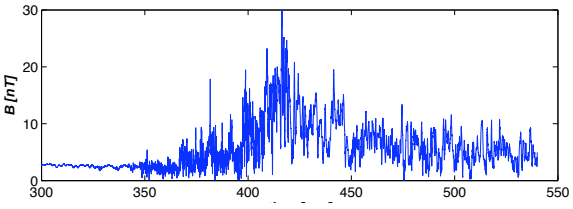
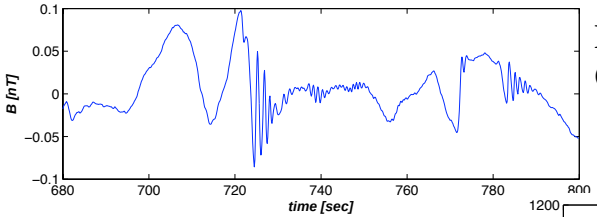
Assumptions

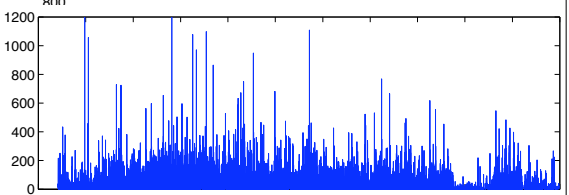
All we've been taught about data analysis is based on some implicit assumptions



Stationarity
(bow shock crossing by CLUSTER)



Linearity
(solar wind by AMPTE)



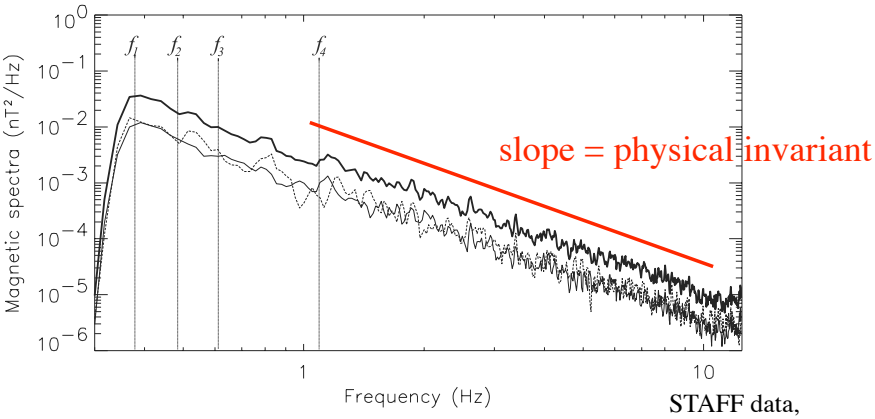
Gaussianity
(energetic electron flux by GOES)

Assumptions

- Any **departure** from these assumptions questions the validity of our analysis
- Many **mistakes** have been made by ignoring some of these assumptions
- But "non-properties" are not necessarily a **nuisance** as they provides deeper insight into the physics !

Power laws

- Power laws are ubiquitous in space physics



STAFF data,
(Sahraoui et al., JGR 2003)

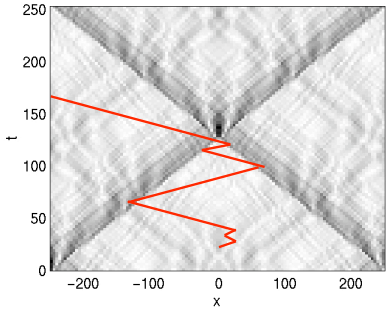
- Power law = self-similarity

Example

- We traditionally use the Fourier Transform to study such power laws

this does not always give the correct answer !

