COSPAR Capacity Building
Sinaia, 5 June 2007
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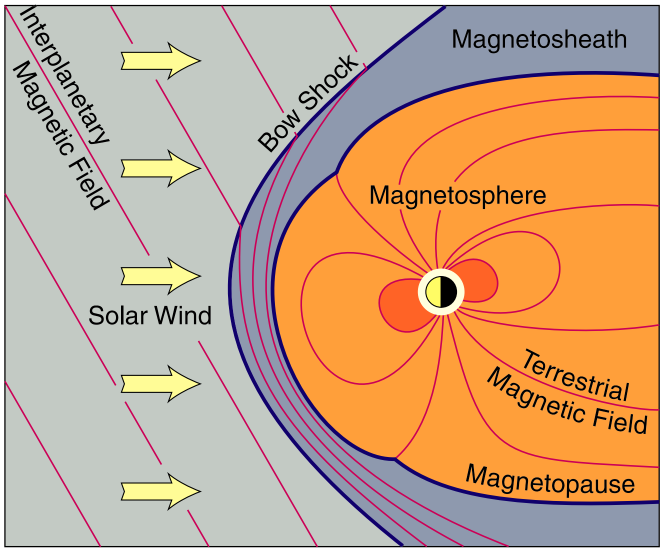
Basic Analysis Techniques & Multi-Spacecraft Data

This lecture:

- ▷ Introduction: single-spacecraft vs. multi-spacecraft
- ▷ Single S/C data: minimum variance analysis
- ▷ Multi S/C data: reciprocal vectors

Computer session: select your assignment(s)

1 Introduction: Magnetospheric structure



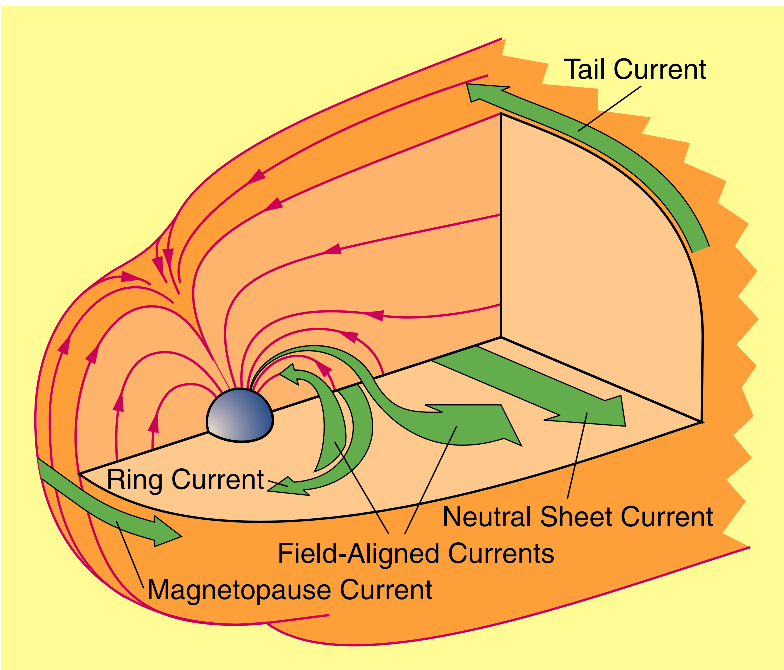
Baumjohann/Treumann/Mayr-Ihbe

Boundary layers,
sheet structures:

- ▷ bow shock,
- ▷ magnetopause,
- ▷ current sheets.

Orientation, motion, shape,
. . . of the surface ?

Magnetospheric structure (cont'd)



Baumjohann/Treumann/Mayr-Ihbe

Magnetospheric currents:

- ▷ magnetopause current,
- ▷ neutral sheet current,
- ▷ field-aligned currents:

Orientation of current sheets, current direction and density, . . . ?

Single spacecraft vs. multi-spacecraft

Single-spacecraft data:

- ▷ one-dim. track in 3+1 dim. space-time
  $\longrightarrow$ ambiguity,
- ▷ additional information required
- ▷ popular approach: minimum variance analysis.

Multi-spacecraft data:

- ▷ several trajectories in 3+1 dim. space-time,
- ▷ analysis of spatio-temporal processes,
- ▷ additional information still useful,
- ▷ various approaches.

