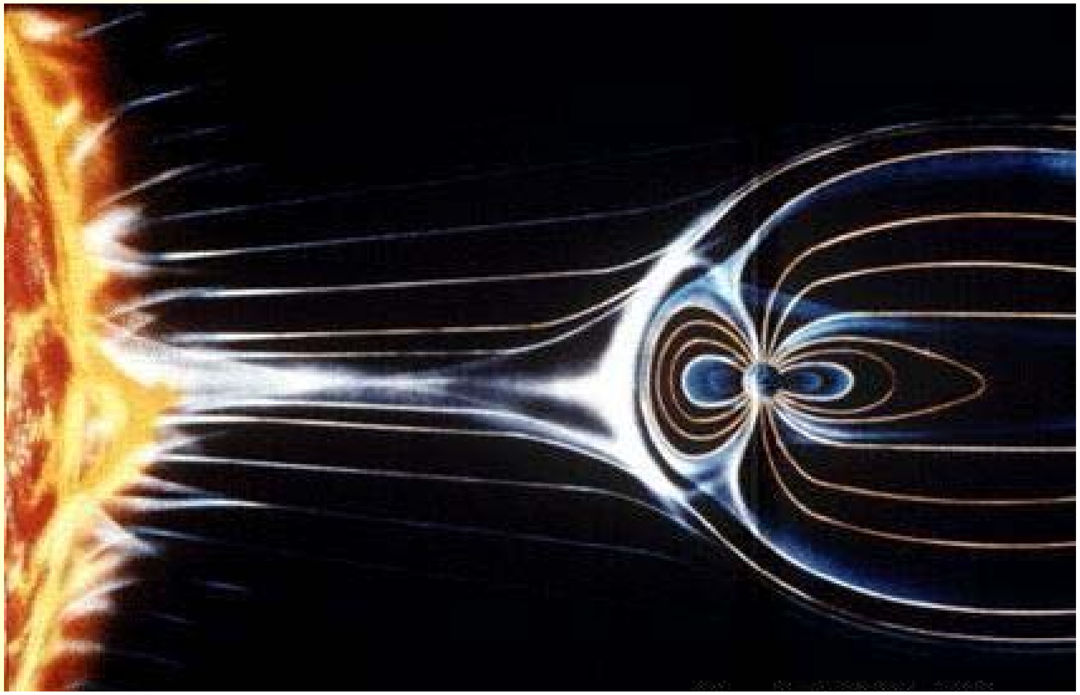
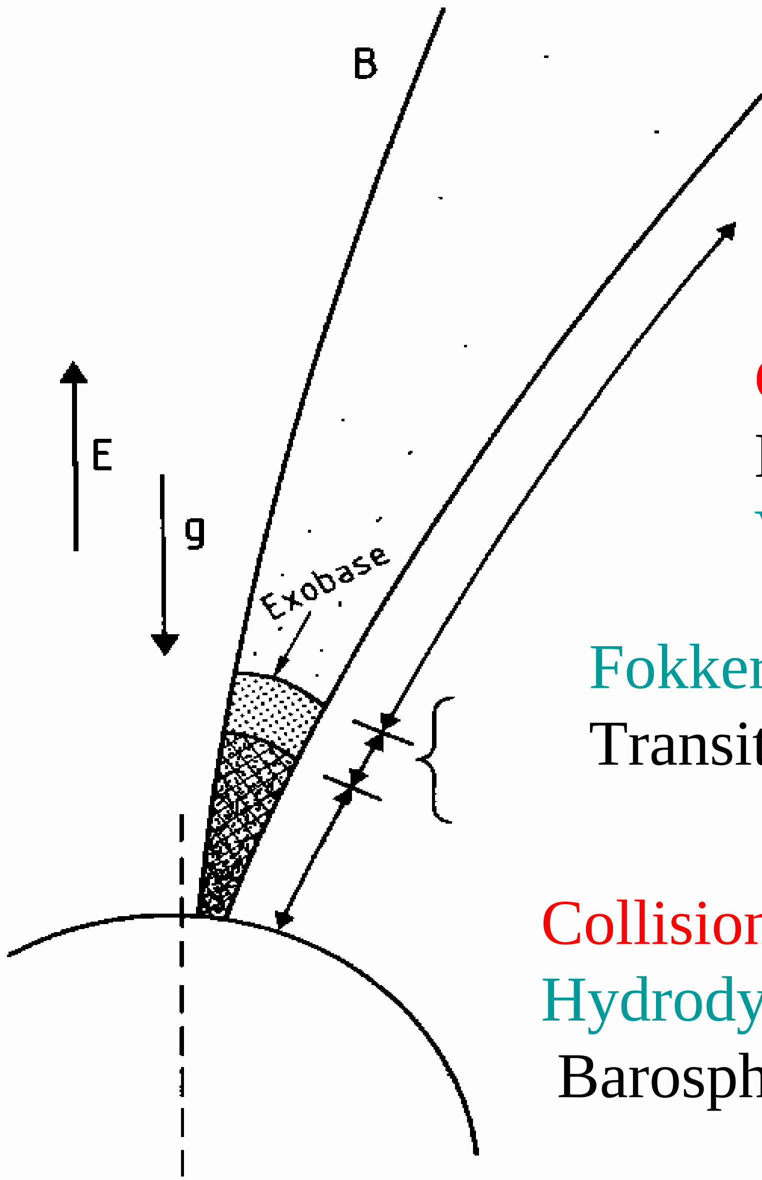
Fokker-Planck modeling of the solar and polar wind flow

Viviane Pierrard and Joseph Lemaire
Belgian Institute for Space Aeronomy



aeronomie.be

$$Kn = \frac{\text{mean free path}}{\text{density scale height}}$$



Collisionless region Kn>>1
Exosphere
Vlasov equation

Fokker-Planck equation
Transition region

Collision-dominated region Kn<<1
Hydrodynamic equations
Barosphere

Kinetic approach

- Velocity distribution function

$f(\vec{r}, \vec{v}, t)$=number of particles with a velocity in [$\vec{v}$, $\vec{v}+d\vec{v}$] and a position in [$\vec{r}$, $\vec{r}+d\vec{r}$] at an instant t

- Fokker-Planck equation (non linear):

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} + \vec{a}\cdot\frac{\partial f}{\partial \vec{v}} = -\frac{\partial}{\partial \vec{v}}\cdot\left[\vec{A}f - \frac{1}{2}\frac{\partial}{\partial \vec{v}}\cdot\left(\vec{D}f\right)\right] \quad [1]$$