- *Example* : test particle simulations in a 2D magnetic field with coherent structures (Hada & Kuramitsu, 1999)



ion trajectory vs amplitude of magnetic field

Spectral analysis

- We are dealing with a self-similar process

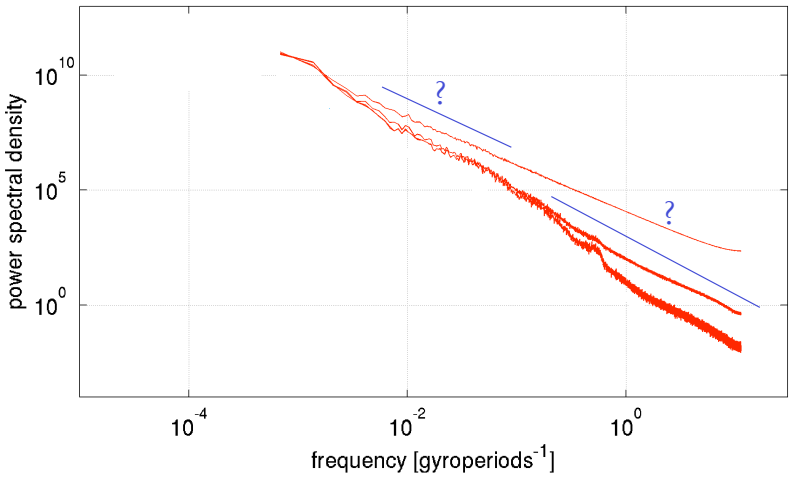
$$\Rightarrow u(x) = \lambda^{\alpha} u(\lambda x)$$

$$\Rightarrow u(k) = \mathcal{F}u(x) \sim k^{-\beta}$$

- The power spectral density should give a power law

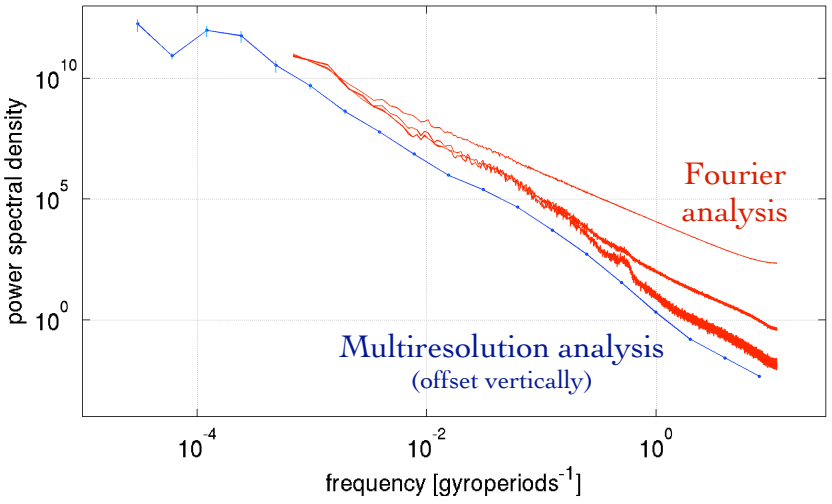
Spectral analysis

power spectral density of using windowed Fourier transform with various types of windows and detrendings

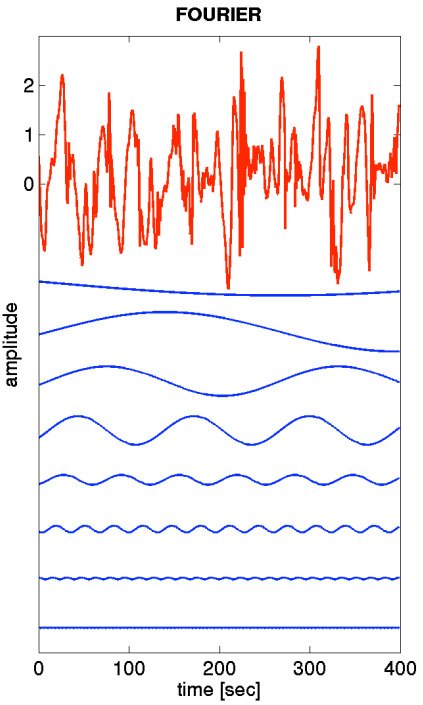


Spectral analysis

Multiresolution (wavelet) analysis provides a better estimate of the spectral index