2 Single-Spacecraft Data: MinVar Approach

To resolve space-time ambiguity,

- ▷ additional information is needed,
- ▷ choose an appropriate **physical model**,
- ▷ use physical constraints to estimate **model parameters**.

Phenomena	Models	Parameters
Boundaries	Planar structures	Normal vector $\hat{\mathbf{n}}$
FACs	Current sheets	Orientation, $\mathbf{j}_{\parallel}$
Waves	Plane waves	Prop. vector $\mathbf{k}$

Key technique: **minimum variance analysis**

Minimum variance principle

Physical model for **boundary analysis**: planar 1D structure with normal vector $\hat{\mathbf{n}}$, use physical constraint

$$0 \;=\; \nabla \cdot \mathbf{B} \;\stackrel{\text{1D}}{=}\; \hat{\mathbf{n}} \cdot \nabla(\mathbf{B} \cdot \hat{\mathbf{n}}) \quad \Rightarrow \quad \mathbf{B} \cdot \hat{\mathbf{n}} \;=\; \text{const}$$

In practice: model only approximately valid.

Least squares approach: as an estimator for the sheet normal direction take the direction $\hat{\mathbf{n}}$ where data set $\{\mathbf{B}^{(\ell)}\}_{\ell=1,\ldots,L}$ shows the **smallest quadratic variation (variance)**:

$$\left\langle |(\mathbf{B} - \langle \mathbf{B} \rangle) \cdot \hat{\mathbf{n}}|^2 \right\rangle \;\equiv\; \frac{1}{L} \sum_{\ell} \left| \left(\mathbf{B}^{(\ell)} - \langle \mathbf{B} \rangle \right) \cdot \hat{\mathbf{n}} \right|^2 \;=\; \text{Min!}$$

Minimum variance principle (cont'd)

Result: $\hat{\mathbf{n}}$ is the normalized **eigenvector** associated with the smallest (positive) eigenvalue λ_3 of the **(co)variance matrix**:

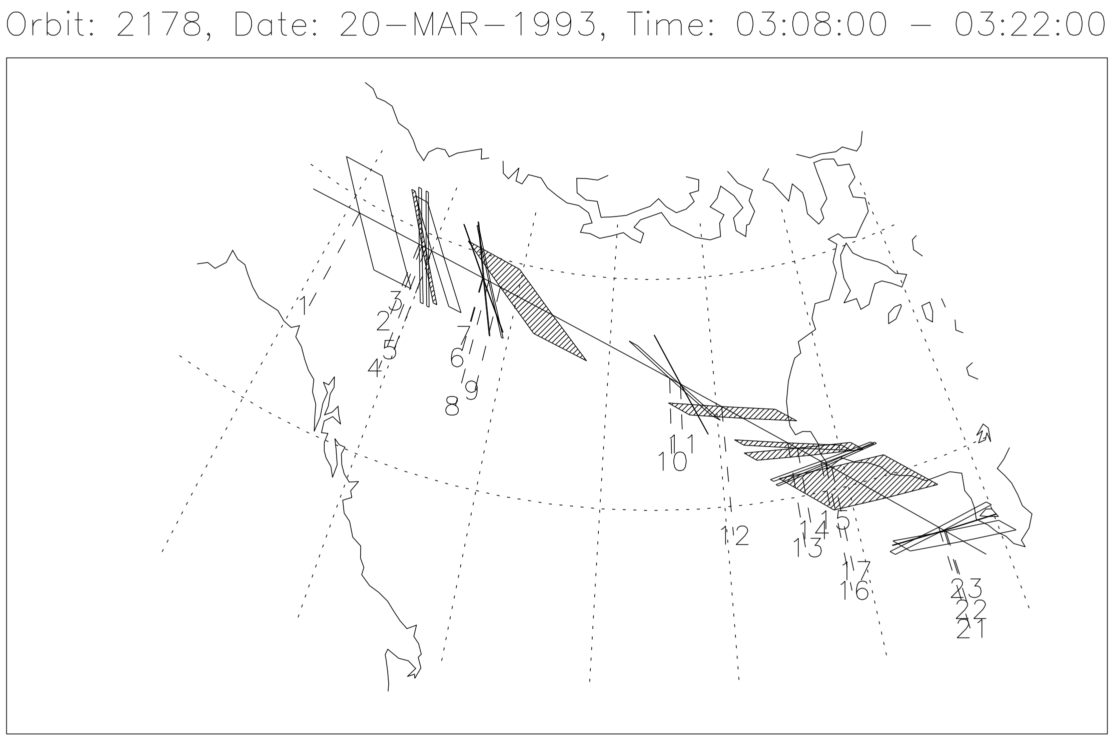
$$\mathsf{M} = \begin{pmatrix} \mathrm{cov}(B_x, B_x) & \mathrm{cov}(B_x, B_y) & \mathrm{cov}(B_x, B_z) \\ \mathrm{cov}(B_y, B_x) & \mathrm{cov}(B_y, B_y) & \mathrm{cov}(B_y, B_z) \\ \mathrm{cov}(B_z, B_x) & \mathrm{cov}(B_z, B_y) & \mathrm{cov}(B_z, B_z) \end{pmatrix}$$

