

Application of multi-moment transport equations to space plasmas simulation

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Moment-closure transport equations

- Starting point
$$\frac{\partial f}{\partial t} + \vec{\nabla}_{\vec{r}} \cdot (\vec{v}f) + \vec{\nabla}_{\vec{v}} \cdot \left(\frac{\vec{F}}{m} f \right) = \frac{\delta f}{\delta t} \Big|_{coll}$$
- Assumptions
 - thermal equilibrium
 - closed system
$$f_o(\vec{r}, \vec{v}) = n_s \left(\frac{m_s}{2\pi k_b T_s} \right)^{3/2} e^{-\frac{m_s(\vec{v}-\vec{u}) \cdot (\vec{v}-\vec{u})}{2k_b T_s}}$$
- Regressive approach

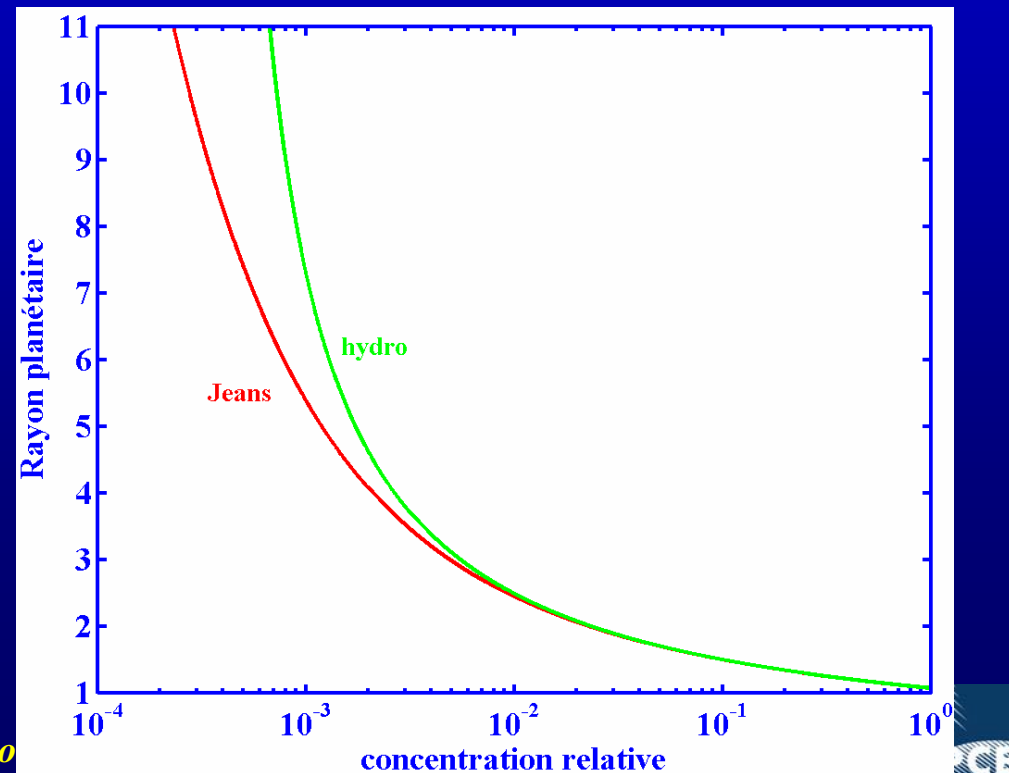
$$f(\vec{r}, \vec{c}_s) = f_o(\vec{r}, \vec{c}_s) \left[1 - \left(1 - \frac{m_s c_s^2}{5k_b T_s} \right) \frac{m_s}{k_b T_s \rho_s} \vec{q}_s \cdot \vec{c}_s \right]$$
 - moments of the distribution function
 - development around the thermal equilibrium
 - transport equation for successive order moments
- Consequence
 - loss of « energy dependent » effects
 - collision frequency, finite Larmor radius, wave particle interaction,...
 - space-velocity coupling \Rightarrow coupled transport equations

Moment-closure transport equations

- In fact
 - open system
 - non thermal equilibrium
- Problem in the representation
 - transport equations
 - unphysical behaviour
 - Hyperbolicity
 - distribution function
 - expansion failure
 - closure relationship
- Mathematics are not physics

Open system

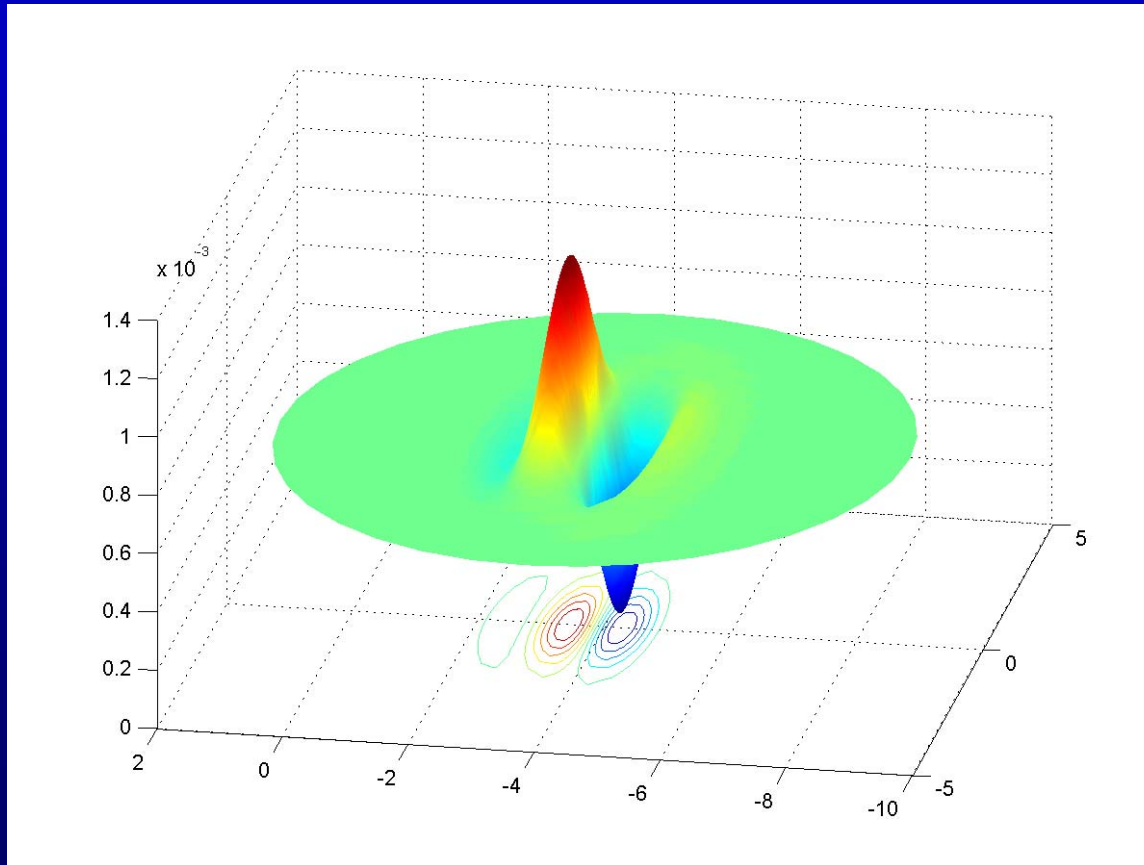
- In the plasmasphere
 - Non relevant
- On open flux tube
 - Similarity with exospheric models
 - loss of particles
 - Altitudinal limitation