Fourier vs multiresoltion analysis



Wavelet transform : definition

Fourier transform

$$u(\omega) = \int_{-\infty}^{+\infty} u(t) e^{j\omega t} dt$$

Continuous wavelet transform

$$u(t,a) = \int_{t_a}^{t_b} u(t') \phi_{t,a}(t')\, dt'$$

mother wavelet

$$\phi_{t,a}(t') = \frac{1}{\sqrt{a}} \phi\left(\frac{t'-t}{a}\right)$$

scale

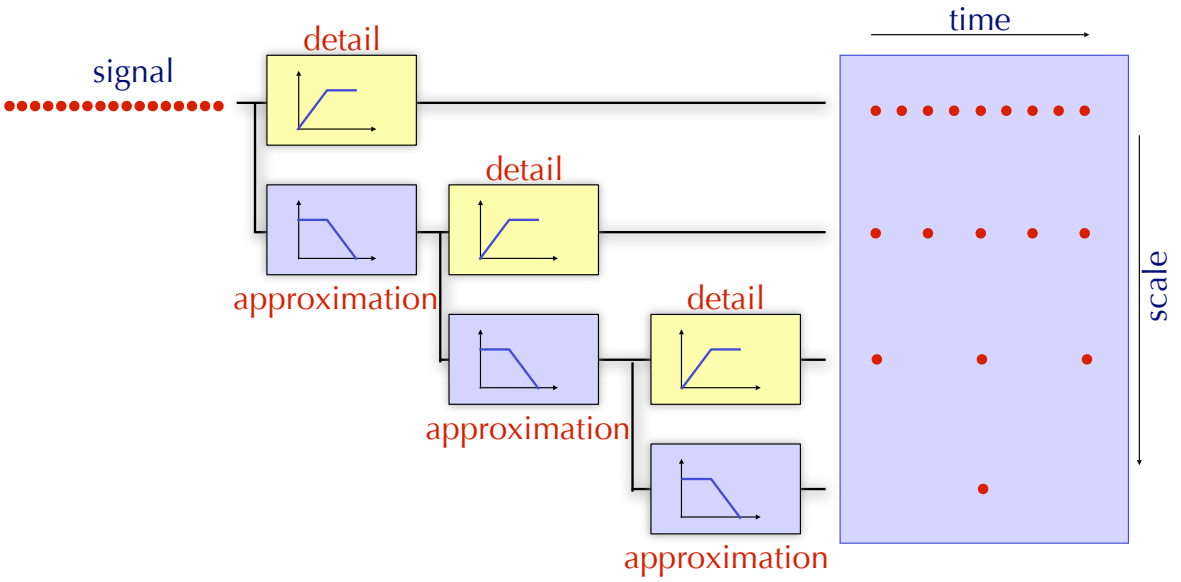
Discrete wavelet transform

$$u_{j,k} = \int_{t_a}^{t_b} u(t') \phi_{j,k}(t')\, dt'$$

mother wavelet

$$\phi_{j,k}(t') = \frac{1}{2^{j/2}} \phi\left(2^{-j} t' - k\right)$$

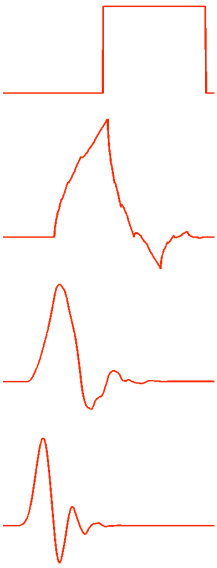
Multiresolution analysis



The discrete wavelet transform can be implemented by means of a fast pyramidal recursive filter bank

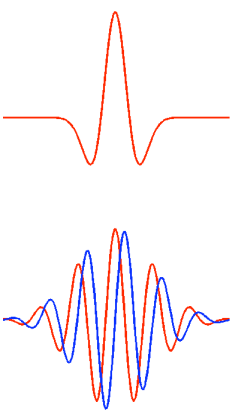
Mother wavelets : examples

Discrete wavelets (here Daubechies wavelets)