Assessment of model quality: **eigenvalue ratios** λ_3/λ_2 and λ_3/λ_1 $\rightarrow$ should be small.

Hodogram plots: project magnetic field components onto eigenvectors of M, then plot (B_1 vs. B_2) and (B_1 vs. B_3).

Details: see chapter 8 of the ISSI Cluster data analysis book.

Sheet current estimation using FREJA data



Current density in sheets: $j_{\|} = \frac{\Delta_{\perp} B}{\mu_0 \, \mathbf{v} \cdot \hat{\mathbf{n}} \, \Delta t}$.

3 Multi-Spacecraft Data

More information than in single-spacecraft missions **but** spatial resolution still much less than temporal resolution.

Different approaches to data analysis and interpretation:

(1) Apply single-spacecraft techniques separately, estimate model parameters for each satellite, and compare.

- ▷ Assess evolution/variation of parameters.
- ▷ Consistency checks.

Different approaches to data analysis (cont'd)

(2) Build refined models based on the degree of coherency/correlation between measurements at different spacecraft.

- ▷ Use timing information of boundary crossings to perform **boundary parameter estimation**.
- ▷ Interpret differences in measurements as (linear) spatial variations to estimate **spatial derivatives (curl, div, grad)**.
- ▷ Interpret differences in measurements as phase variations of plane waves to estimate **wave parameters (k-filtering = wave telescope technique)**.

Boundary analysis: crossing times approach

Boundary velocity $\mathbf{V} = V\hat{\mathbf{n}}$, crossings at t_α, $\mathbf{r}_\alpha$.

For convenience, choose time and space origin such that

$$\sum_\alpha \mathbf{r}_\alpha = 0 \quad , \quad \sum_\alpha t_\alpha = 0 \ .$$

Now **minimize the cost function**

$$\sum_\alpha [\hat{\mathbf{n}} \cdot \mathbf{r}_\alpha - V t_\alpha]^2 \ = \ \text{Min!} \quad \Leftrightarrow \quad \sum_\alpha [\mathbf{m} \cdot \mathbf{r}_\alpha - t_\alpha]^2 \ = \ \text{Min!}$$

w.r.t. $\hat{\mathbf{n}}$ and V or $\mathbf{m} = \hat{\mathbf{n}}/V$ ('slowness').

Crossing times approach (cont'd)

Result: $\underbrace{\left(\sum_\alpha \mathbf{r}_\alpha \mathbf{r}_\alpha^\dagger\right)}_{=\mathsf{R}} \mathbf{m} \;=\; \sum_\alpha t_\alpha \mathbf{r}_\alpha \quad \Rightarrow \quad \mathbf{m} \;=\; \mathsf{R}^{-1} \sum_\alpha t_\alpha \mathbf{r}_\alpha$.

Cluster-II mission: four satellites, then

$$\mathsf{R}^{-1} \;=\; \mathsf{K} \;=\; \sum_\alpha \mathbf{k}_\alpha \mathbf{k}_\alpha^\dagger$$

where the vectors $\mathbf{k}_\alpha$ are the **reciprocal vectors**.

Explicit formula: $\boxed{\mathbf{m} \;=\; \sum_\alpha t_\alpha \mathbf{k}_\alpha} \rightarrow V = 1/|\mathbf{m}|, \hat{\mathbf{n}} = V\mathbf{m}.$

Reciprocal vectors of the Cluster-II tetrahedron

Cluster-II mission: four satellites, tetrahedral configuration.

