STIINTE-02 : Methods

S. Haaland :
Discontinuity Analysis with Cluster – an overview

A. Blagau :
Nonplanar discontinuities

J. Vogt :
Estimation of gradients from 3 spacecraft

D. Constantinescu :
Wave detection with sensor arrays

E. Yordanova :
Turbulence (talk originally scheduled for Wednesday)

Discontinuity Analysis with Cluster

S. Haaland, B.U.Ö. Sonnerup, G. Paschmann,

A. Blagau, E. Georgescu

with additional inputs from

The Cluster CIS, FGM and EFW teams,

The QSAS team,

Organization

- Overview

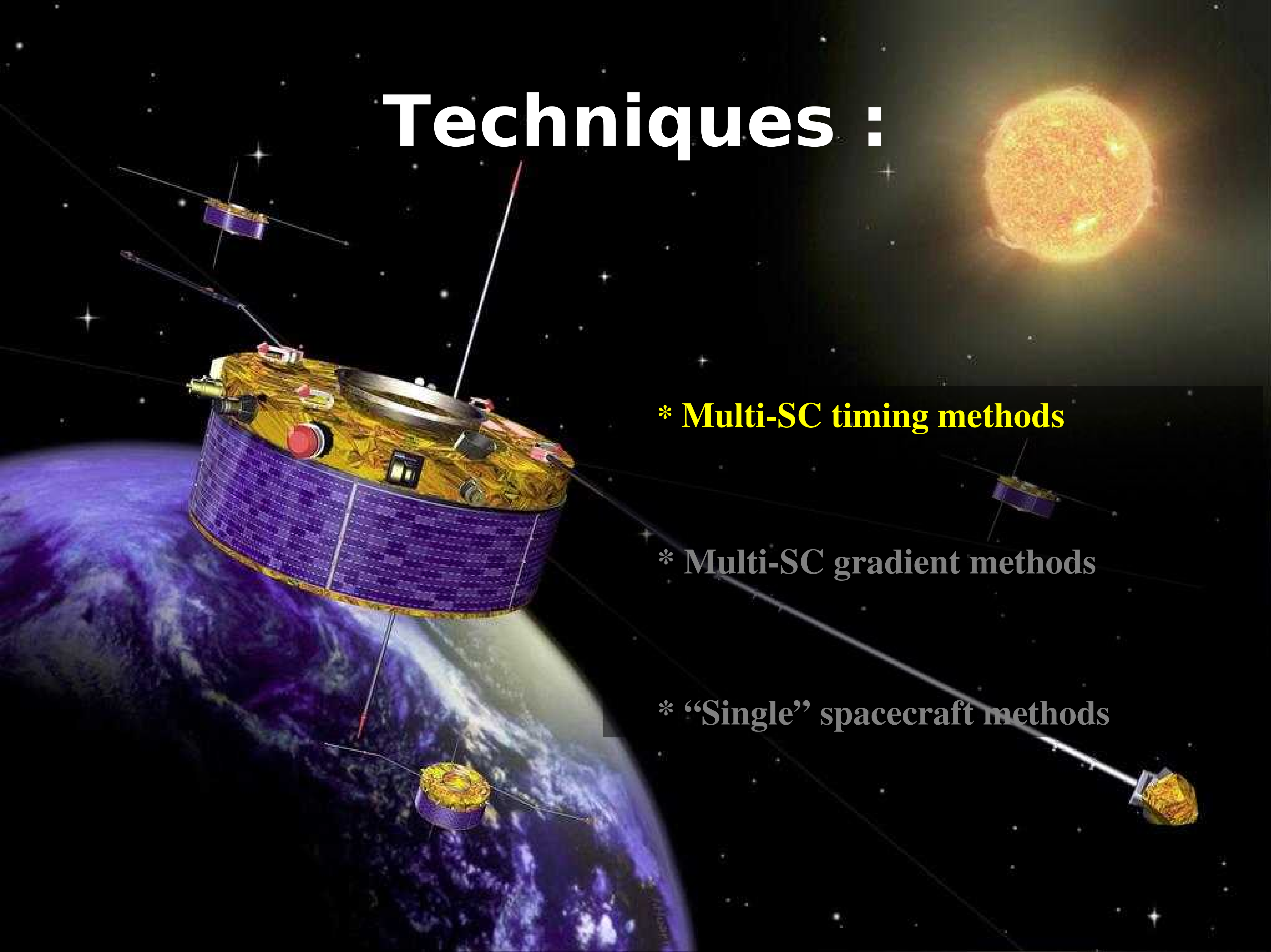
- Techniques
  - multi- versus single spacecraft methods

- Science results
  - Orientation, thickness, current density

- Error analysis for timing methods

Timing Methods			**Normal**	**Velocity**	**Jahr**
CVA	**(Constant Velocity Approach)**	**Russell, 1982**	*V*	*V*	*1982*
DA	**(Discontinuity Analyzer)**	**Dunlop, 1998**	*V*	*V*	*2001*
CTA	**(Constant Thickness Approach)**	**Haaland, 2004**	*V*	*V*	*2004*
MTV	**(Minimum Thickness Variation)**	**Paschmann, 2005**	*V*	*V*	*2005*
MVV	**(Minimum Velocity Variation)**	**(Haaland, 2006)**	*V*	*V*	*2005*
Gradient Methods					
GRA	**(Gradient of any Quantity)**	**Shen, 2003**	*V*	*X*	*2002*
MVAJ	**(Minimum Variance of J)**	**Haaland, 2004**	*V*	*V*	*2004*
MVAcE	**(Minimum Variance of $\nabla \times$ E)**	**In progress**	*V*	*V*	*2005*
MDD	**(Minimum Directional Derivative)**	**Shi, 2005**	*V*	*X*	*2005*
STD	**(Spatio Temporal Derivative)**	**Shi, 2006**	*X*	*V*	*2005*
Single-spacecraft Methods					
MVA	**(Min/Max Variance of B, V, E, dB/dt)**	**Sonnerup, 1967**	*V*	*X*	*1967*
HT	**(deHoffmann-Teller Analysis)**	**?**	*X*	*V*	*(~1980 ?)*
MFR	**(Minimum Faraday Residue)**	**Khrabrov, 1998**	*V*	*V*	*1998*
MMR	**(Minimum Massflow Residue)**	**Sonnerup, 1998,2004**	*V*	*V*	*2005*
MLMR	**(Minimum Linear Momentum Residue)**	**Sonnerup, 2005**	*V*	*V*	*2006*
MTER	**(Minimum Total Energy Residue)**	**“**	*V*	*V*	*2006*
MER	**(Minimum Entropy Residue)**	**“**	*V*	*V*	*2006*
COM	**(Combination of above)**	**“**	*V*	*V*	*2006*
Remote sensing					
ASYM	**..in the magnetotail**	**Roelof, 1976**	*X*	*V*	*1976*
ASYM	**..in the magnetosheath**	**West & Buck, 1976**	*X*	*V*	*1976*
ASYM	**..in the distant tail**	**Scholer 1983,1984**	*X*	*V*	*1983*
ASYM	**..ionospere (DMSP, < 1000 km)**	**Newell, 1991**	*X*	*V*	*1991*