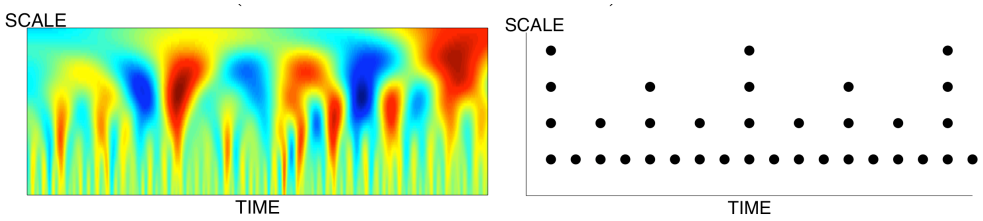


The number of vanishing moments (= order of the wavelet) is a key parameter

Continuous wavelets



Discrete or continuous transform ?



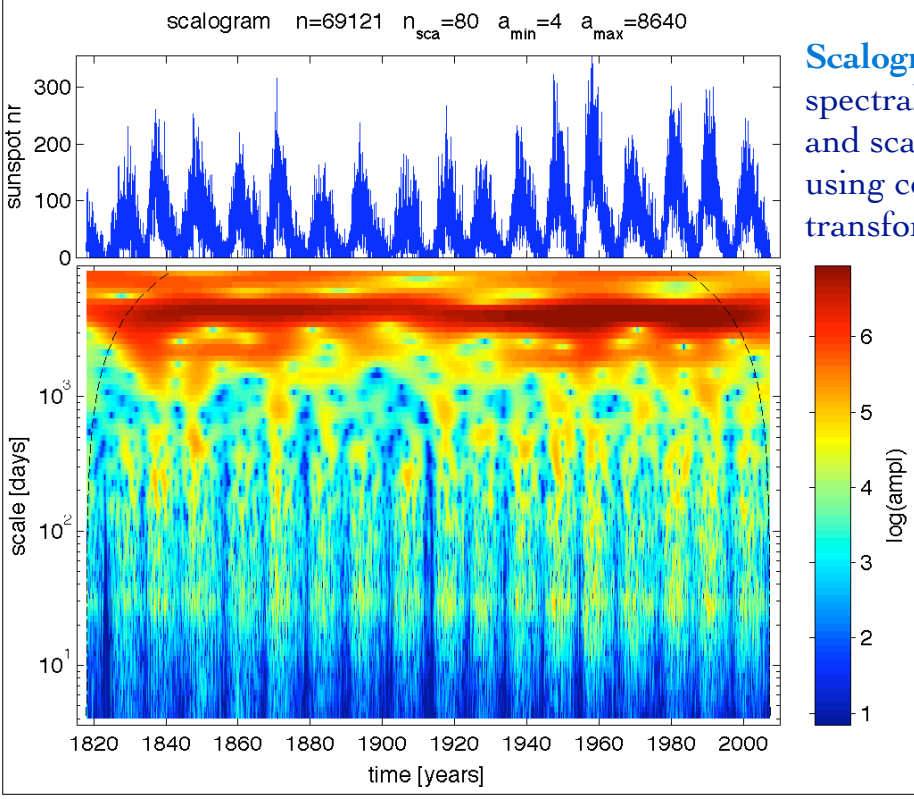
Continuous transform

- highly redundant
- good for data analysis
- scales can be freely chosen
- computationally expensive

Discrete transform

- non-redundant and uses orthogonal bases
- useful for multiresolution analysis (denoising) and compression
- scales are imposed
- very fast algorithms (faster than FFT)