Reciprocal base of the tetrahedron: $\{\mathbf{k}_\alpha\}$, defined through

$$\mathbf{k}_\alpha = \frac{\mathbf{r}_{\beta\gamma} \times \mathbf{r}_{\beta\lambda}}{\mathbf{r}_{\beta\alpha} \cdot (\mathbf{r}_{\beta\gamma} \times \mathbf{r}_{\beta\lambda})}$$

where $\mathbf{r}_{\alpha\beta} = \mathbf{r}_\beta - \mathbf{r}_\alpha$ are relative position vectors, $(\alpha, \beta, \gamma, \lambda)$ must be a permutation of $(0, 1, 2, 3)$.

Very useful for **boundary analysis**, estimation of **spatial derivatives** (curl, div, grad), characterization of the **geometric quality** of the tetrahedron.

Current estimation using multi-spacecraft data

Electrical currents from magnetic field data: estimate $\nabla \times \mathbf{B}$.

Linear estimators for spatial derivatives:

$$\boxed{\begin{array}{rcl} \nabla \times \mathbf{B} & \simeq & \sum_\alpha \mathbf{k}_\alpha \times \mathbf{B}_\alpha \\ \nabla \cdot \mathbf{B} & \simeq & \sum_\alpha \mathbf{k}_\alpha \cdot \mathbf{B}_\alpha \equiv \sum_\alpha \mathbf{k}_\alpha^\dagger \mathbf{B}_\alpha \\ \nabla \mathbf{B} & \simeq & \sum_\alpha \mathbf{k}_\alpha \mathbf{B}_\alpha^\dagger \end{array}}$$

Gradient of scalar field: $\nabla p \simeq \sum_\alpha \mathbf{k}_\alpha p_\alpha$

Error analysis of spatial derivative estimation

Sources of error:

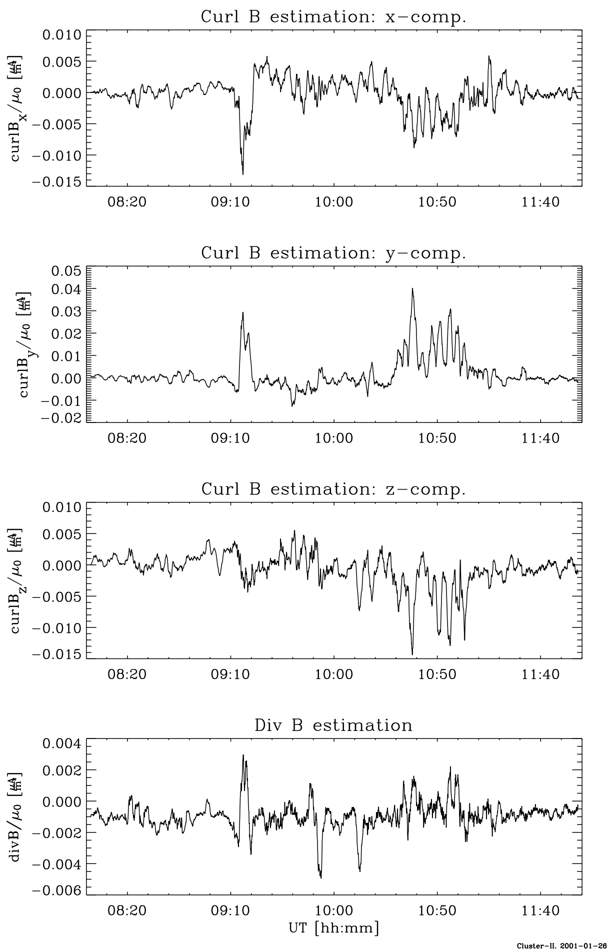
- ▷ FGM measurement errors δB,
- ▷ position inaccuracies δr,
- ▷ nonlinear field variations.