Distribution function

- The Taylor expansion does not ensure positivity

$$f(\vec{r}, \vec{c}_s) = f_o(\vec{r}, \vec{c}_s) \left[1 + \frac{m_s}{2k_b T_s p_s} \tau_s : \vec{c}_s \vec{c}_s - \left(1 - \frac{m_s c_s^2}{5k_b T_s} \right) \frac{m_s}{k_b T_s p_s} \vec{q}_s \cdot \vec{c}_s \right]$$

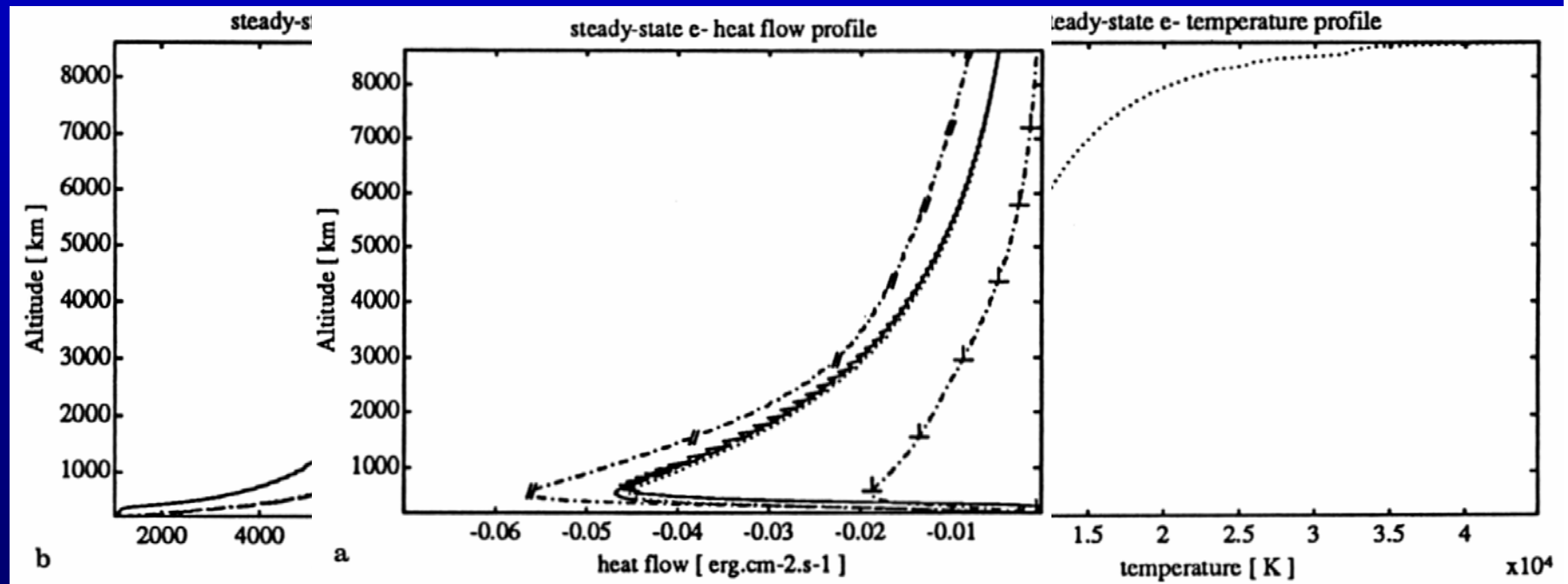


Unphysical behaviour 1

- Ex : 13 moments closure

- Electron temperature equation
$$\frac{\partial T_e}{\partial t} + \vec{\nabla} \cdot \vec{q}_e = 0$$

- Steady state
$$\vec{\nabla} \cdot \vec{q}_e = 0$$



- high electron temperatures

Unphysical behaviour 2

- In 1D
$$\frac{\partial q_e}{\partial r} + \frac{q_e}{A} \frac{\partial A}{\partial r} = 0$$

- For the electron temperature

$$\frac{\partial T_e^{\parallel}}{\partial t} + \frac{6}{5n_e k_b} \frac{\partial q_e}{\partial r} + \frac{2}{5n_e k_b} \frac{q_e}{A} \frac{\partial A}{\partial r} = 0$$

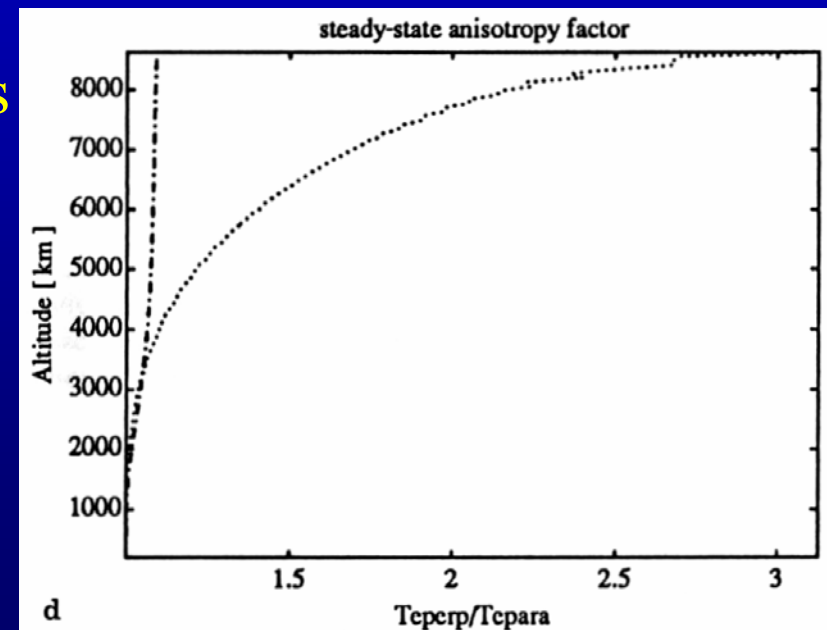
$$\frac{\partial T_e^{\perp}}{\partial t} + \frac{2}{5n_e k_b} \frac{\partial q_e}{\partial r} + \frac{4}{5n_e k_b} \frac{q_e}{A} \frac{\partial A}{\partial r} = 0$$