$\frac{\partial f}{\partial t}$ → 0

$\vec{a}$ → $\vec{g}$, $\vec{E}$, $\vec{B}$

$\vec{A}f$ → friction

$\vec{D}f$ → diffusion

- We search stationary solutions

Method of Solution

- Spectral method developed by Shizgal (1984). Expansion of the solution in polynomials:

$$f(z, y, \mu) = \exp(-y^2) \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} P_l(\mu) S_j(y) L_k(z)$$

$$y^2 = \frac{mv^2}{2kT}$$

μ=cos θ

$$Z = -\int_{r}^{r_{top}} \sigma n(r') dr'$$

P(μ): Legendre polynomials

S(y): Speed polynomials

L(z): Modified Legendre polynomials

l = 1,…,10 j=1,…16 k=1,…,10

At each radial distance, f(v,μ) is represented by 2*10*16=320 points.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \vec{a} \cdot \frac{\partial f}{\partial \vec{v}} = -\frac{\partial}{\partial \vec{v}} \cdot \left[\vec{A} f - \frac{1}{2} \frac{\partial}{\partial \vec{v}} \cdot \left(\vec{D} f \right) \right] \quad [1]$$

Solar wind model

Assumptions for self collisions in the transition region

- B radial
- We solve this equation for electrons
- We consider collisions with electrons and protons
- Self collisions simulated by a convergent iterative process:
- 1st step: VDF of the background particles assumed to be known (Maxwellian with n and T from exospheric model)
- Successive iterations: use of VDF found for the test electrons with the previous iteration.
- The solution depends on the boundary conditions!

Boundary conditions at 2 different altitudes

Pierrard, Maksimovic and Lemaire, JGR, 107, 29305, 2001

Bottom (collision-dominated):

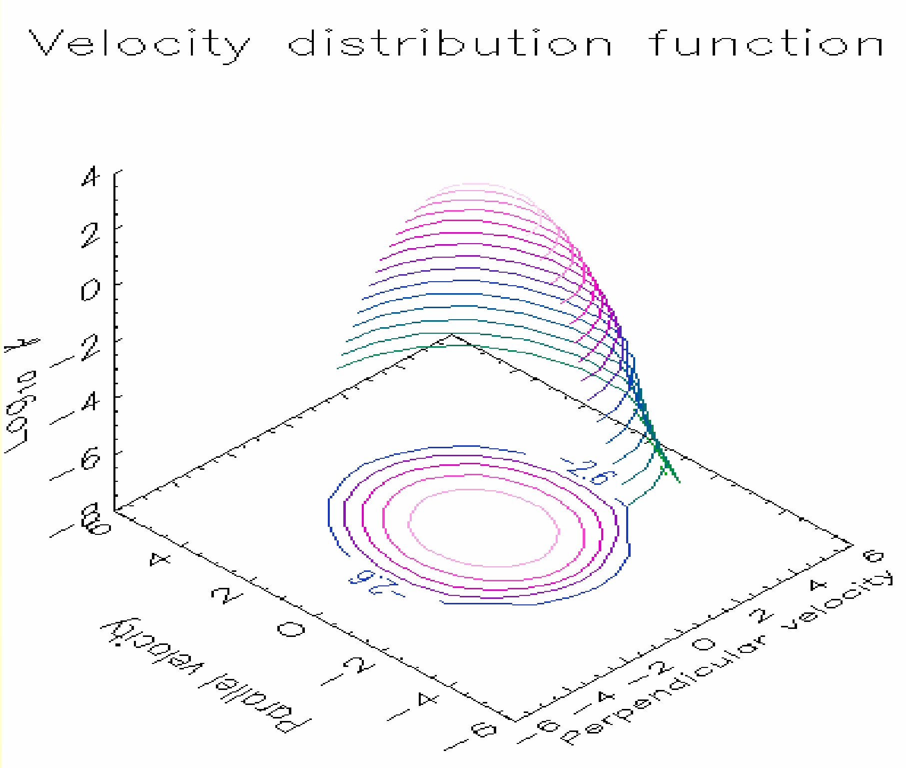
$f(2\ R_s, \mu>0, v)$ = maxwellian

Top (exospheric conditions):

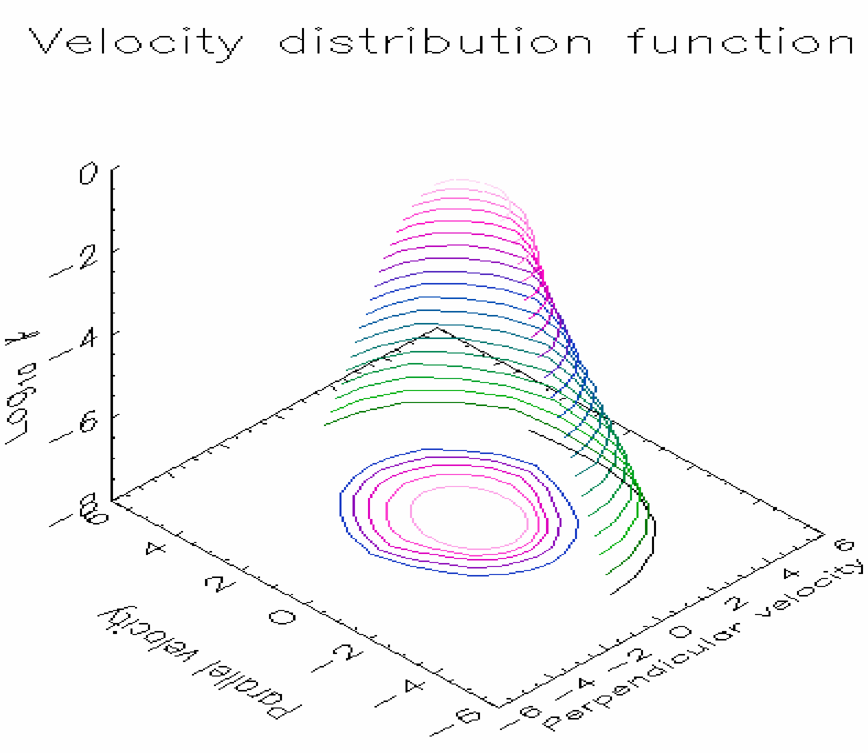
$f(14\ R_s, \mu<0, v<v_e) = f(14\ R_s, \mu>0, v<v_e)$