Techniques :

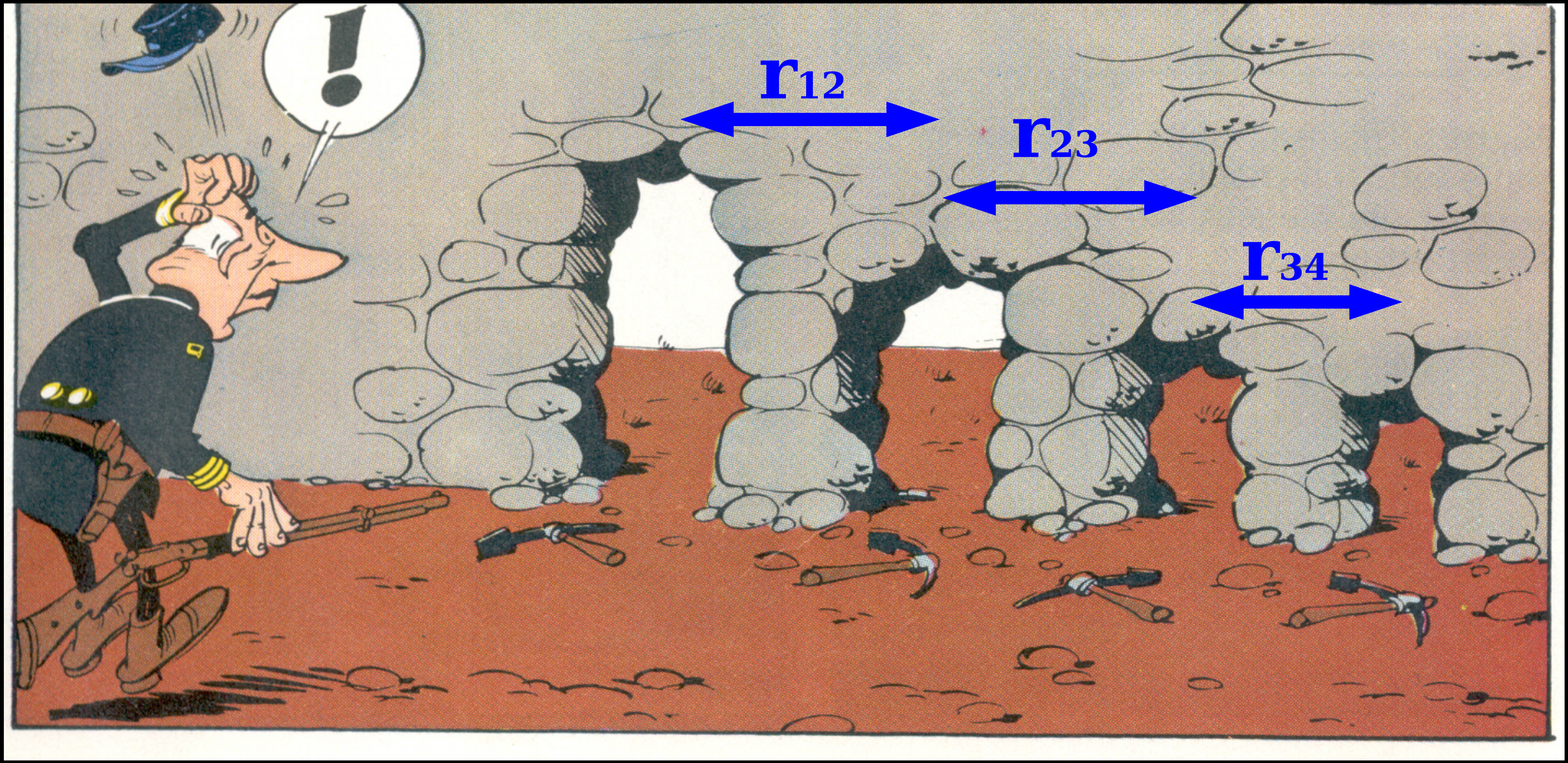


* **Multi-SC timing methods**

* **Multi-SC gradient methods**

* **“Single” spacecraft methods**

Timing Methods :



Orientation, Velocity :

- find out WHEN each SC crossed discontinuity

Thickness, Acceleration :

- find DURATION of crossing

Multi-SC Timing Methods I :
Constant Velocity (CVA)

$$\begin{bmatrix} \mathbf{r}_{12} \\ \mathbf{r}_{13} \\ \mathbf{r}_{14} \end{bmatrix} \cdot \frac{1}{V} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} t_{12} \\ t_{13} \\ t_{14} \end{bmatrix}$$

Caveats :

- no acceleration

-> not good for wavy MP, cusp, magnetotail

Multi-SC Timing Methods II :