- If $q_e < 0$

- Perpendicular temperature increases
- Parallel temperature decreases

- Strong anisotropy

- inconsistency



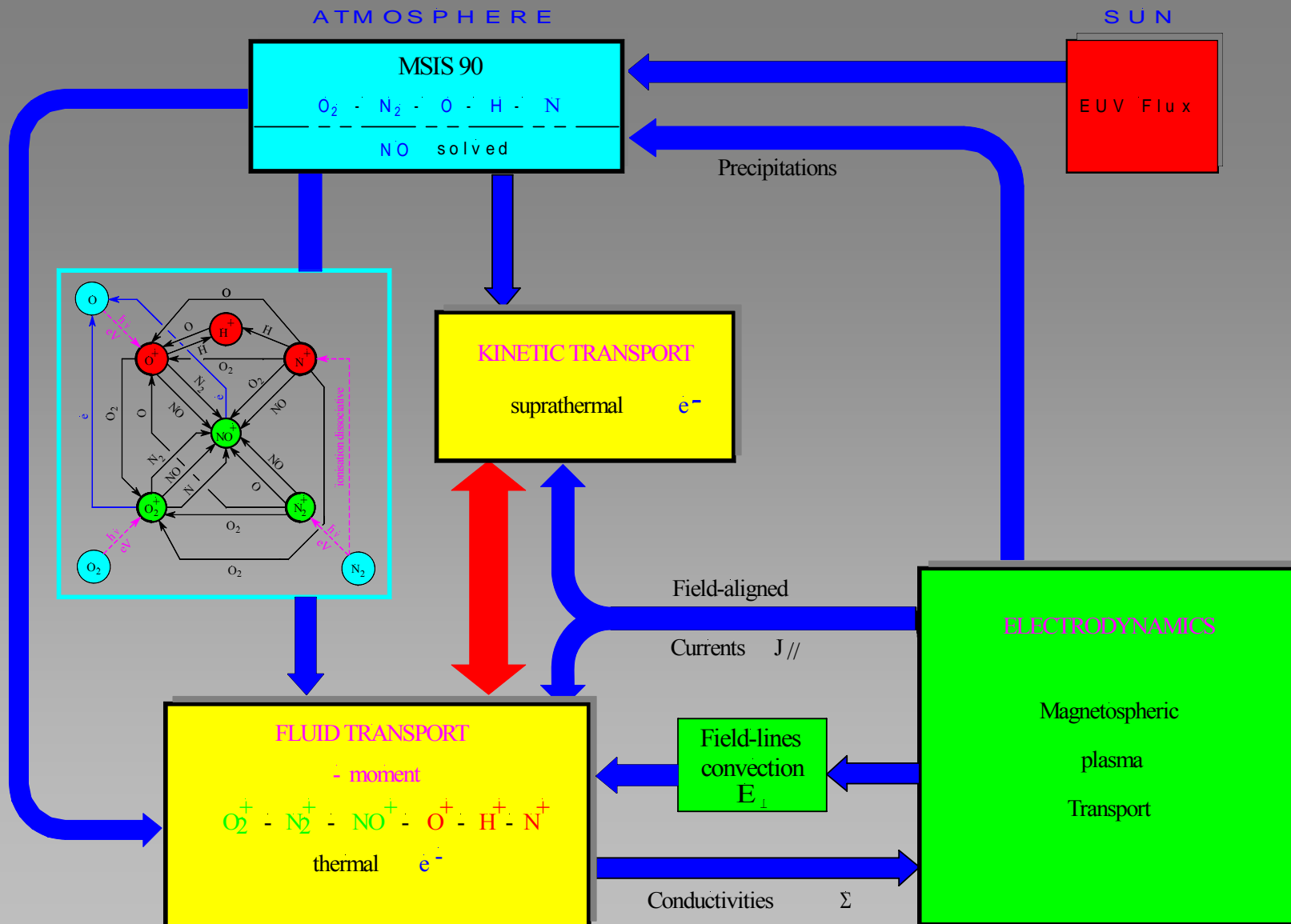
Hyperbolicity 1

- Arises from the coupled set of equations

$$\frac{\partial}{\partial t} \begin{pmatrix} n \\ \bar{u} \\ T^{\parallel} \\ T^{\perp} \\ \vec{q}^{\parallel} \\ \vec{q}^{\perp} \end{pmatrix} + \underbrace{\begin{pmatrix} \bar{u} & n & 0 & 0 & 0 & 0 \\ \frac{k_b T^{\parallel}}{mn} & \bar{u} & \frac{k_b}{m} & 0 & 0 & 0 \\ \dots & \bar{u} & \dots & \dots & \dots & \dots \\ & & \bar{u} & & & \\ & & & \bar{u} & & \\ & & & & \bar{u} & \\ & & & & & \bar{u} \end{pmatrix}}_M \vec{\nabla} \begin{pmatrix} n \\ \bar{u} \\ T^{\parallel} \\ T^{\perp} \\ \vec{q}^{\parallel} \\ \vec{q}^{\perp} \end{pmatrix} = 0$$

- Hyperbolicity if all eigenvalues of M are real (in 1D)
 - Fourier analysis
- If one eigenvalue is imaginary
 - Instabilities
- The moment equations are not unconditionally stable