$f(14\ R_s, \mu<0, v>v_e) = 0$

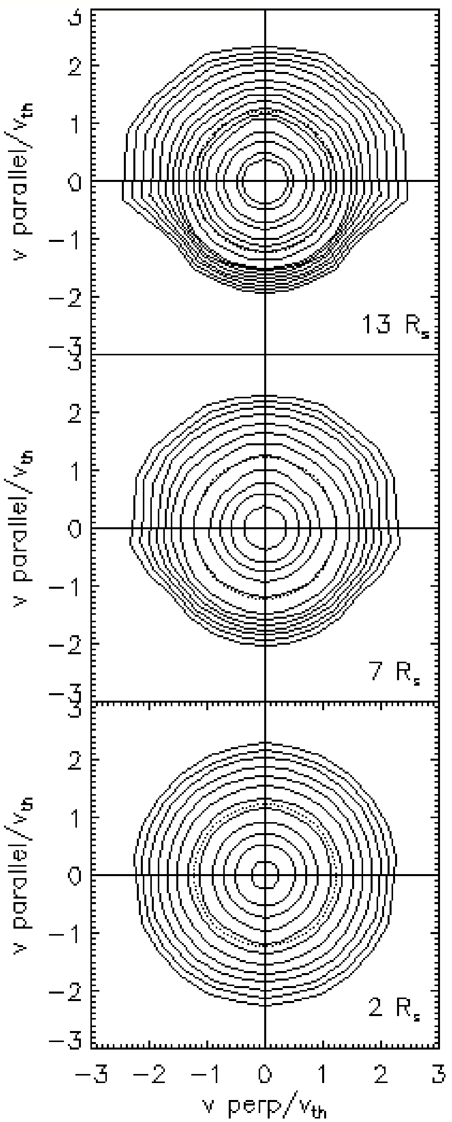
2 R_s



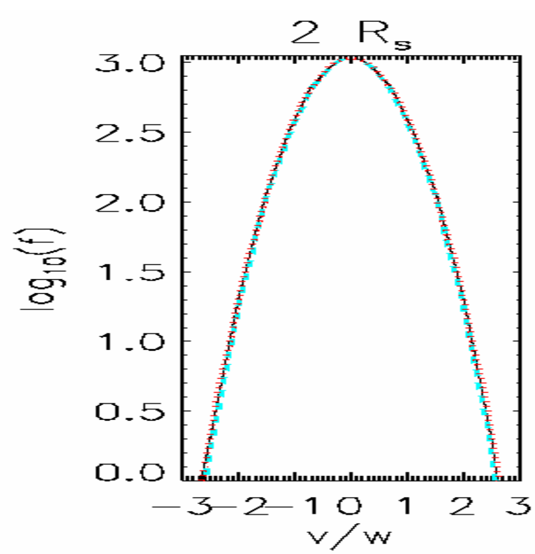
13 R_s

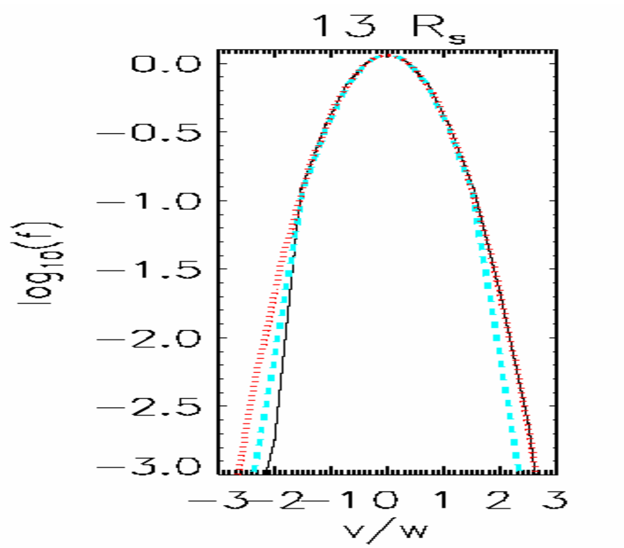


In the transition region, the electron velocity distribution function becomes anisotropic

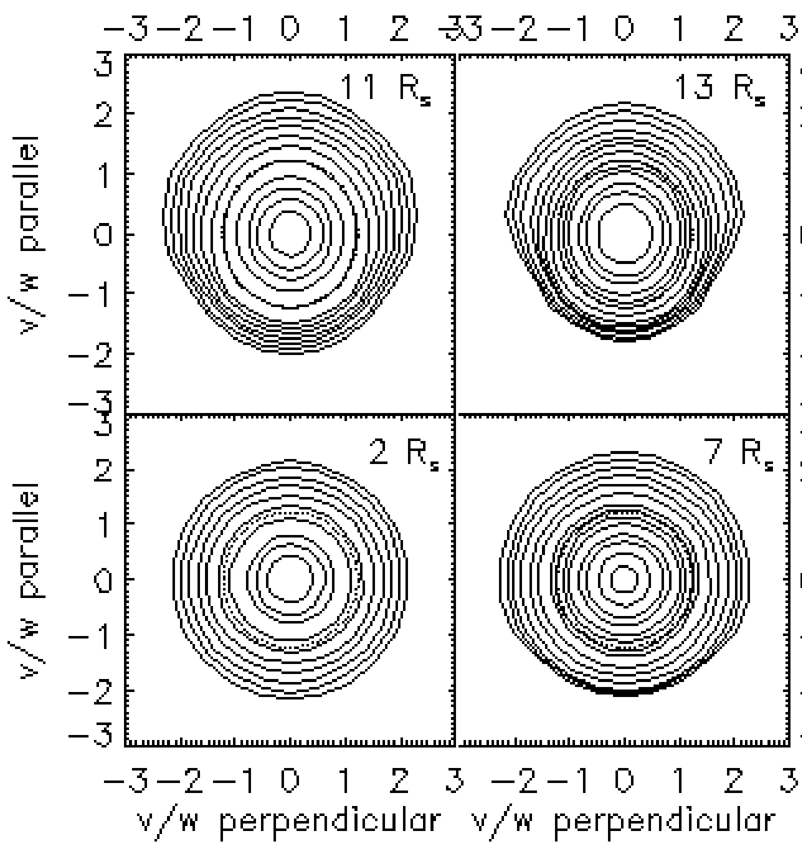


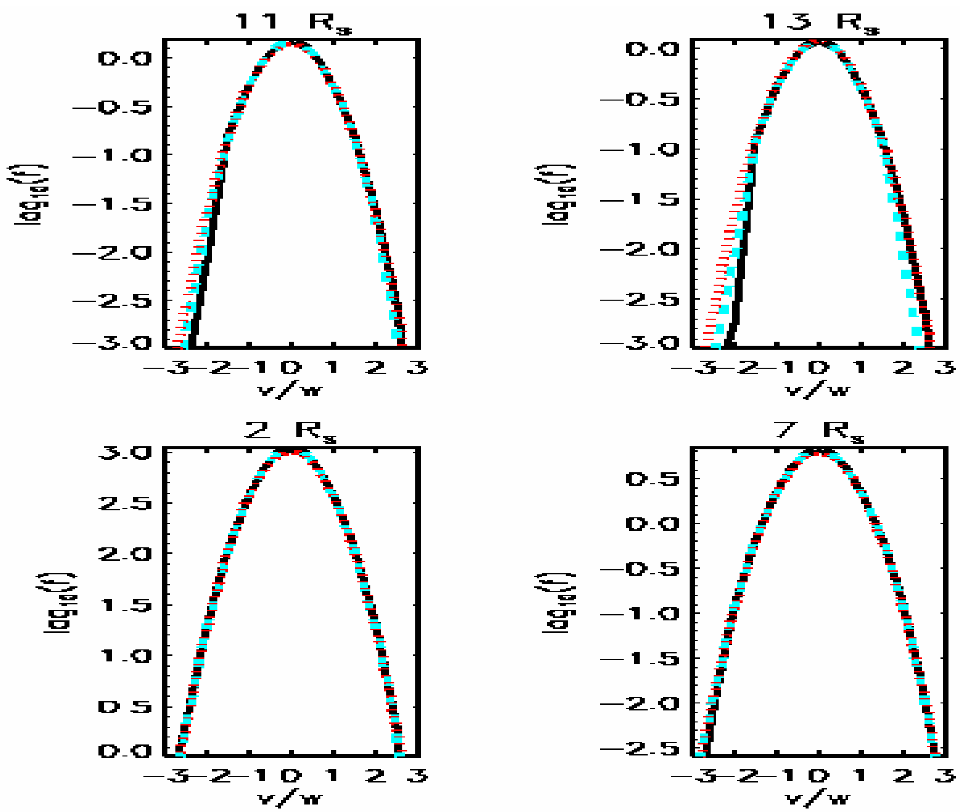
Electron velocity distribution function found by solving the Fokker-Planck equation