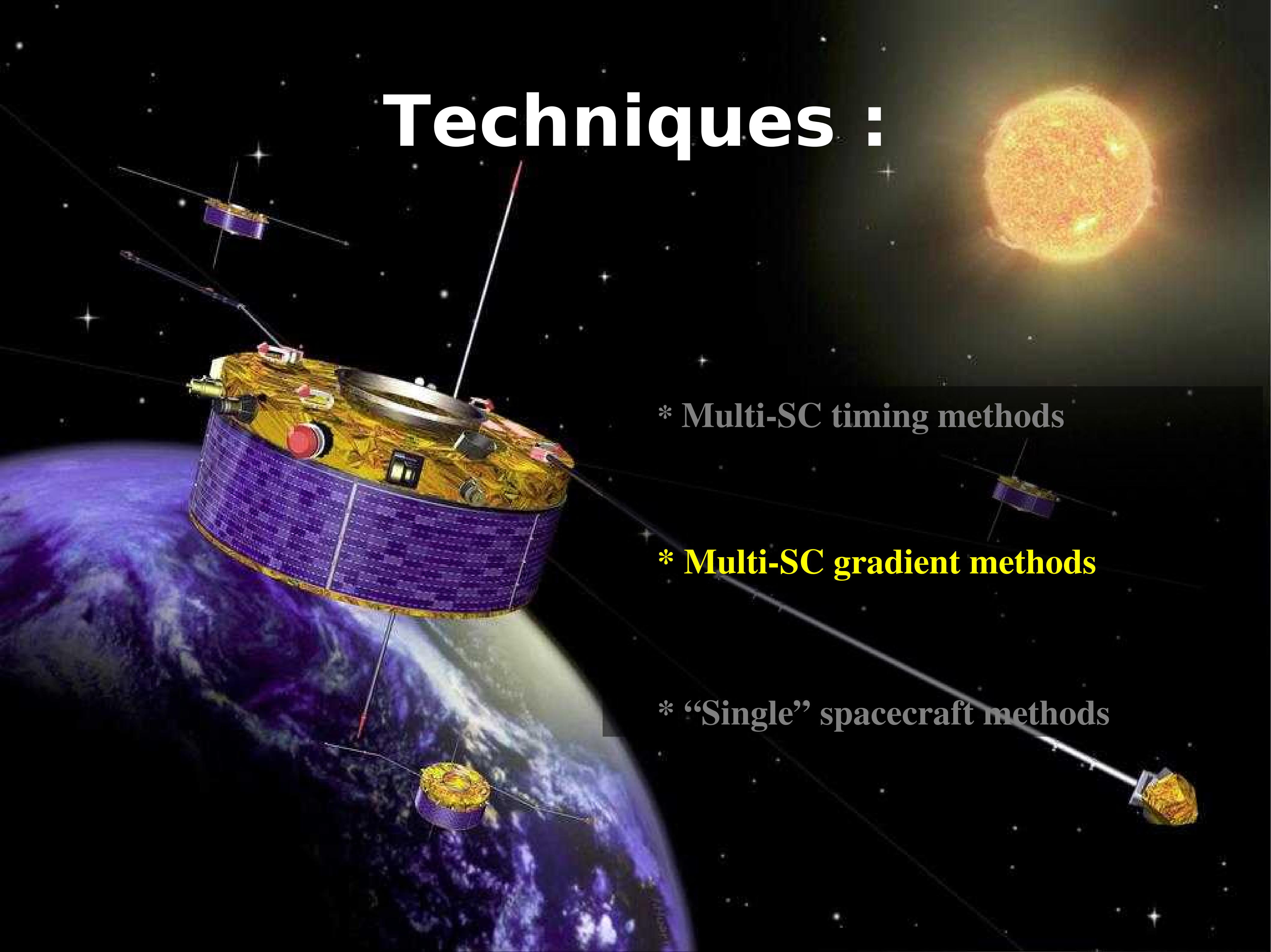
Acceleration (CTA)

1) $V(t) = V_0 + a_1 t + a_2 t^2 + a_3 t^3$

2) $d = \int_{T - \tau}^{T + \tau} V(t)\, dt$

3) $\mathbf{R}_i \cdot \mathbf{n} = \int_{t = 0}^{t = t_i} V(t)\, dt$

Techniques :



* Multi-SC timing methods

* Multi-SC gradient methods

* “Single” spacecraft methods

Gradient of a scalar quantity (GRA)

(e.g., Shen et al, 2003)

$$\boldsymbol{n} = \nabla f(t) \ / \ \left|\nabla f(t)\right|$$

Where f(t) is a scalar quantity, e.g., the magnetic pressure or plasma density (from EFW)

Spatio Temporal Derivative (STD)

(Shi et al, GRL, 2006)

$$\frac{d\mathbf{B}}{dt} = \frac{\partial \mathbf{B}}{\partial t} - \mathbf{V} \cdot \nabla \mathbf{B}$$

For a quasi stationary structure :

$$\frac{d\mathbf{B}}{dt} + \mathbf{V} \cdot \nabla \mathbf{B} = 0$$

Minimal Directional Derivative (MDD)

(Shi et al, GRL, 2005)

$$\boldsymbol{D}(t)=\boldsymbol{n}(t)\cdot\nabla\boldsymbol{B}(t)$$

$$=\left[\frac{\partial B_x}{\partial n},\ \frac{\partial B_y}{\partial n},\ \frac{\partial B_z}{\partial n}\right]$$

$$L_{ij}=(\nabla\boldsymbol{B})(\nabla\boldsymbol{B})^T=\sum_{k=1}^{3}\frac{\partial B_k}{\partial x_i}\frac{\partial B_k}{\partial x_j},\qquad(i,j=1..3)$$

MVAJ: Minimum variance of current

(Haaland et al, 2004)

$\nabla \cdot \boldsymbol{B} = 0$ (conservation of magnetic poles)

$\nabla \cdot \boldsymbol{J} = 0$ (conservation of charge)

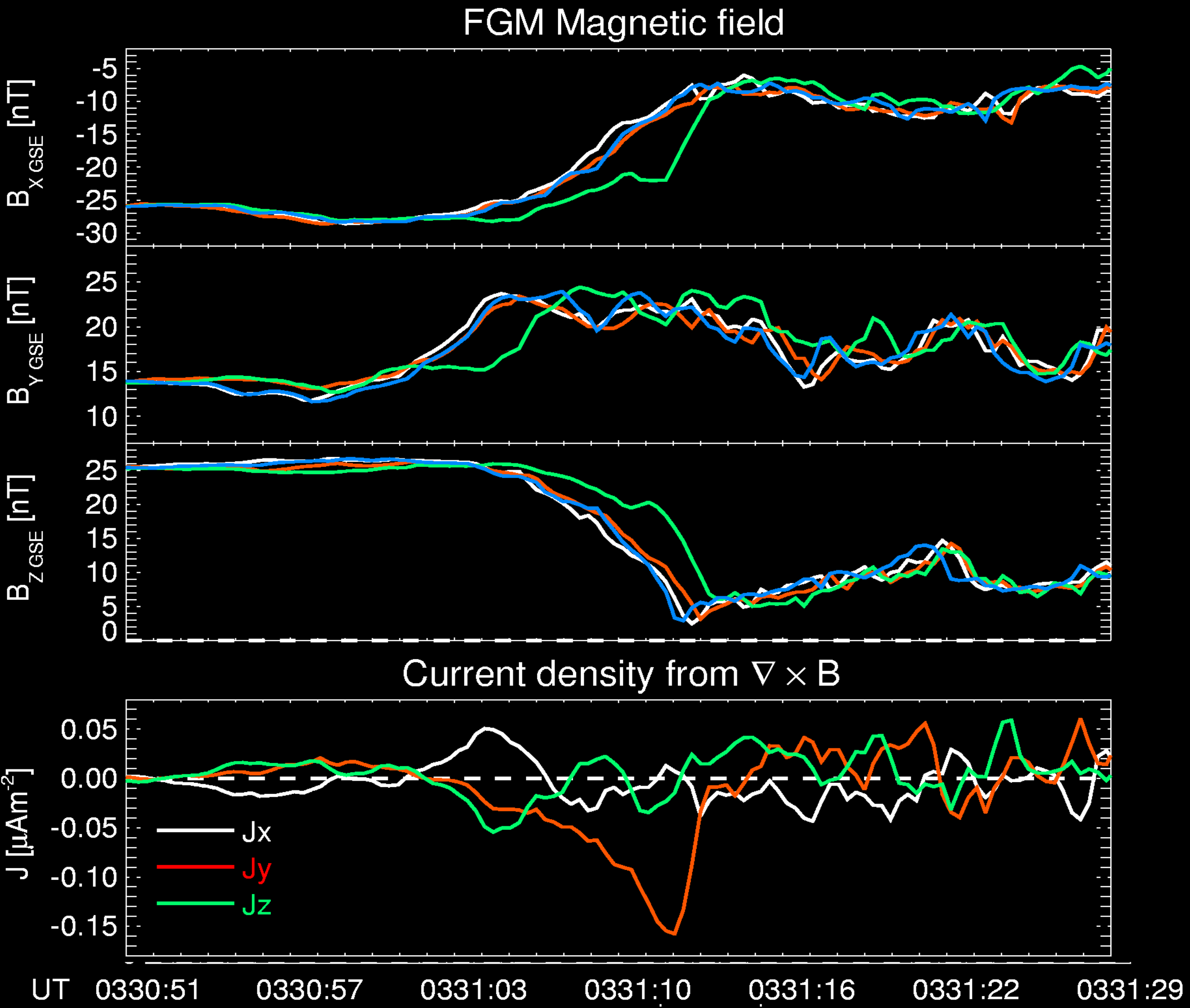
$$\boxed{Q_{ij}^{J} = \langle J_i \; J_j \rangle - \langle J_i \rangle \langle J_j \rangle}$$