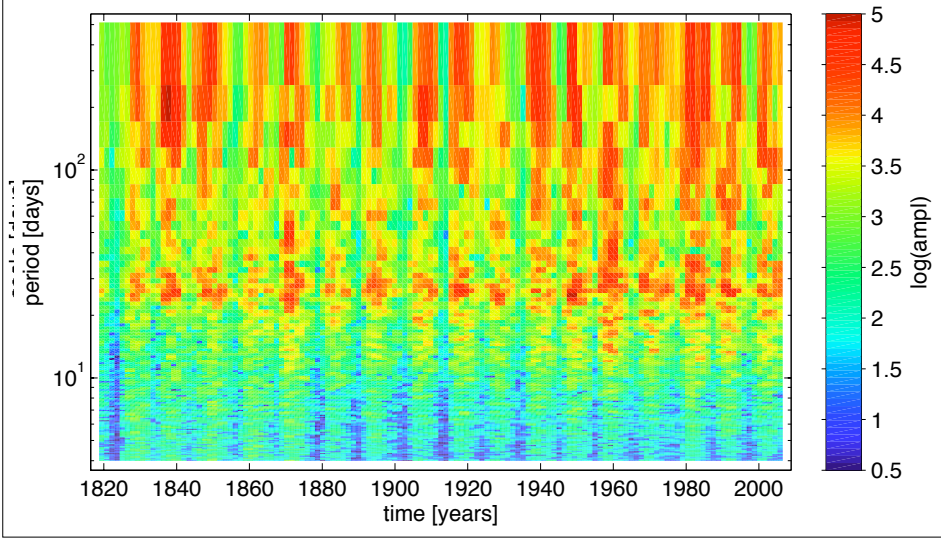
Scalograms



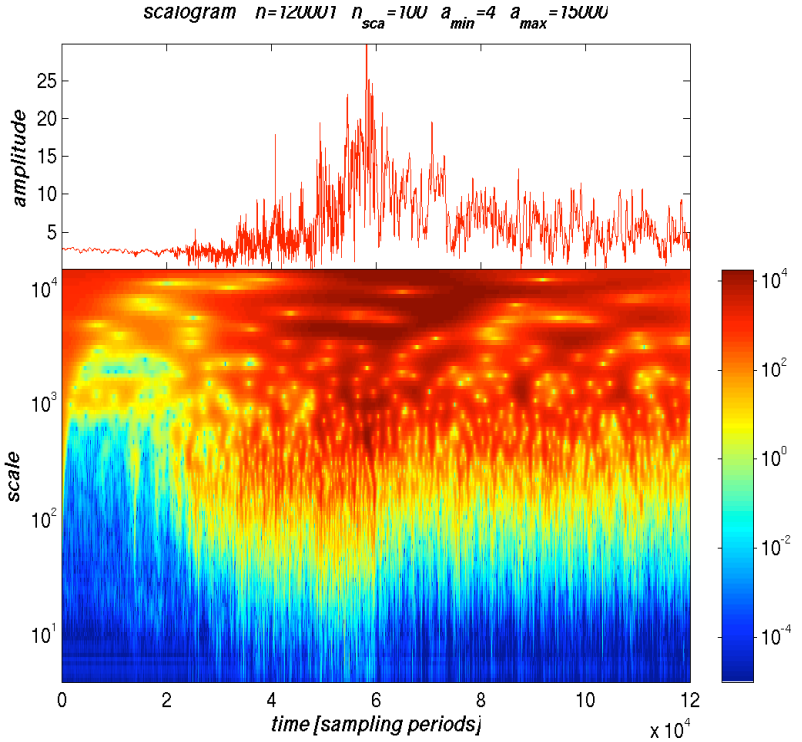
Scalogram = power spectral density vs time and scale (or frequency), using continuous wavelet transform.