Adding the collisions with the protons:





The moments of f

Number density [m^{-3}]

$$n(\vec{r}) = \int_{-\infty}^{\infty} f(\vec{r},\vec{v})\, d\vec{v}$$

Particle flux [$m^{-2}\ s^{-1}$]

$$\vec{F}(\vec{r}) = \int_{-\infty}^{\infty} f(\vec{r},\vec{v})\,\vec{v}\, d\vec{v}$$

Bulk velocity [$m\ s^{-1}$]

$$\vec{u}(\vec{r}) = \frac{\vec{F}(\vec{r})}{n(\vec{r})}$$

Pressure [Pa]

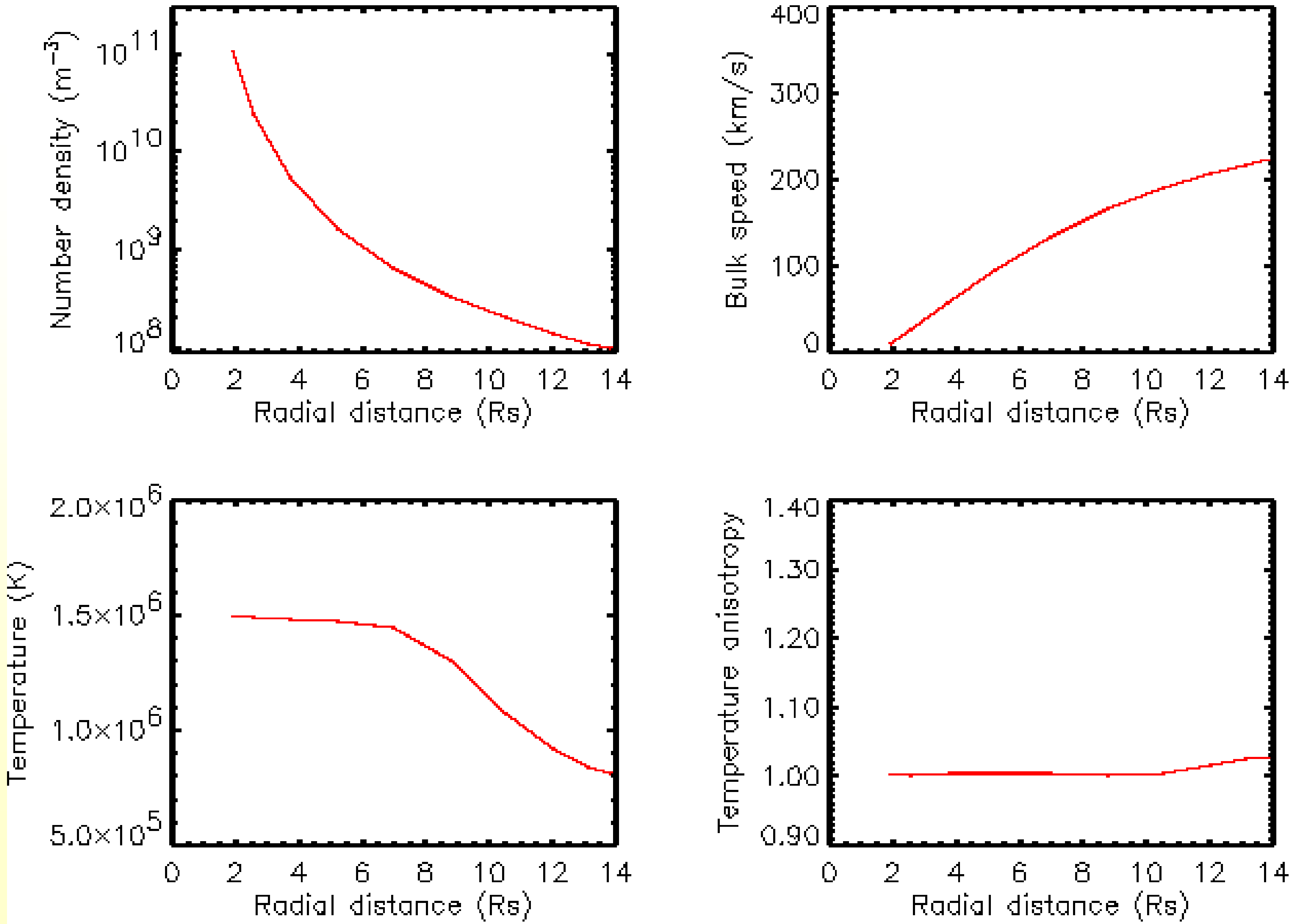
$$\vec{P}(\vec{r}) = m\int_{-\infty}^{\infty} f(\vec{r},\vec{v})(\vec{v}-\vec{u})(\vec{v}-\vec{u})d\vec{v}$$

Temperature [K]

$$T(\vec{r}) = \frac{m}{3k\,n(\vec{r})}\int_{-\infty}^{\infty} f(\vec{r},\vec{v})|v-u|^2 d\vec{v}$$

Energy flux [$Jm^{-2}\ s^{-1}$]