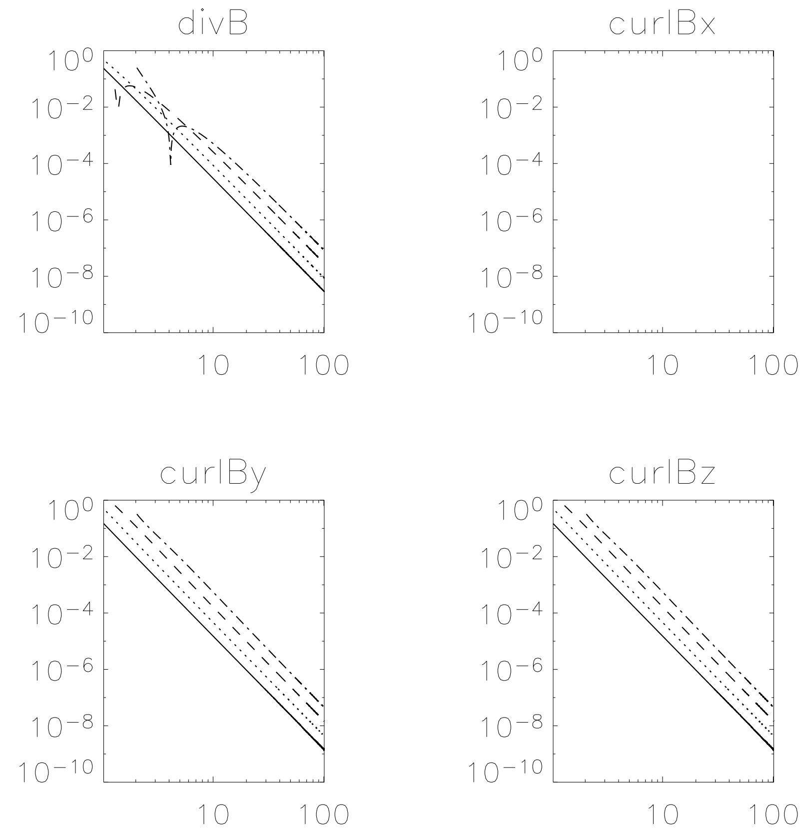
Effect of δB on gradient estimation errors, approx. formula:

$$\boxed{\left\langle (\delta\,|D\mathbf{B}|)^2 \right\rangle \;=\; \frac{f}{3} \sum_{\alpha} |\mathbf{k}_\alpha|^2 \, (\delta B)^2}$$

Here $f = 3$ for $\nabla \cdot \mathbf{B}$, $f = 2$ for $\nabla \times \mathbf{B}$, $f = 1$ for $\hat{\mathrm{e}} \cdot \nabla\mathbf{B}$.

Computer session assignments





Model field: radial3d. noise level: 0.00000

Estimation of inhomogeneity length scales

Unit vector field: $\hat{\mathbf{B}} = \mathbf{B}/B = \mathbf{B}/|\mathbf{B}|$.

Associated gradient matrix: $\nabla\hat{\mathbf{B}} \simeq \sum_\alpha \mathbf{k}_\alpha \hat{\mathbf{B}}_\alpha^\dagger$.

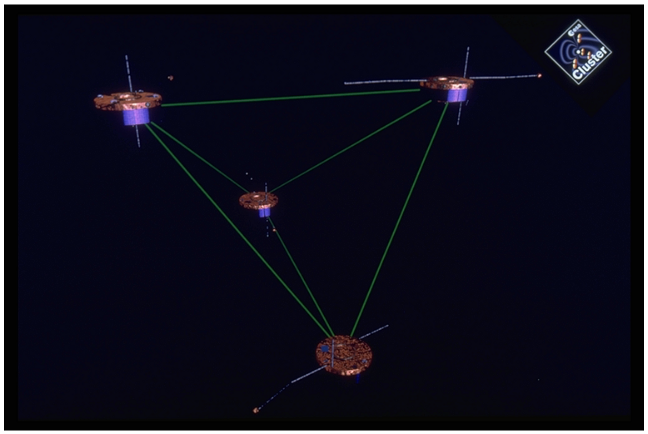
Curvature: $\hat{\mathbf{B}} \cdot \nabla\hat{\mathbf{B}}$, curvature radius

$$R_{\mathsf{curv}} \ = \ |\hat{\mathbf{B}} \cdot \nabla\hat{\mathbf{B}}|^{-1} \ .$$

Other inhomogeneity length scales measure the **convergence** of field lines ($\hat{\mathbf{V}} \perp \hat{\mathbf{B}}$):

$$R_{\mathsf{conv}} = |\hat{\mathbf{V}} \cdot \nabla\hat{\mathbf{B}}|^{-1}$$

Wave identification using Cluster-II



[Image credit: ESA]

Cluster-II mission

Four point measurements allow to study spatiotemporal phenomena: Alfvén waves, surface waves, turbulence . . .