Scalograms

- But the usefulness of scalograms has been heavily overemphasized... you can do almost the same with the good old Fourier transform



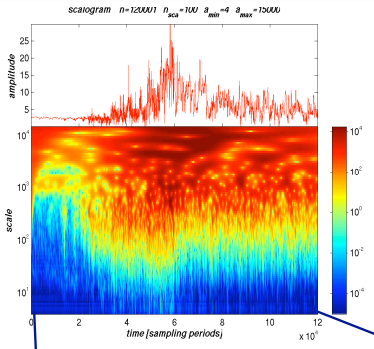
Continous transform : an example



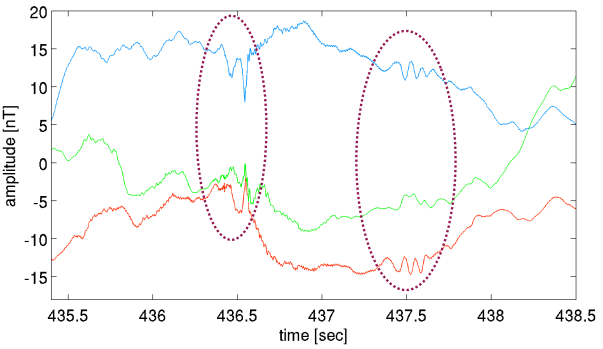
STAFF magnetic field data

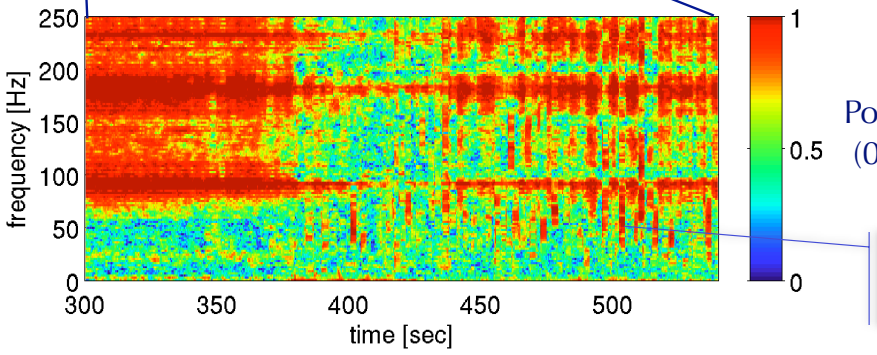
Scalogram of B_y

Continous transform : an example



STAFF m
field data





Polarisation
$(0 \leq p \leq 1)$

are these patches meaningful ?

Denoising

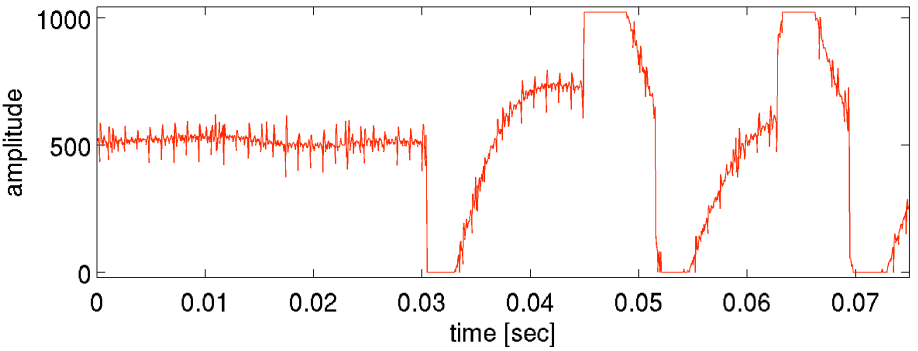
- The **discrete wavelet transform** is one of the most powerful techniques around for denoising

"Wavelet denoising is like removing weed while saving daisies" (M. Wickerhauser)

- Principle
  - decompose your data into wavelet coefficients
  - keep only those coefficients that are interest (the largest ones)
  - set all other coefficients to zero
  - recompose your data set

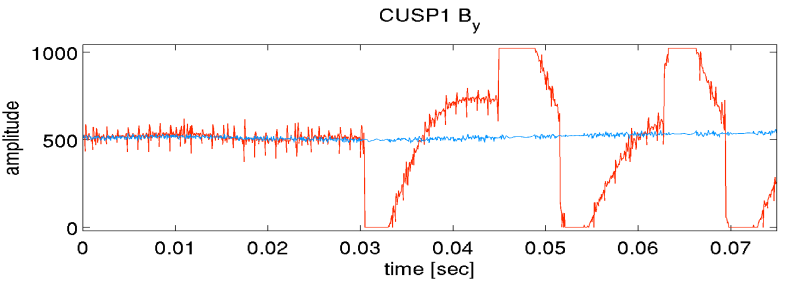
Denoising : example

Example : AC magnetic field measurements with strong interference from active particle experiment (CUSP2000 sounding rocket)