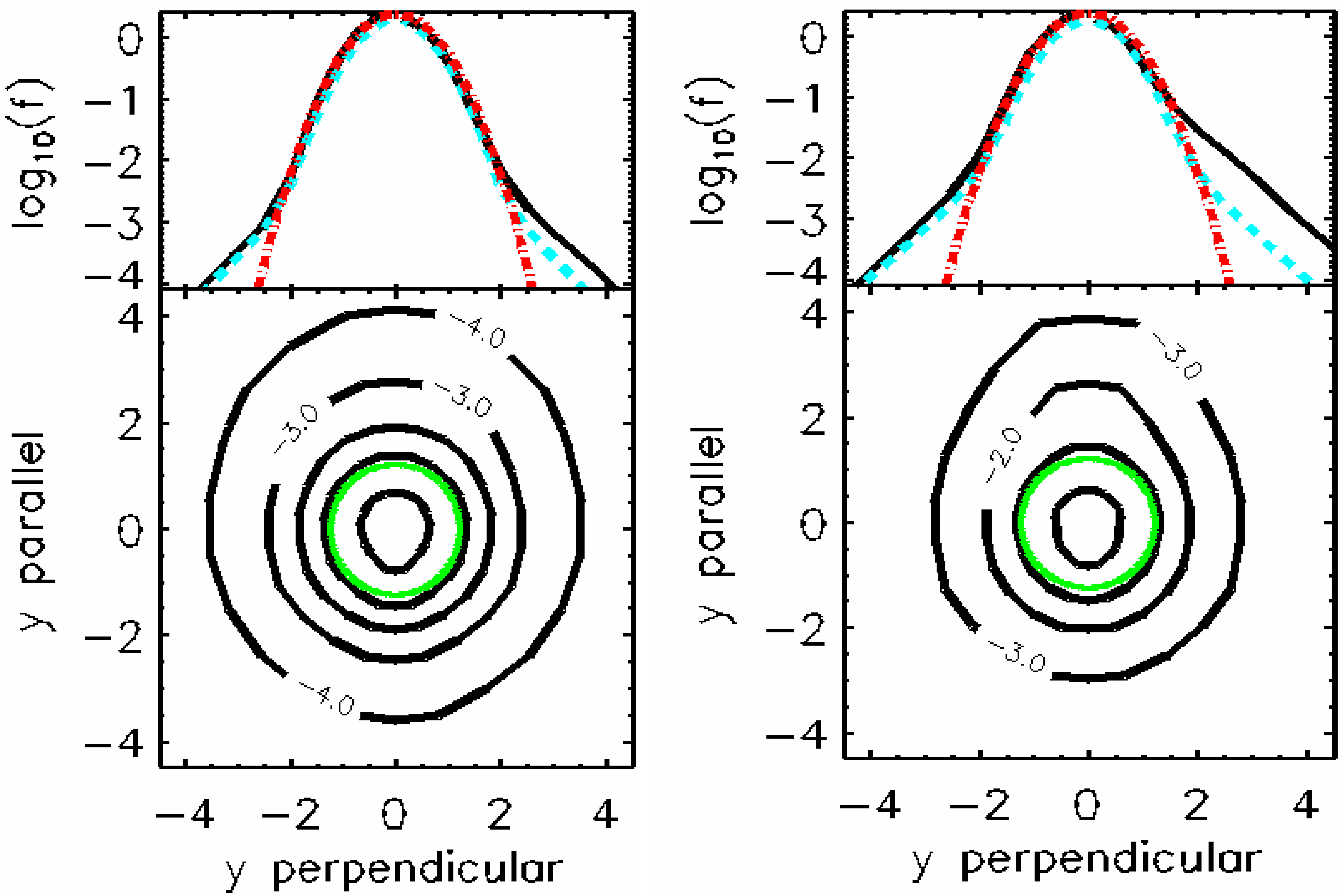
$$\vec{E}(\vec{r}) = \frac{m}{2}\int_{-\infty}^{\infty} f(\vec{r},\vec{v})|v-u|^2(\vec{v}-\vec{u})d\vec{v}$$


Number density (m^{-3})
10^{11}
10^{10}
10^{9}
10^{8}
0 2 4 6 8 10 12 14
Radial distance (Rs)
Bulk speed (km/s)
400
300
200
100
0
0 2 4 6 8 10 12 14
Radial distance (Rs)
Temperature (K)
2.0×10^{6}
1.5×10^{6}
1.0×10^{6}
5.0×10^{5}
0 2 4 6 8 10 12 14
Radial distance (Rs)
Temperature anisotropy
1.40
1.30
1.20
1.10
1.00
0.90
0 2 4 6 8 10 12 14
Radial distance (Rs)

Solar wind model with observed boundary conditions

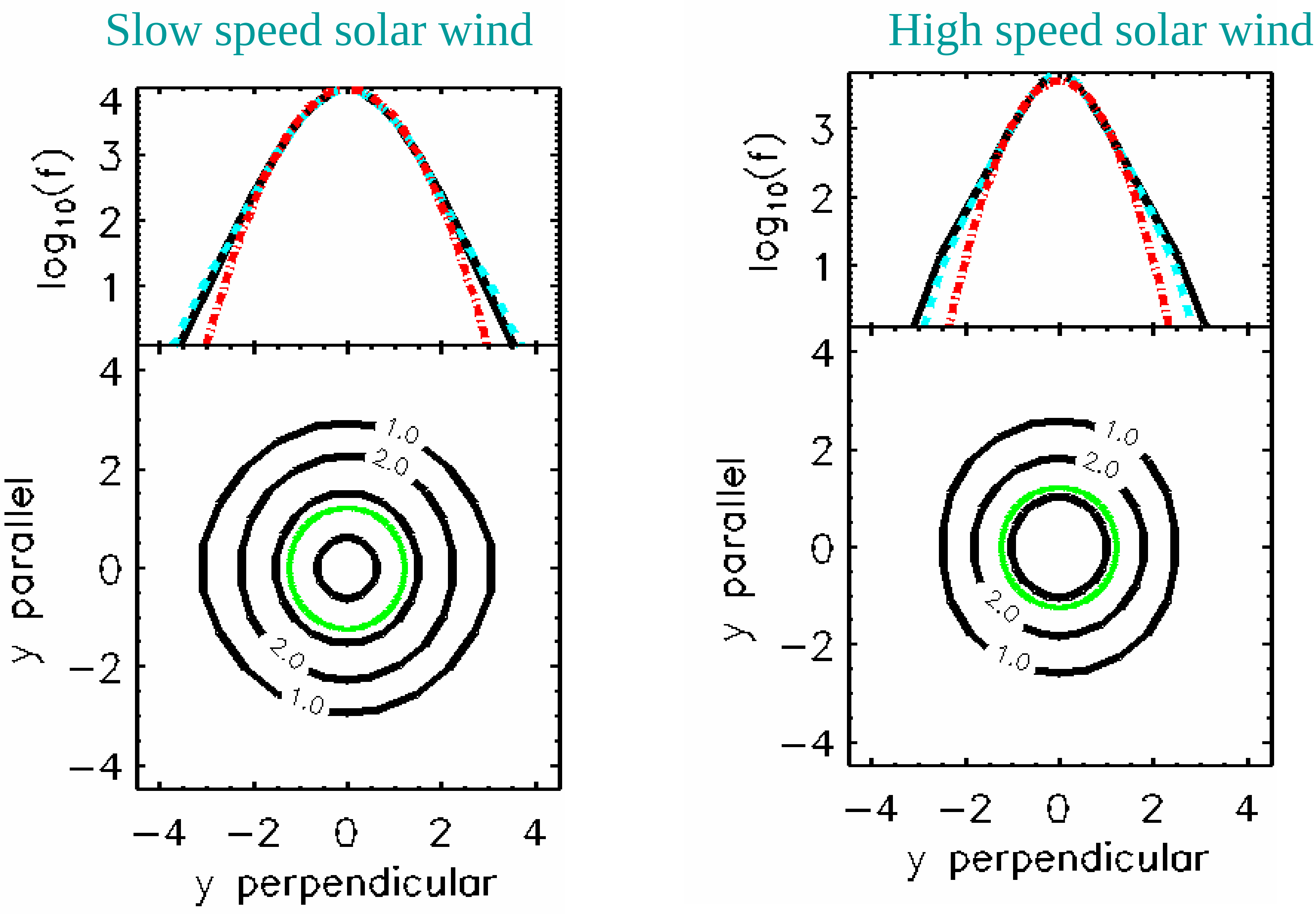
Electron velocity distribution functions observed by WIND at 215 Rs