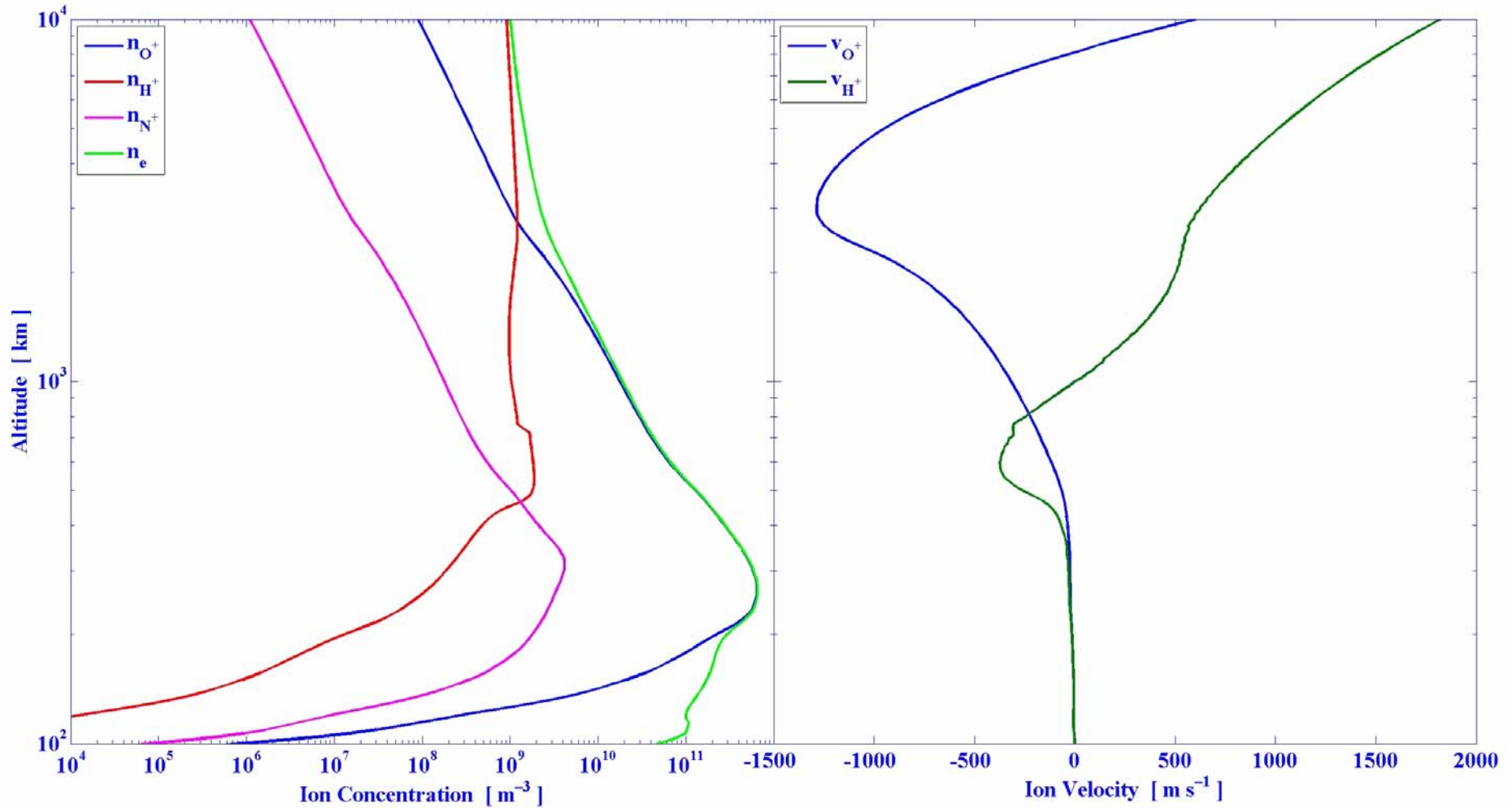
Synopsis of TRANSCAR model



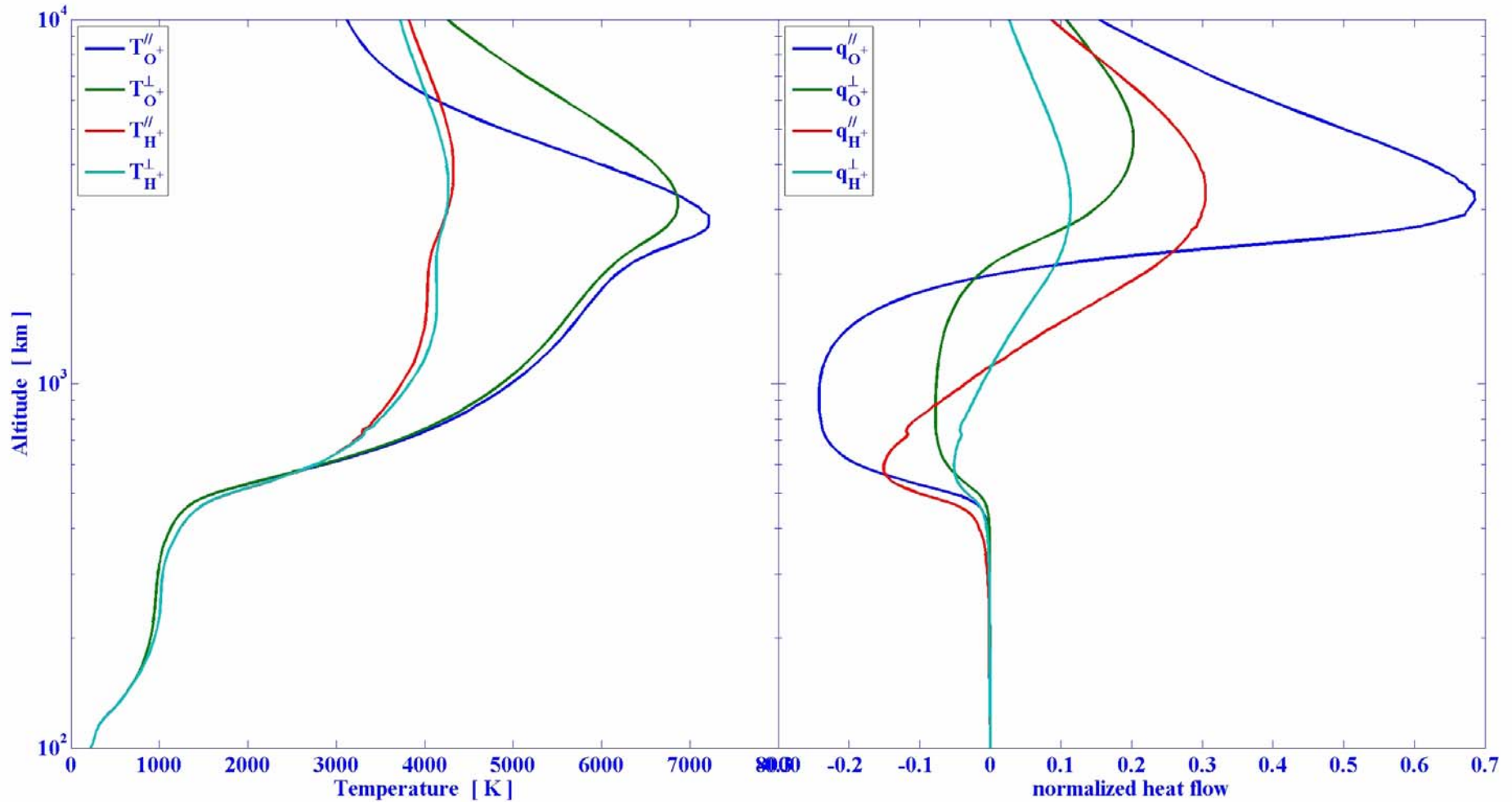
Case study

- Opening of a closed flux tube ($L=8$) in polar ionosphere
- 16 moment-closure transport equations
 - Six ions O^+ , H^+ , N^+ , N_2^+ , NO^+ , O_2^+
 - 1 D system (field aligned) n u T^{\parallel} T^{\perp} q^{\parallel} q^{\perp}
- Transition
 - diffusive equilibrium \Rightarrow polar wind outflow
 - H^+ dominant ion \Rightarrow O^+ dominant ion