MVAJ : **eigenvalues** and **eigenvectors** of Q_{ij}^{J}

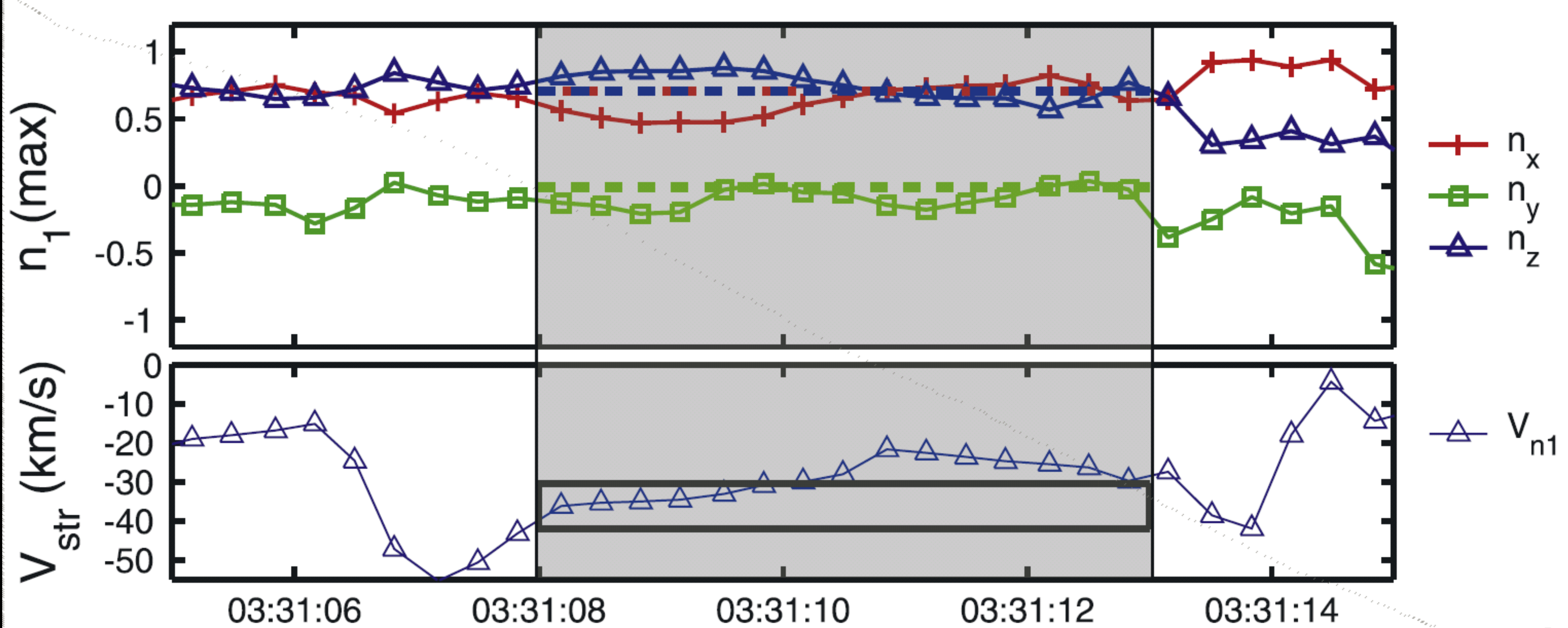
Velocity from MVAJ : Integrate Ampère's law

$$\mu_0 J_i = (\nabla \times \boldsymbol{B})_i = -\frac{\partial B_j}{\partial x_k} = -\frac{dB_j}{dt}\frac{1}{v_k}$$

$$\Delta B_j = -\mu_0 v_k \int_0^t J_i \, dt$$



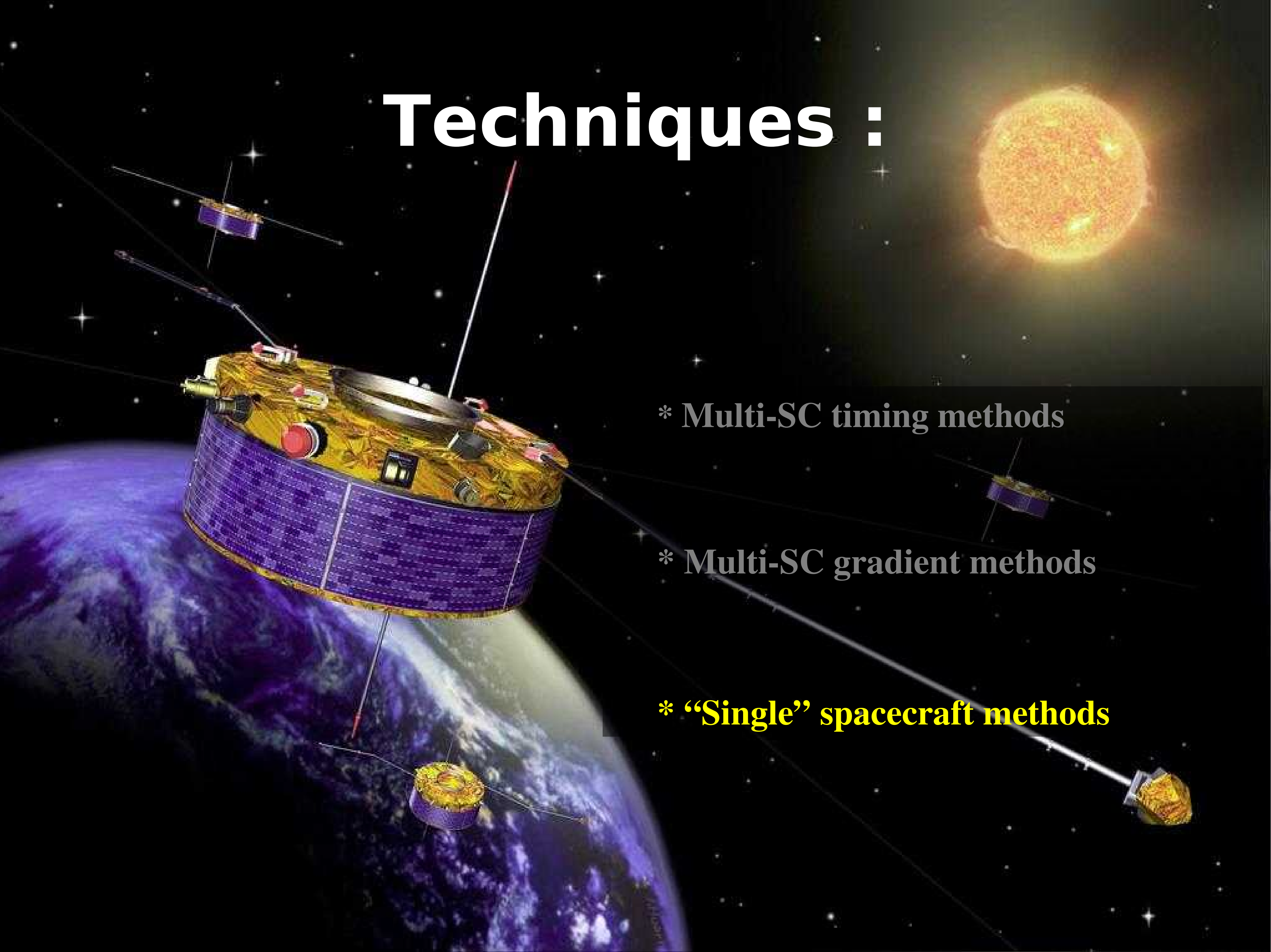
[From Haaland et al, 2004]