Slow speed solar wind　　　　High speed solar wind



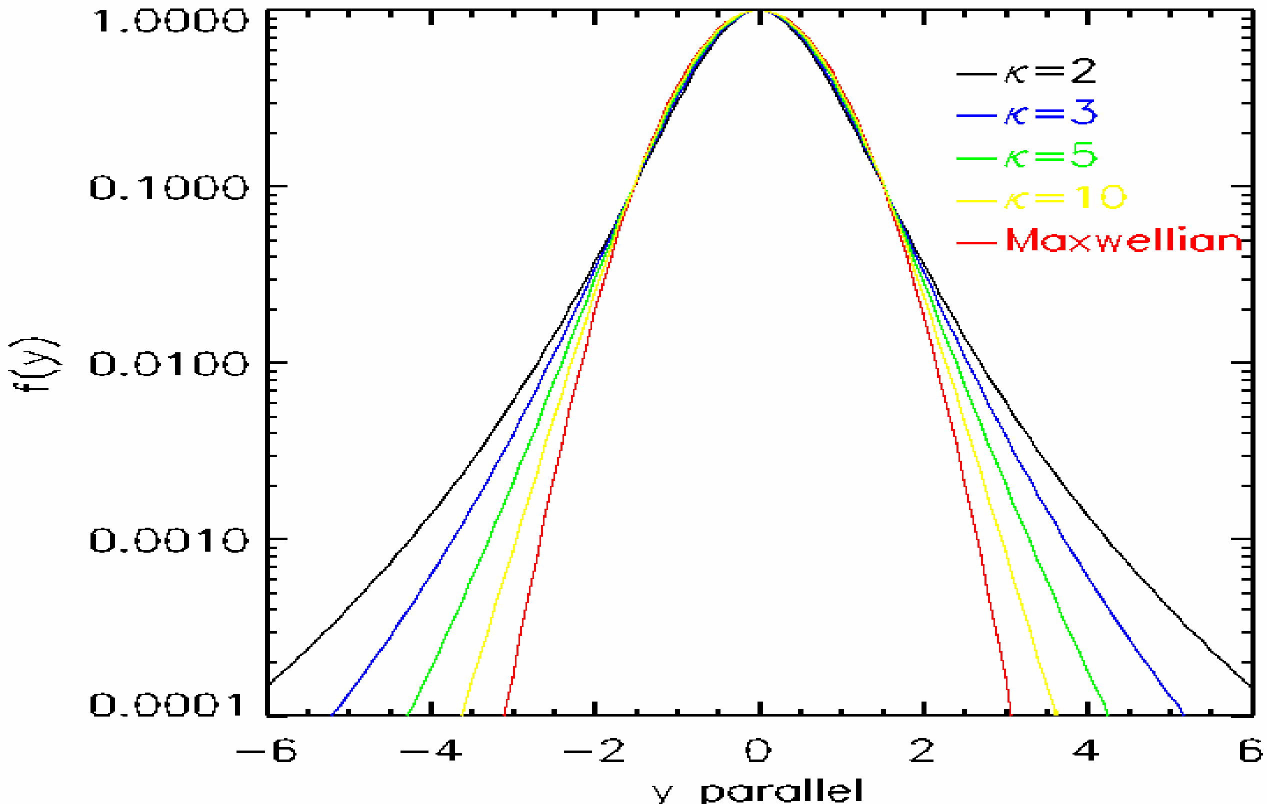
Pierrard, Maksimovic and Lemaire, JGR, 104, 17021, 1999

Electron velocity distribution functions found at 4 Rs by solving the Fokker-Planck equation

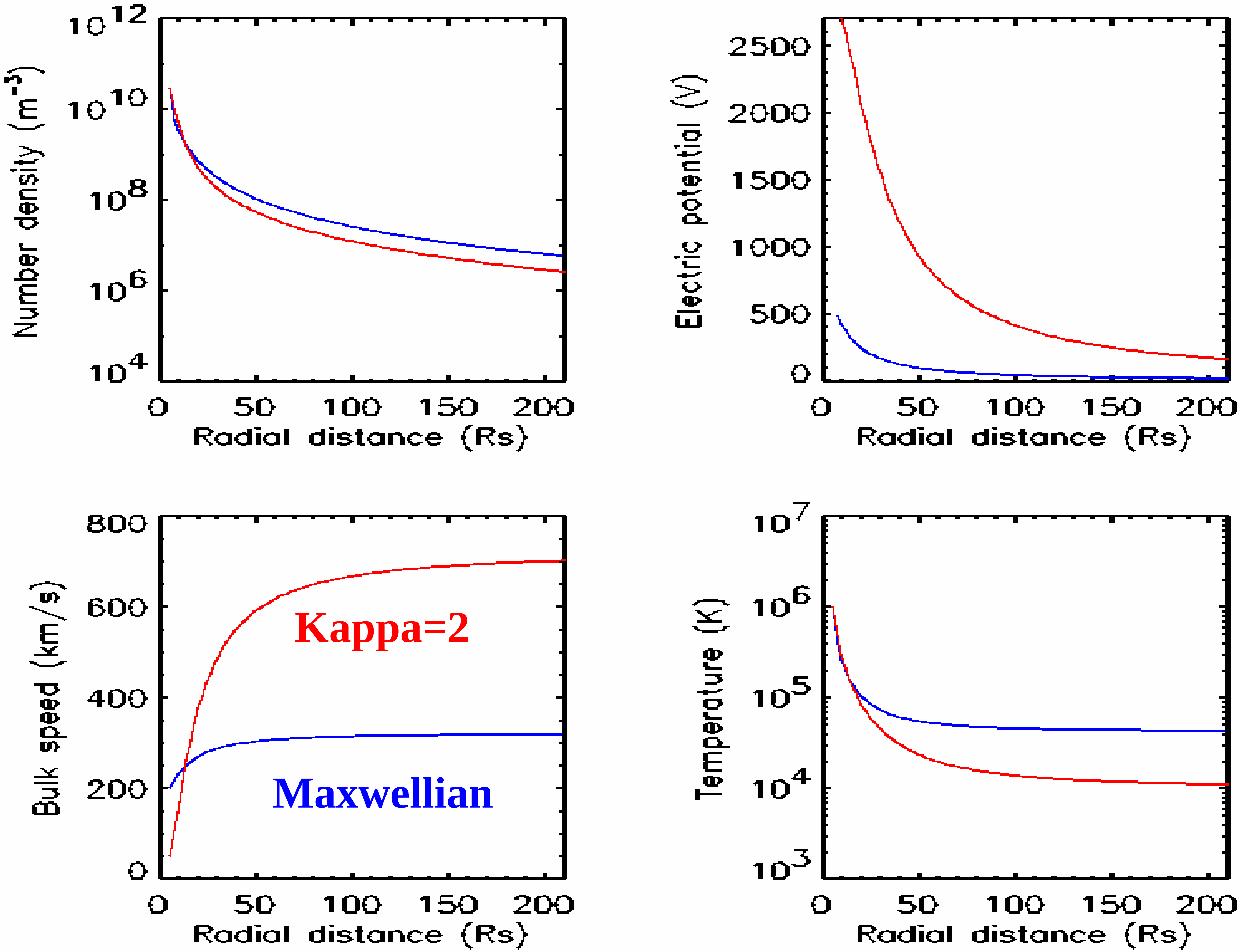


A small amount of additional suprathermal electrons can accelerate the solar wind to high velocities