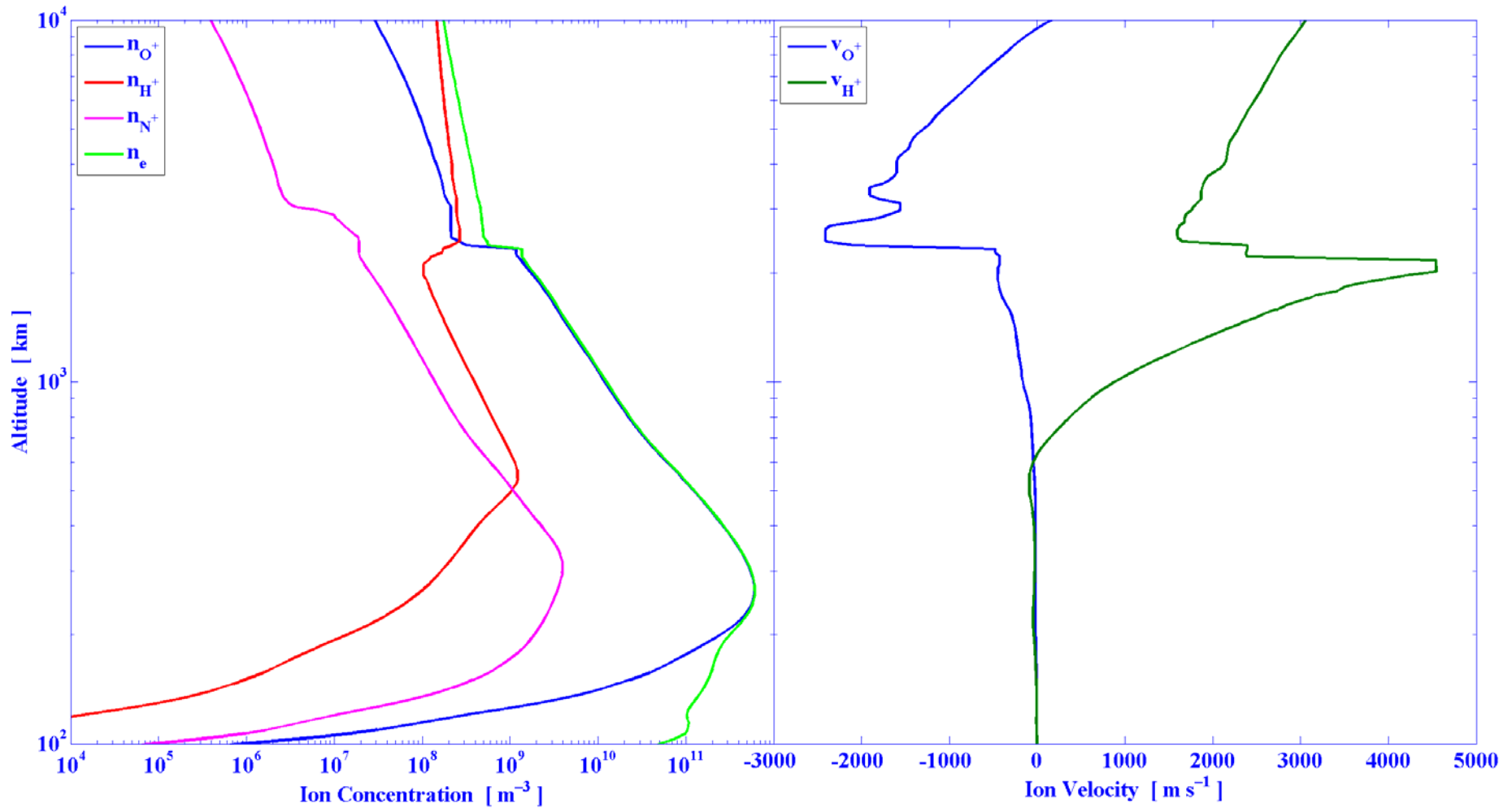
Strong outflow



Strong heating

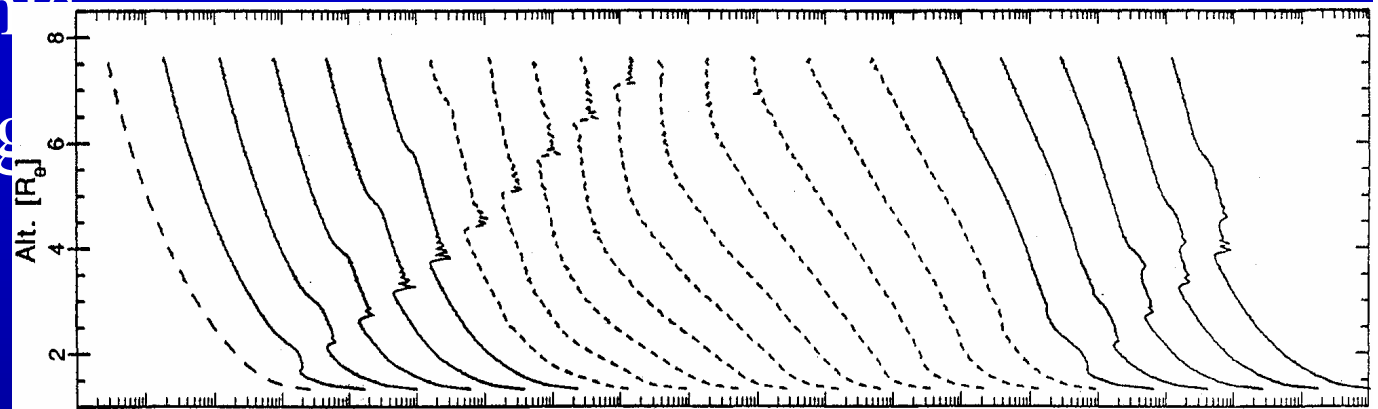


Shock formation



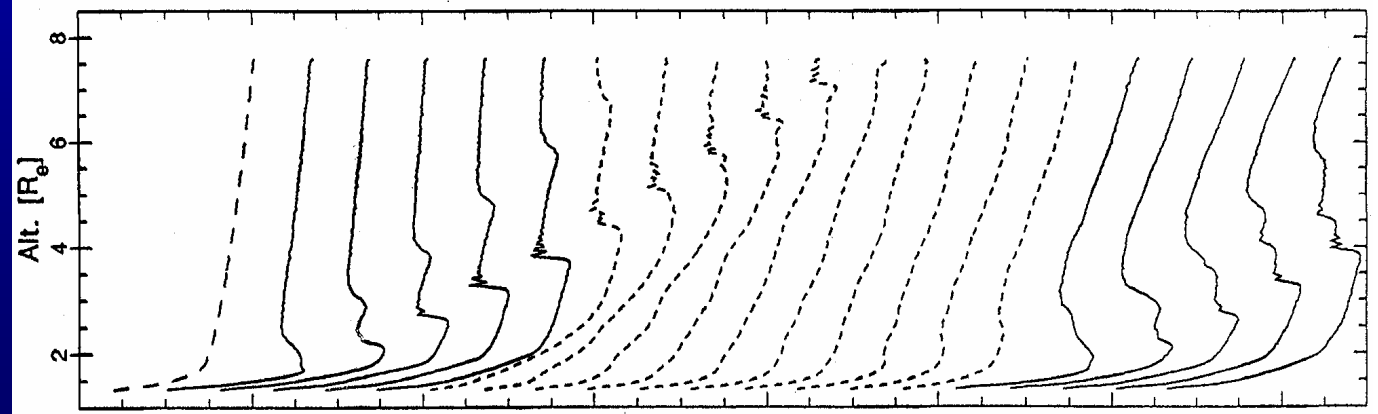