Solid lines/symbols = MDD/STD (Shi et al, 2005,2006)
Dashed line, box : MVAJ, CVA, CVA, MVAB, MFR,HT,MMR (Haaland et al, 2004)

[From Shi et al, 2006]

Techniques :

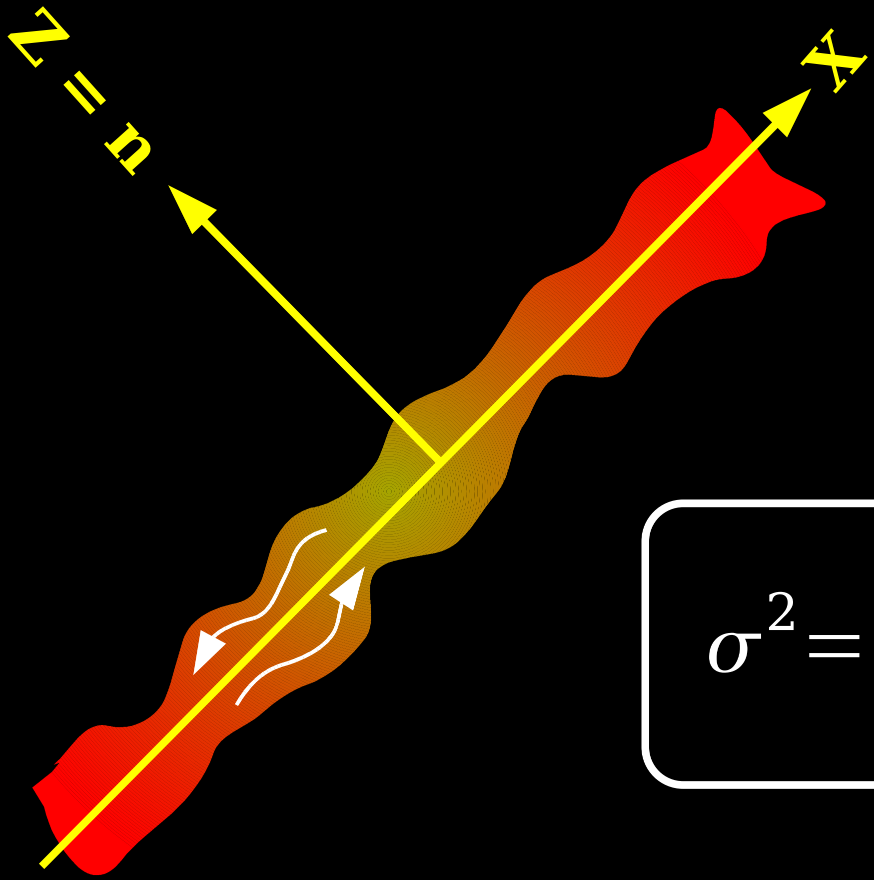


* Multi-SC timing methods

* Multi-SC gradient methods

* “Single” spacecraft methods

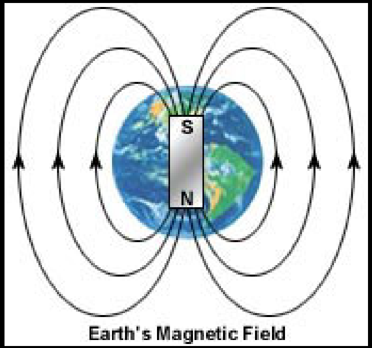
Minimum variance (e.g. B-field)



$$\sigma^2 = \frac{1}{M}\sum_{m=1}^{M}\left|\left(\boldsymbol{B}^{(m)} - \langle\boldsymbol{B}\rangle\right)\cdot\boldsymbol{n}\right|^2$$

- Find direction with **minimum variance** in **B**

Minimum variance – other applications



Earth's Magnetic Field

$$\nabla \cdot \boldsymbol{B} = 0$$

(conservation of magnetic poles)

$$\nabla \cdot \boldsymbol{J} = 0$$

(conservation of charge)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0$$

(conservation of mass flux)

. . .

$$\frac{\partial \eta}{\partial t} + \nabla \cdot q = 0$$

(conservation of any quantity q)

Generic Residue Analysis

$$Q_{ij}^{GENERIC} = \left\langle \left(\Delta q_{ki} - U_i \Delta \eta_k\right) \left(\Delta q_{kj} - U_j \Delta \eta_k\right) \right\rangle$$

$$\left.\begin{array}{l} Q^{MVAB} \\ Q^{MVAJ} \\ Q^{MMR} \\ Q^{MFR} \\ Q^{MTER} \\ \cdots \end{array}\right\}$$

Orientation of discontinuity from any method :

Find eigenvalues and eigenvectors of Q

Combining covariance matrices

1) Combing Q matrices from more spacecraft

$$\boldsymbol{Q_{ALL}} = w_1 \boldsymbol{Q_1} + w_2 \boldsymbol{Q_2} + w_3 \boldsymbol{Q_3} + w_4 \boldsymbol{Q_4}$$