$$f_{kappa} = \frac{n}{2\pi\kappa^{3/2}}\left(\frac{m}{2kT}\right)^{3/2} A_k \left(1+\frac{m v_e^2}{2kT\kappa}\right)^{-(\kappa+1)}$$



Maksimovic, Pierrard and Lemaire, Astron. Astrophys., 324, 725, 1997
Pierrard and Lemaire, JGR, 101, 7923, 1996



Number density (m^{-3})
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
0 50 100 150 200
Radial distance (Rs)
Electric potential (V)
2500
2000
1500
1000
500
0
0 50 100 150 200
Radial distance (Rs)
Bulk speed (km/s)
800
600
400
200
0
Kappa=2
Maxwellian
0 50 100 150 200
Radial distance (Rs)
Temperature (K)
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
0 50 100 150 200
Radial distance (Rs)

Comparison with other models

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \vec{a} \cdot \frac{\partial f}{\partial \vec{v}} = -\frac{\partial}{\partial \vec{v}} \cdot \left[\vec{A} f - \frac{1}{2} \frac{\partial}{\partial \vec{v}} \cdot \left(\vec{D} f \right) \right] \quad [1]$$

Exospheric models

No collisions, Vlasov equation, analytic solutions

Hydrodynamic models

$$\int_{-\infty}^{\infty} [1] \; d\vec{v} \quad \rightarrow$$

- Continuity equation

$$\int_{-\infty}^{\infty} [1] \; m\,\vec{v} \; d\vec{v} \quad \rightarrow$$

- Conservation of momentum

$$\int_{-\infty}^{\infty} [1] \; \frac{mv^2}{2} \; d\vec{v} \quad \rightarrow$$

- Conservation of energy

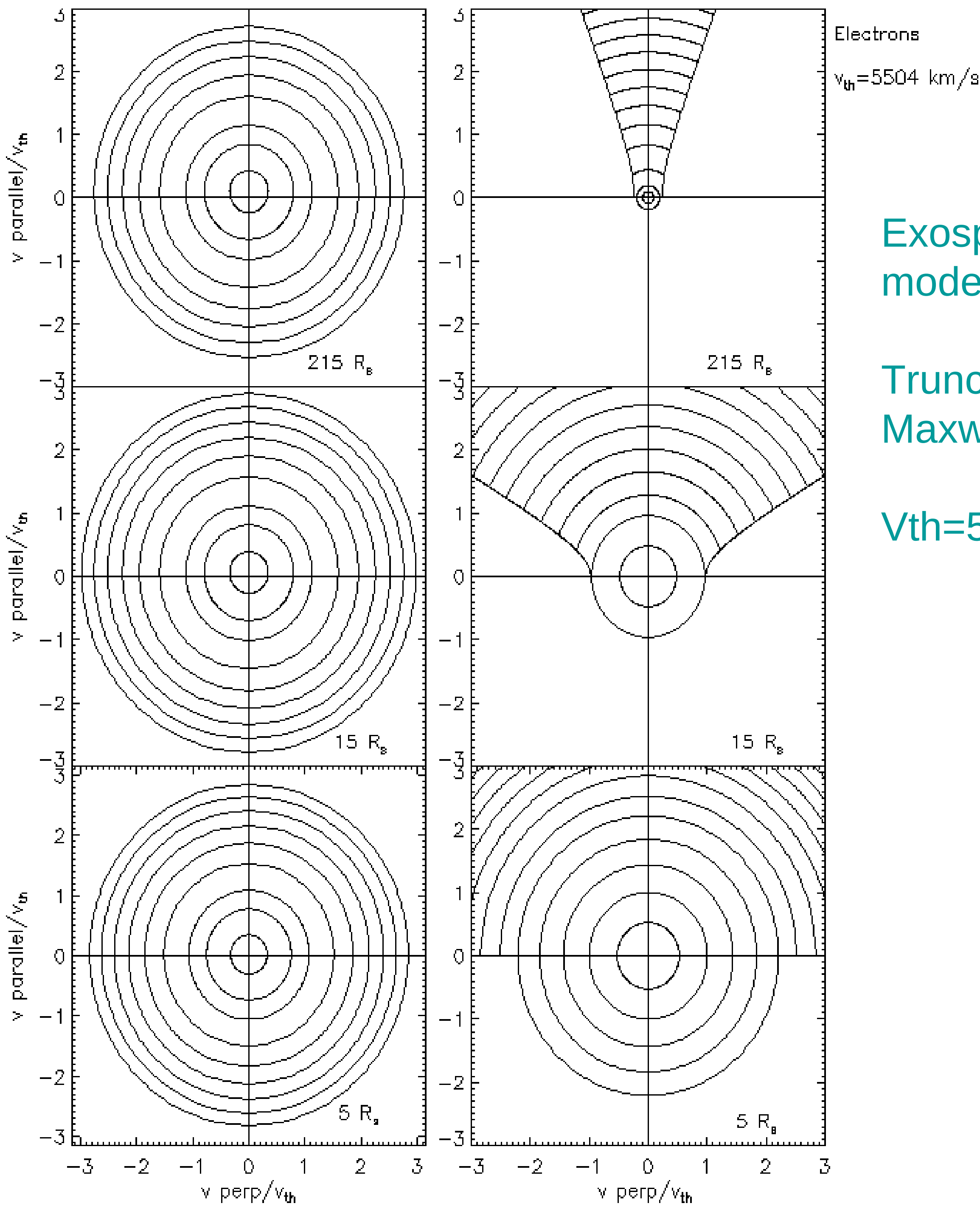
In each equation of order n appears the moment of order n+1.
Assumptions to close the system

Parker (1958): Isotropy of the pressure tensor, T=const.
5 moments: Maxwellian (n, **u**, T, no heat flow)
13 moments: Maxwellian with heat flow and stress
16 moments: Bi-maxwellian with heat flow and stress
20 moments: Grad with heat flow tensor and stress tensor