Fluid Results

- Shocks formation
- Sharp gradients
- Waves propagation



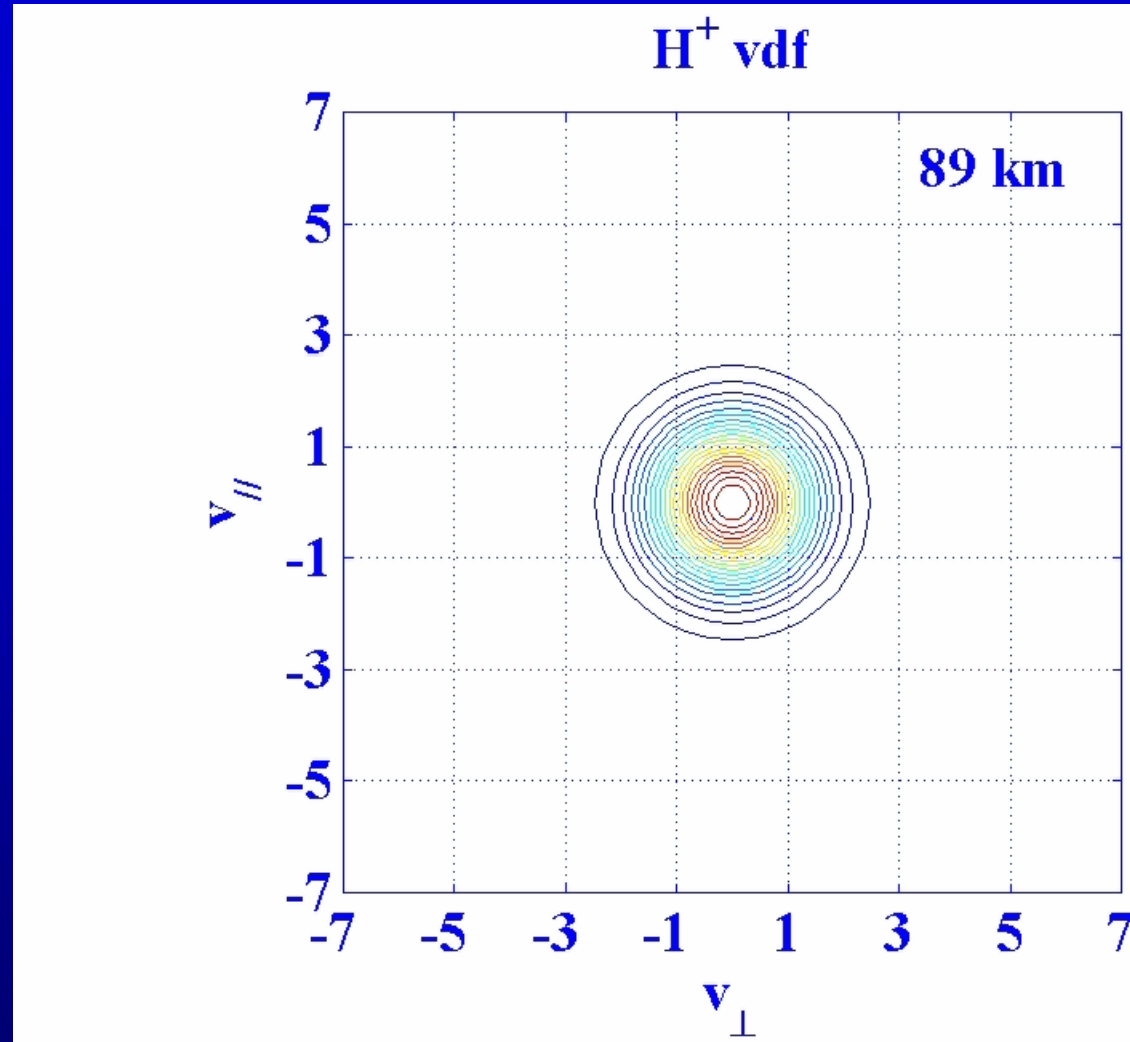
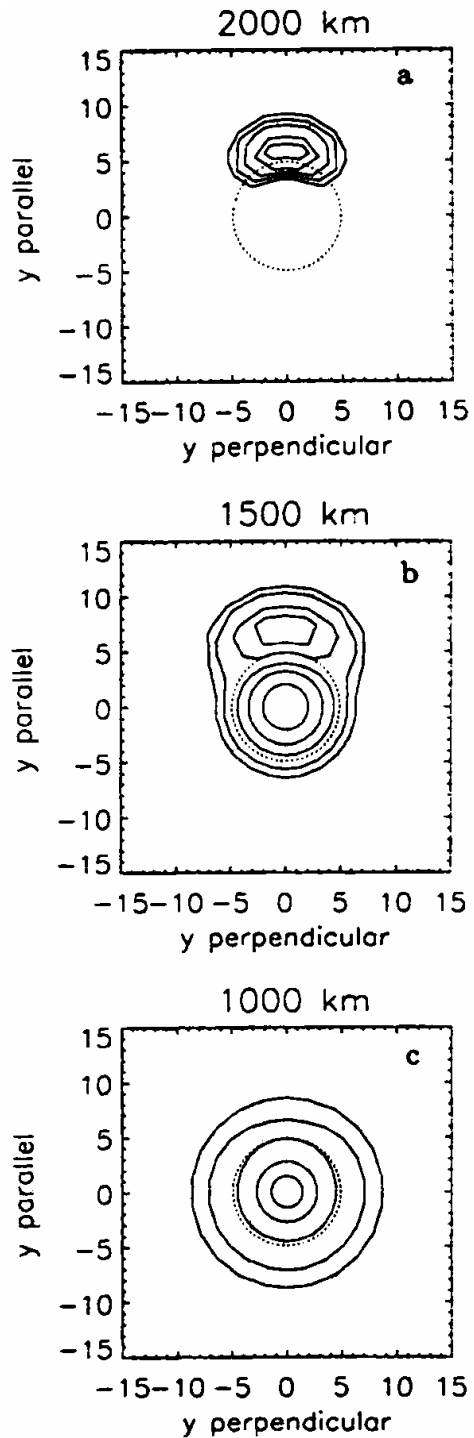
Offset Density Profiles