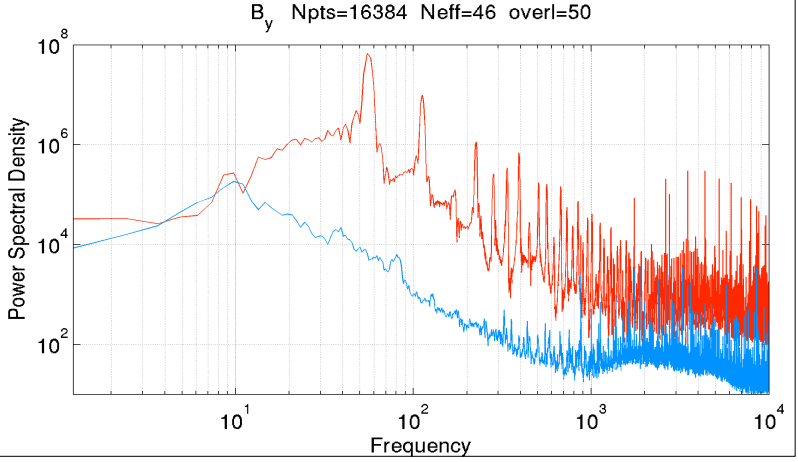


Denoising : example

AC magnetic field data with strong intereference
- raw signal
- denoised

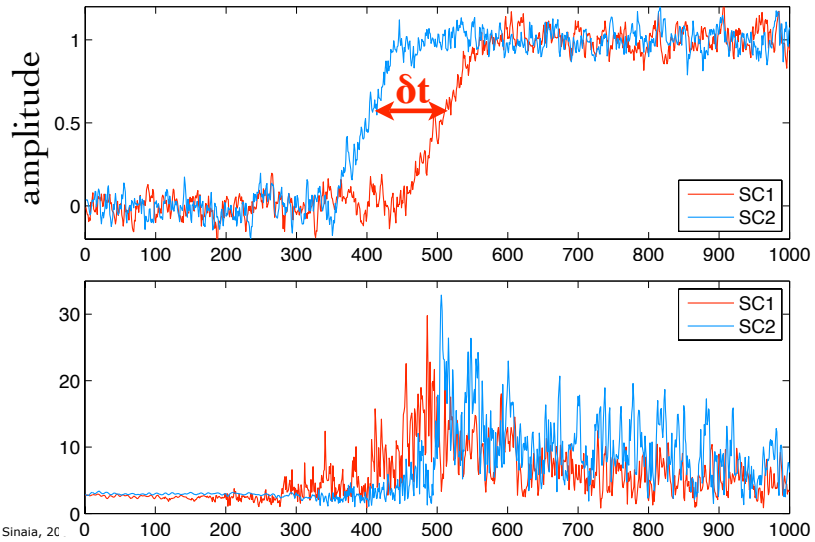


Power spectral density
- raw signal
- denoised



A different application

- There are many other applications...
- One of them is the estimation of **timing differences** between multispacecraft data (Soucek et al., Ann. Geoph., 2004)



To continue

- Computer session (this afternoon) with Matlab
  - exploratory analysis with wavelets
  - estimating the spectral exponent of the AE index
  - wavelet denoising
  - estimating timing differences between Cluster spacecraft
- Further reading
  - C. Torrence & G. Compo, "**A practical guide to wavelet analysis**", hardcopy available here
  - S. Mallat, "**A wavelet tour of signal processing**" (Academic Press, 1998): THE reference
  - J.-L. Starck and F. Murtagh, "**Astronomical image and data analysis**" (Springer, 2006): dedicated to astronomy