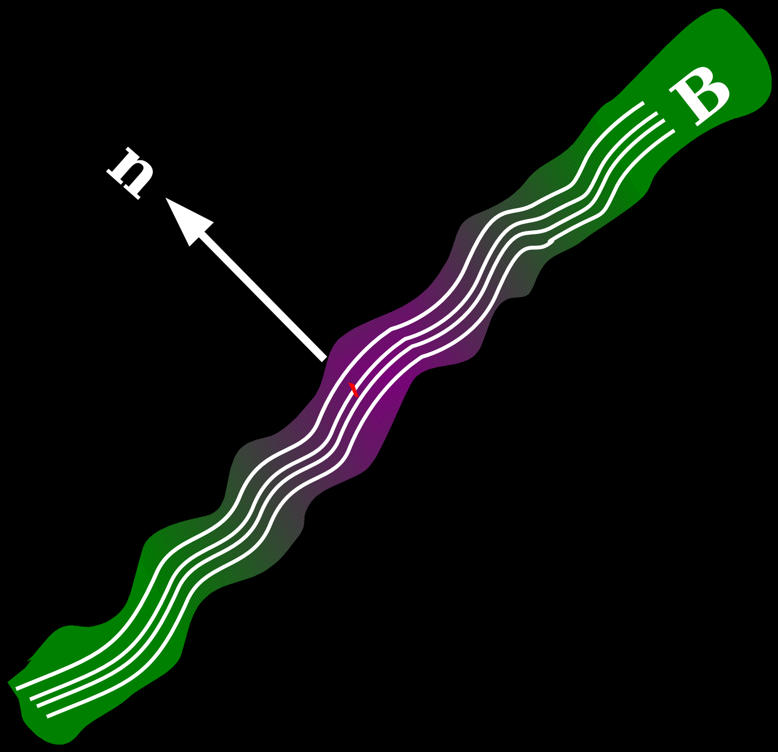
2) Combing Q matrices from several methods

$$\boldsymbol{Q_{COM}} = \sum_{i=0}^{i=N} w_i \boldsymbol{Q_i}, \qquad i = MVAB, MER, MVAJ....$$