Demars et al, 1998



Offset Drift Velocity Profiles

Kinetic results

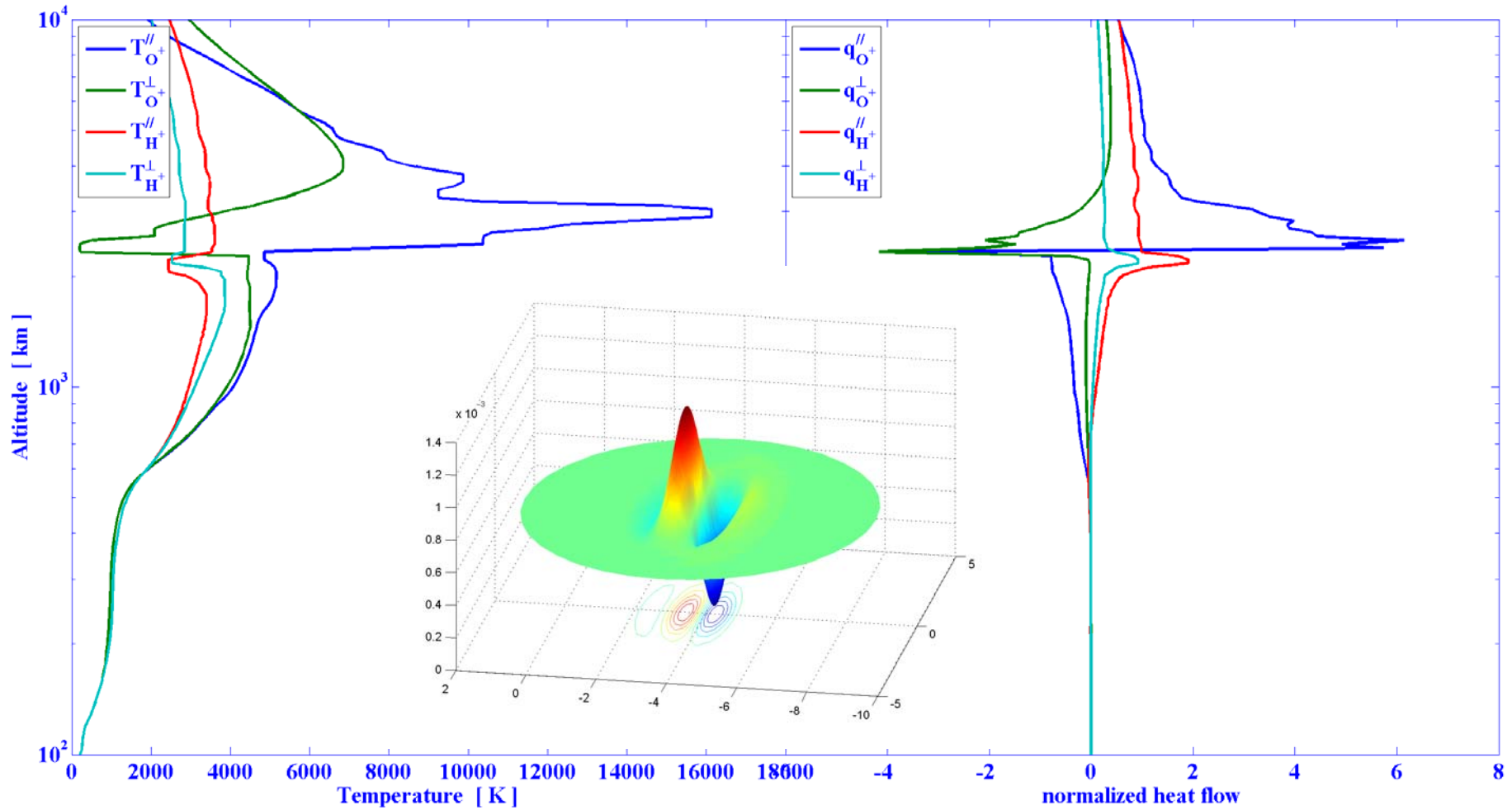


Pierrard and Lemaire, 1998

Terrestrial Interactions from Microscales to global Models - STIMM2007



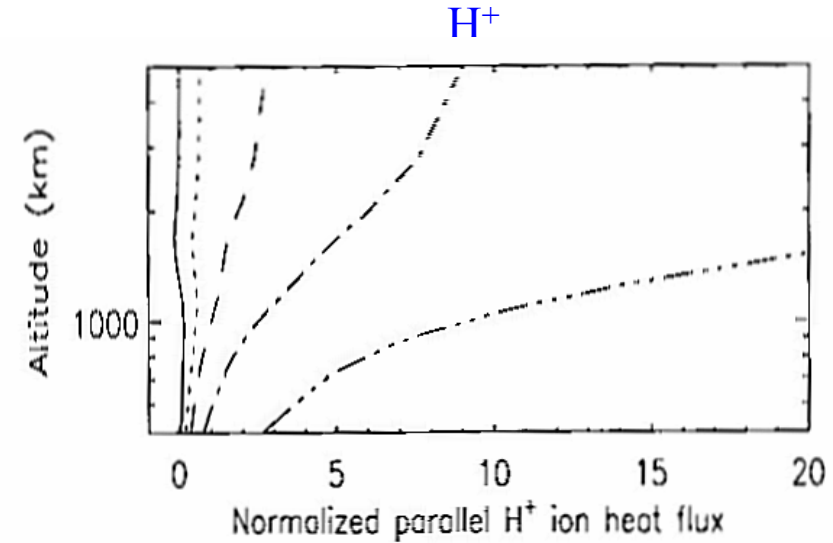
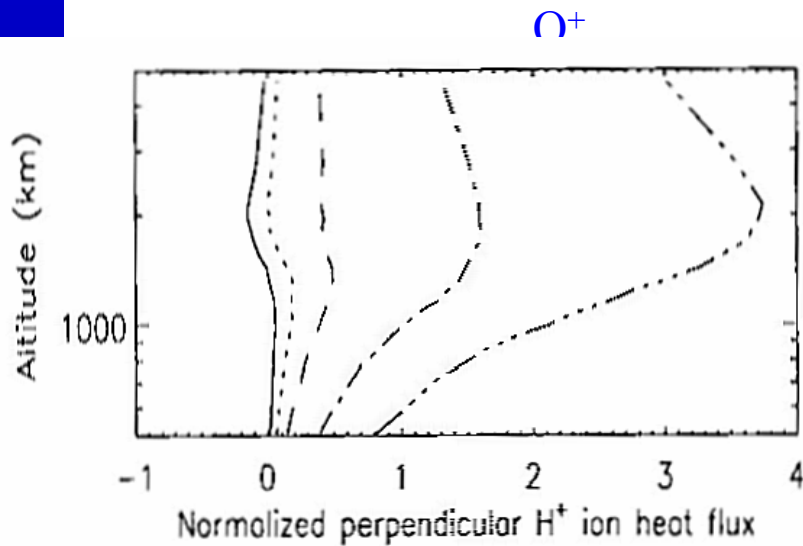
Validity of the expansion