Spatial coverage too bad to determine $\mathbf{k}$-spectrum directly

$\longrightarrow$ use methods from array signal (e.g. seismic data) processing.

Wave identification using Cluster-II (cont'd)

Fourier transformed (scalar) data: $b_\alpha(\omega)$.

Method can be generalized to vector data $\mathbf{B}_\alpha$.

Data vector	**CSD matrix**	**Phase delay vector**
$\mathbf{b}(\omega) = \begin{pmatrix} b_{\alpha=1}(\omega) \\ b_{\alpha=2}(\omega) \\ b_{\alpha=3}(\omega) \\ b_{\alpha=4}(\omega) \end{pmatrix}$	$\mathsf{C} = \langle \mathbf{b}\mathbf{b}^\dagger \rangle$	$\hat{\mathbf{h}}(\mathbf{k}) = \frac{1}{\sqrt{N}} \begin{pmatrix} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}_1} \\ \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}_2} \\ \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}_3} \\ \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}_4} \end{pmatrix}$
	(**measurements**)	(**array geometry**)

Wave identification using Cluster-II (cont'd)

High-resolution beamformers, idea: make the sensor array most sensitive in one "looking direction" through assignment of sensor weights.

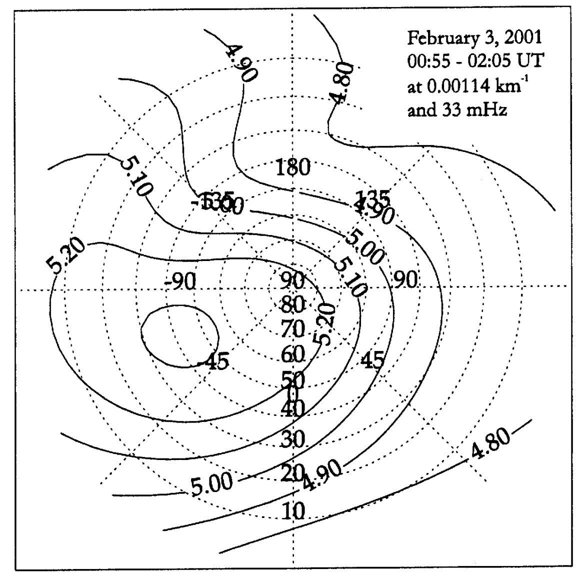
Other names: wave telescope, k-filtering technique, minimum variance estimators, Capon estimators. . .

Resulting estimator for the power spectrum:

$$\boxed{P(\omega, \mathbf{k}) = \left(\hat{\mathbf{h}}^{\dagger} \mathsf{C}^{-1} \hat{\mathbf{h}}\right)^{-1}}$$

Cluster-II wave identification: analysis procedure

- ▷ compute $\mathsf{C}(\omega)$ and the inverse matrix C^{-1},
- ▷ compute $\mathrm{Tr}\{\mathsf{C}(\omega)\}$ (i.e., frequency spectrum) and identify peaks,
- ▷ discretize $\mathbf{k}$-space and compute $\hat{\mathbf{h}}(\mathbf{k})$,
- ▷ construct $P(\omega, \mathbf{k})$ and search for peaks.



[Glassmeier et al., 2001]

4 Summary

Single-spacecraft missions: space-time ambiguity, additional information needed, minimum variance analysis.

Multi-spacecraft missions: much better chance to study spatio-temporal phenomena.

Cluster-II: **reciprocal vectors** useful for the analysis of discontinuities and spatial gradients.

Cluster-II as a **wave telescope**.

Computer session

http://www.faculty.iu-bremen.de/jvogt/cospar/cbw6/intro/

(1) Getting started with IDL

(2) Probability density estimation using the kernel method

(3) Gradient estimation accuracy in model magnetic fields

(4) Gradient estimation in measured magnetic fields

(5) Magnetospheric boundary analysis

Review of basic concepts in time series analysis

See textbooks and web resouces on data analysis, e.g.:

http://www.faculty.iu-bremen.de/jvogt/edu/fall06/c210222/space/

(1) Statistical description of data

(2) Correlation and regression

(3) Fourier transformation and spectral analysis

(4) Basic aspects of time series filtering