3) Constrain Q

$$\boldsymbol{Q_C} = \boldsymbol{P} \cdot \boldsymbol{Q} \cdot \boldsymbol{P}$$

Constraining the variance analysis

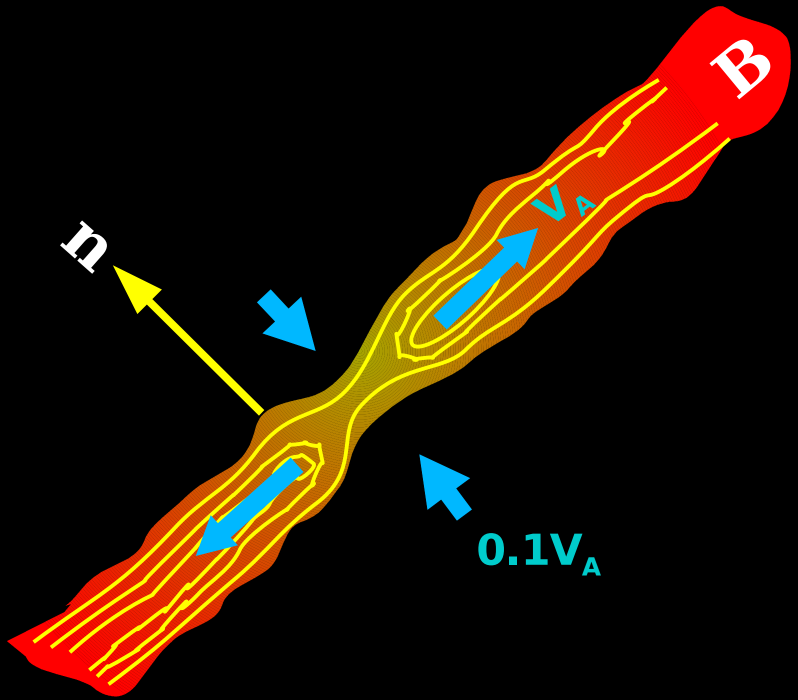


TD – tangentail discontinuity

B•**n** = 0, **V**•**n** = 0, …

Constrain Q matrix so that

B•n == 0



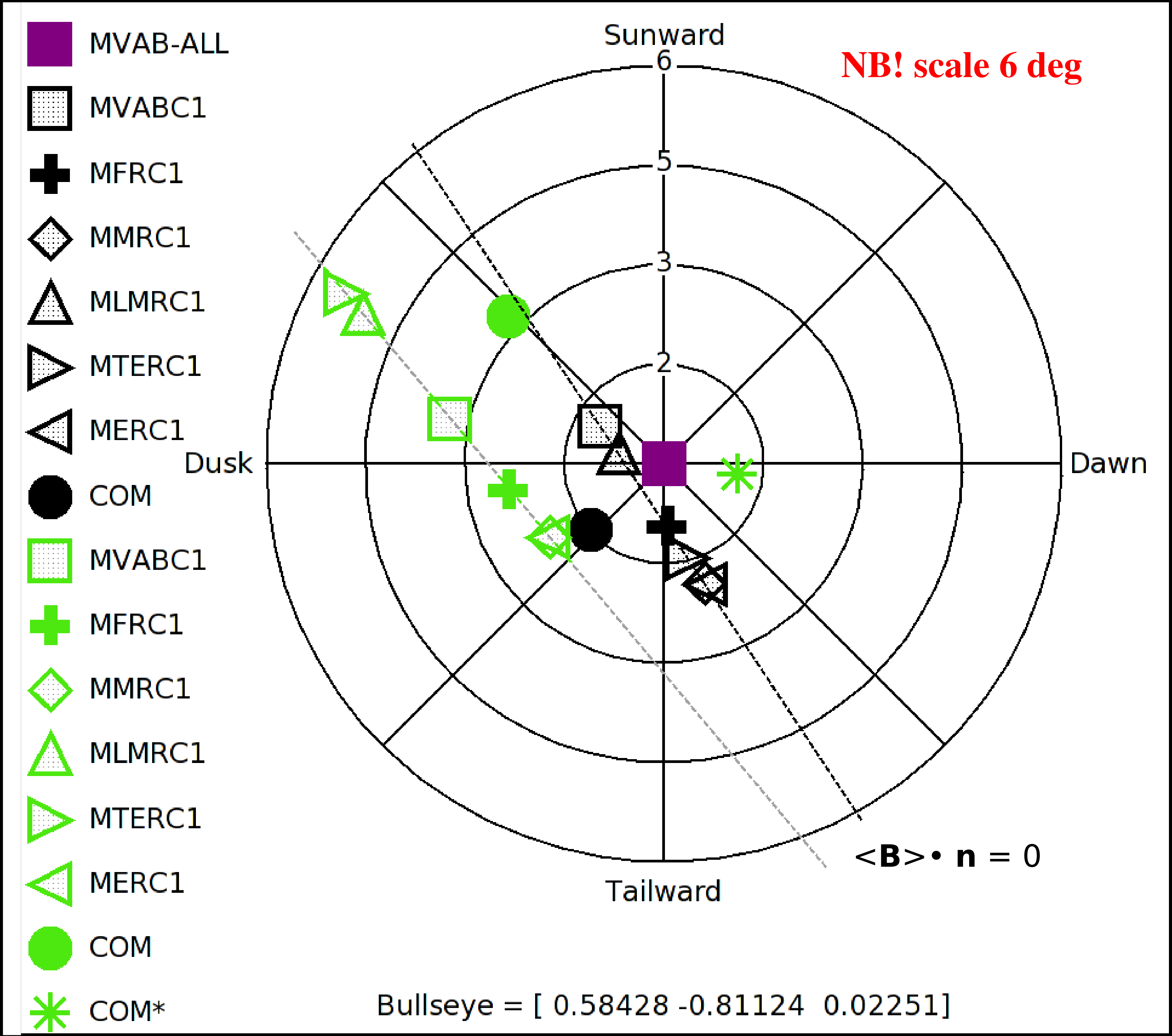
RD – rotational discontinuity

|**B**•**n**| > 0,   **V**•**n** = e.g., V_A

Constrain Q matrix so that

flow across is e.g., Alfvénic

Modified Q matrices - orientation

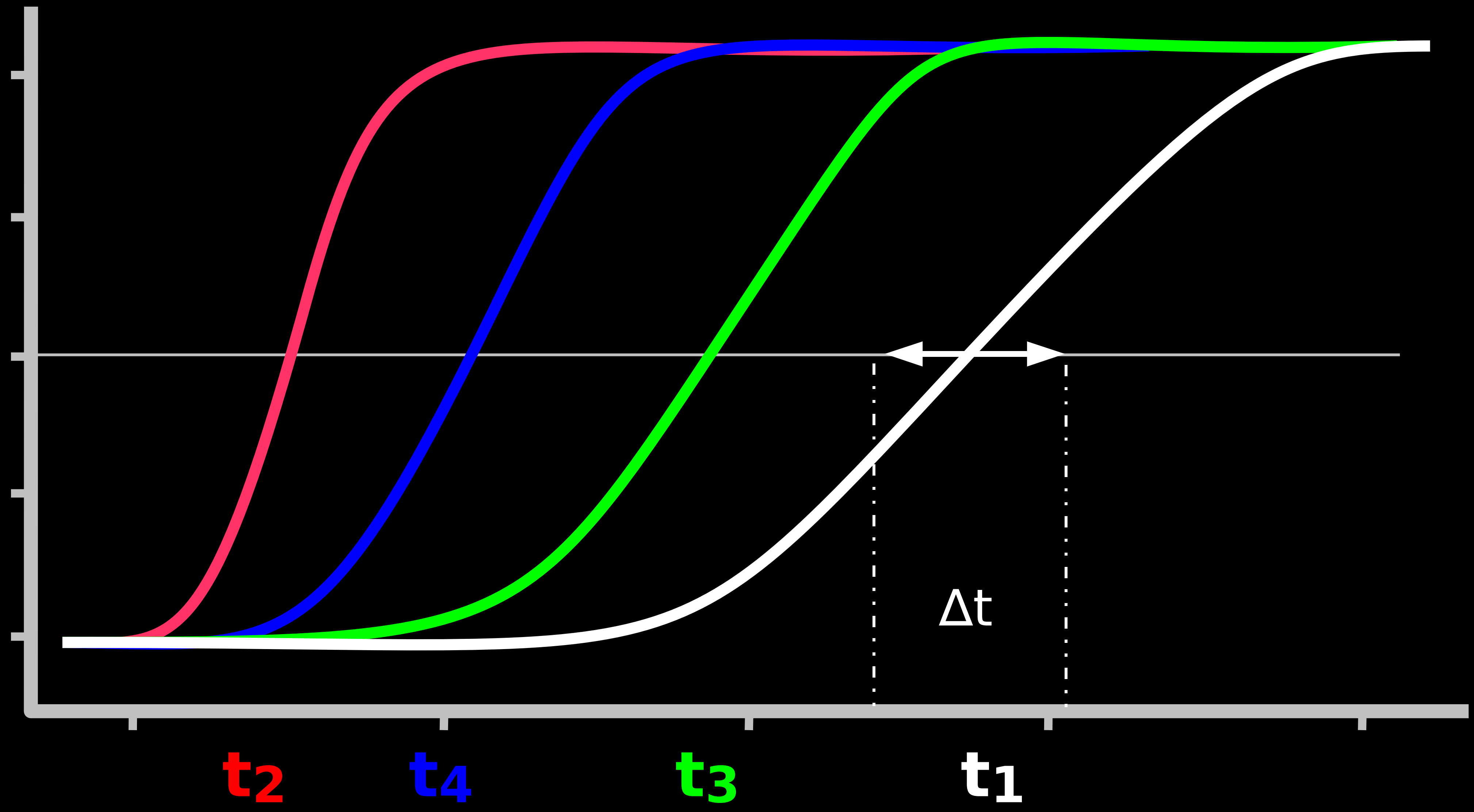


NB! scale 6 deg

Bullseye = [0.58428 -0.81124 0.02251]

Error analysis – timing methods

(based on an idea by T. Knetter)