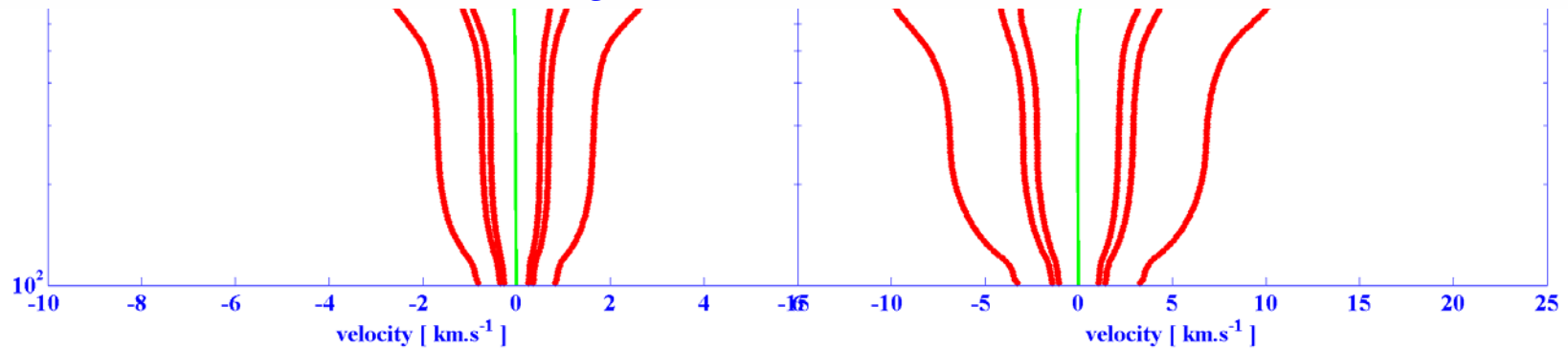
Hyperbolicity 2

- Hyperbolicity drops if

$$\frac{|\vec{q}_s|}{\sqrt{\frac{k_b T_s}{m_s} n_s k_b T_s}} < \alpha \approx 1$$

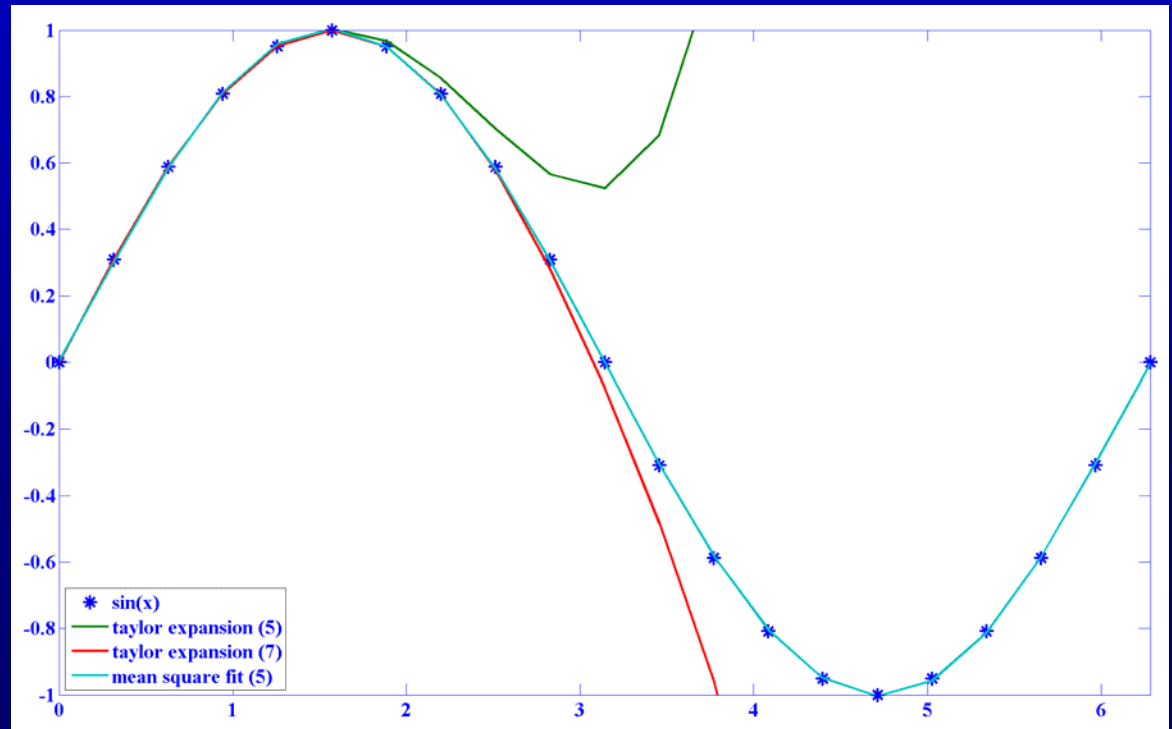


Barghouthi et al, 2001



Keys of the problem

- Heat flow
 - Only thermal contribution
- Closure relationship
 - The high order pressure tensor
- The expansion



Some ideas

- Change from maxwellian to Kappa functions
 - Positivity
 - Relationship between
 - Heat flow
 - High order pressure tensor
- Change the closure relationship
 - Allows for non thermal tail
- Change the expansion
 - Much more accurate
 - But not moment transport equation
- Fluid higher moments as BC for kinetic approach