1) Use t_1, t_2, t_3, $t_4 - \Delta t_4$

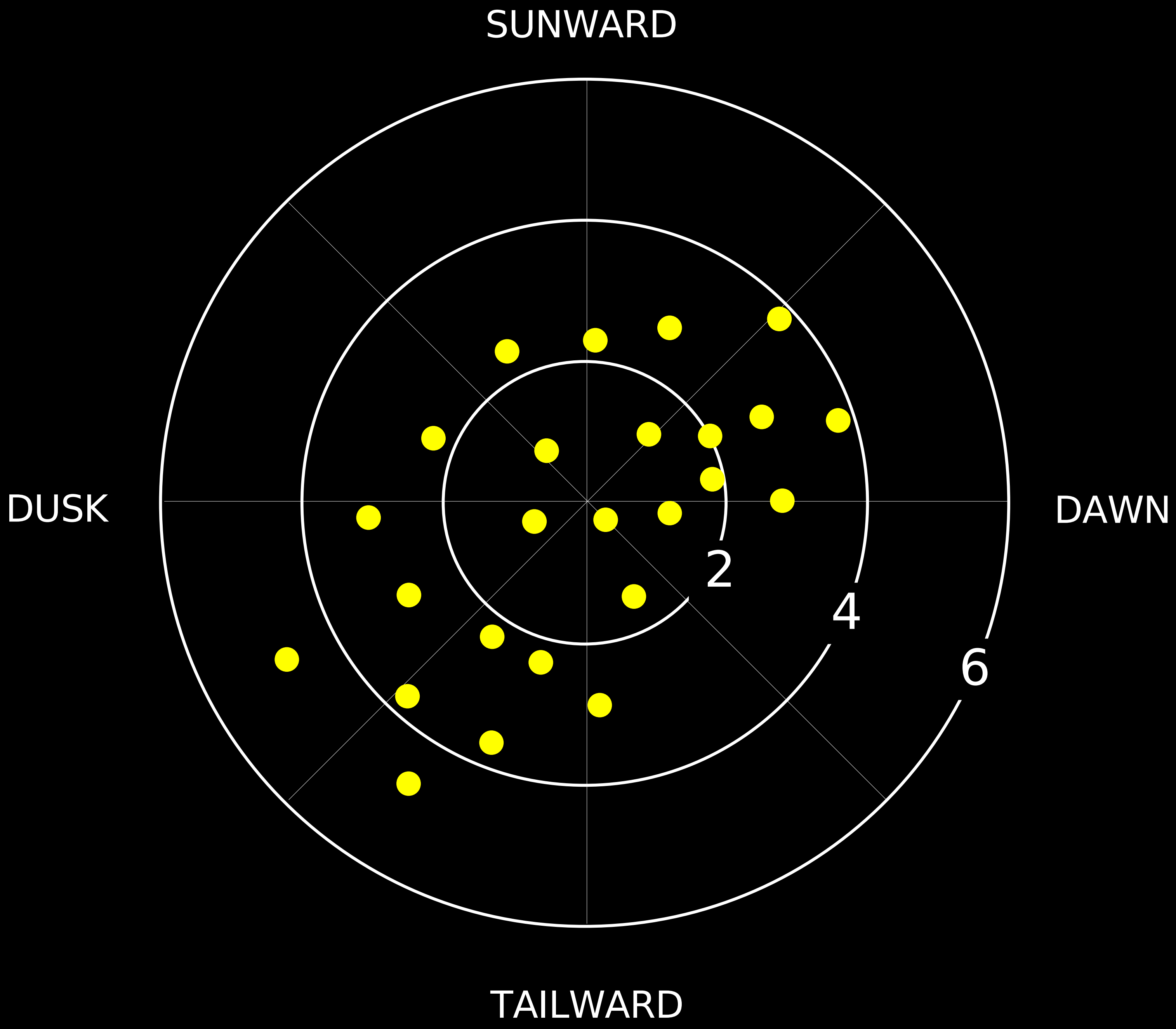
2) Use t_1, t_2, t_1, $t_4 + \Delta t_4$

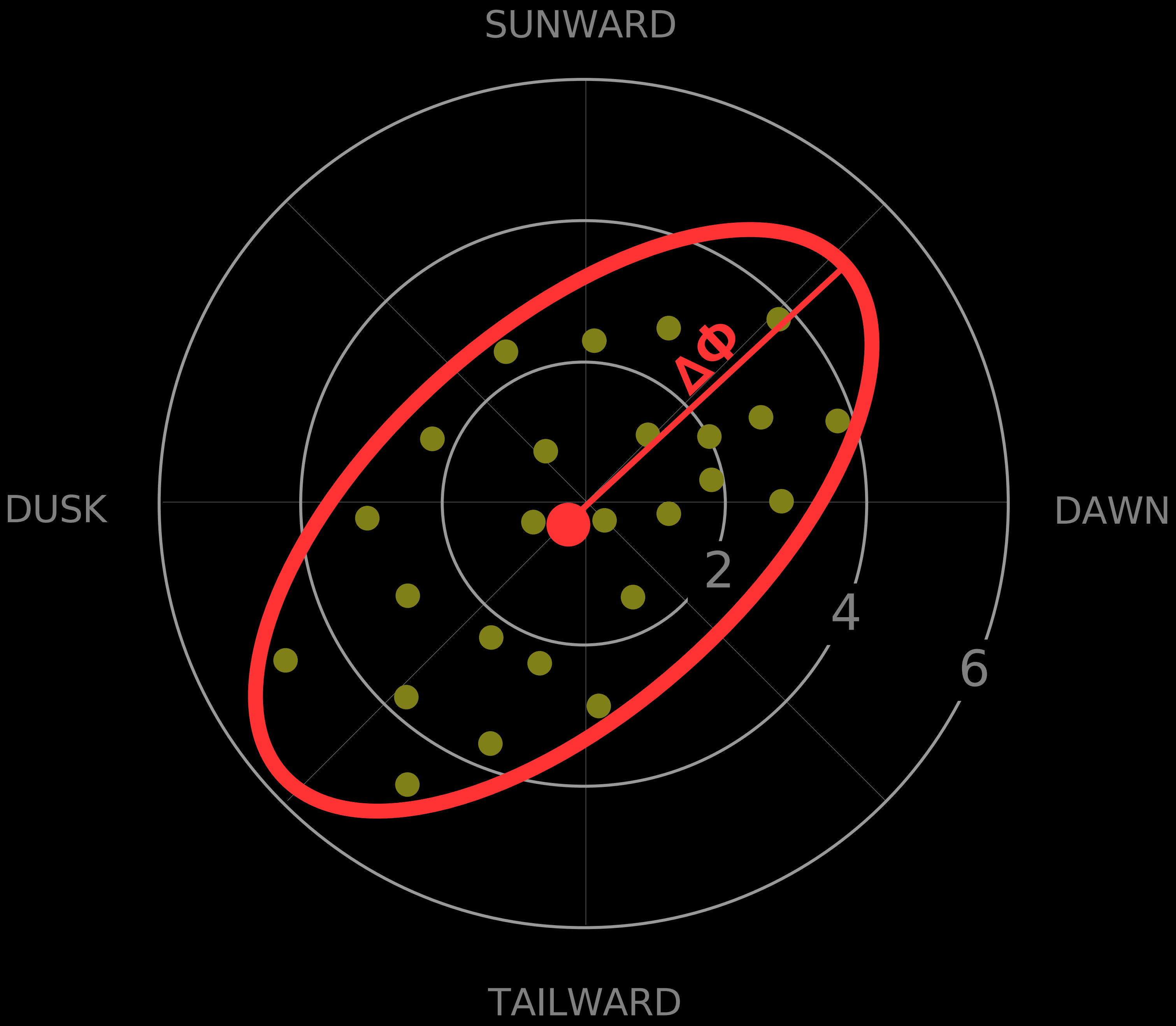
3) Use t_1, t_2, $t_1 + \Delta t_3$, t_4

. . .

. .

81) Use $t_1 + \Delta t_1$, $t_2 + \Delta t_2$, $t_3 + \Delta t_3$, $t_4 + \Delta t_4$


SUNWARD
DUSK
DAWN
TAILWARD
2
4
6


SUNWARD
DUSK
DAWN
TAILWARD
ΔΦ
2
4
6