Hydrodynamic model
5 moments

Displaced Maxwellian

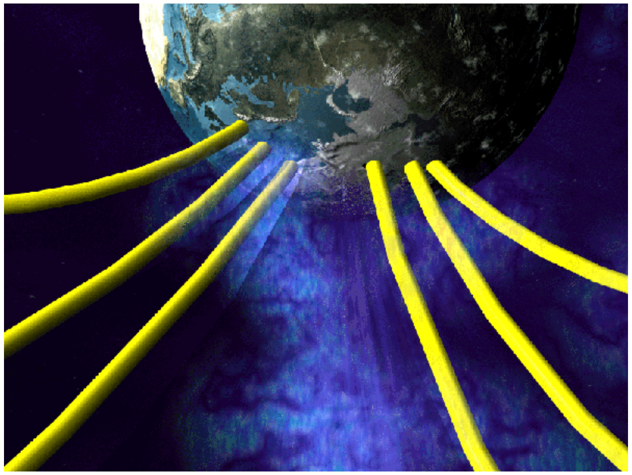
Solar wind electrons



Exospheric model

Truncated Maxwellian

Vth=5504 km/s

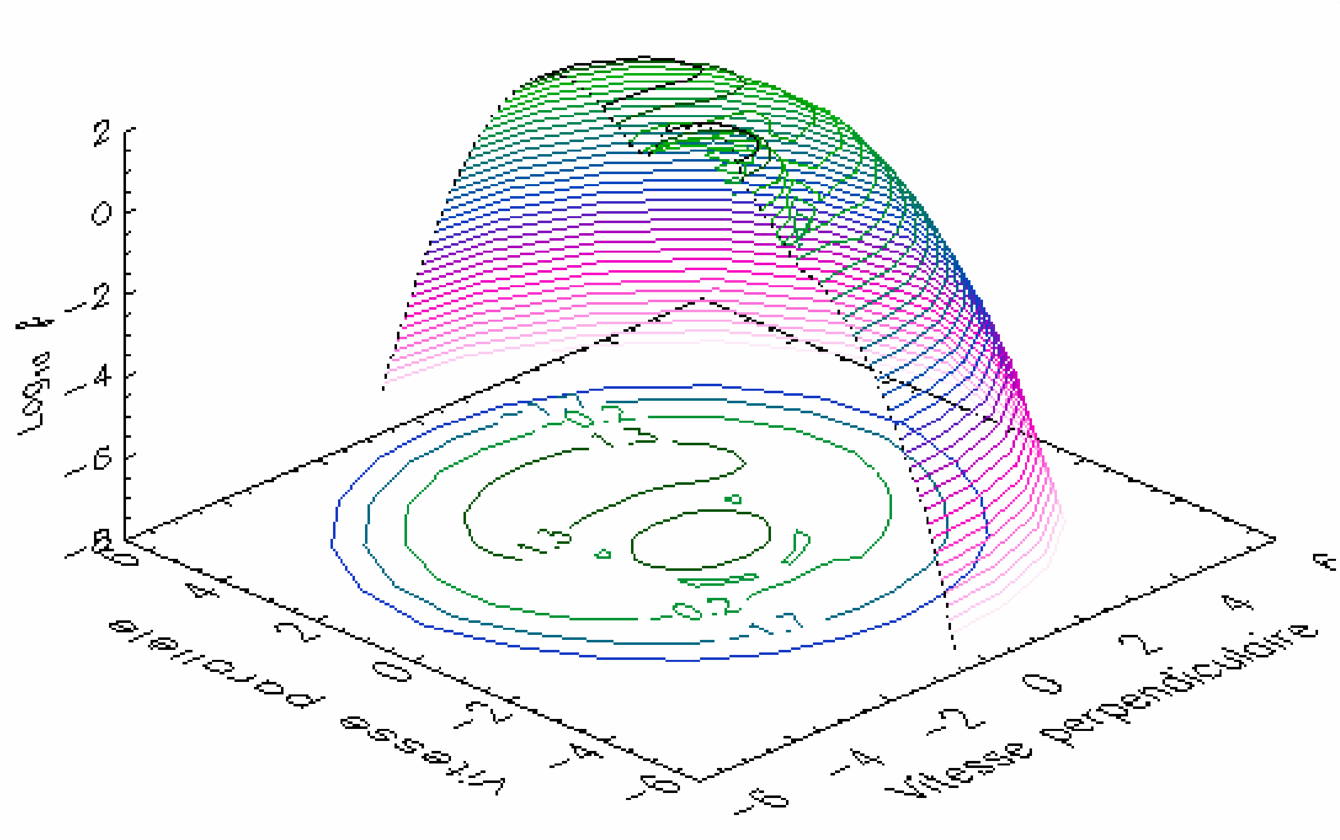


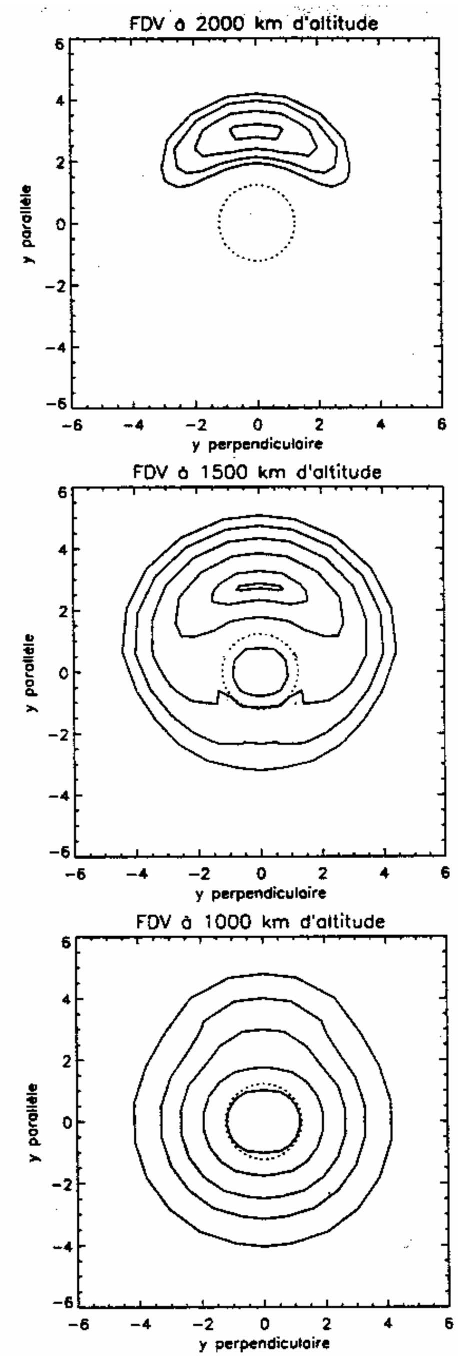
Polar wind model

Assumptions:

- **Kinetic collisional model based on Fokker-Planck equation**
- **We study H^+ (He^+) ions (minor light ions)**
- **Collisions with background O^+ ions**
- **VDF of O^+ ions assumed to be maxwellian**
- **H^+ and He^+ are in a repulsive potential (determined by O^+ ions)**

Pierrard and Lemaire, JGR, A6, 11701, 1998





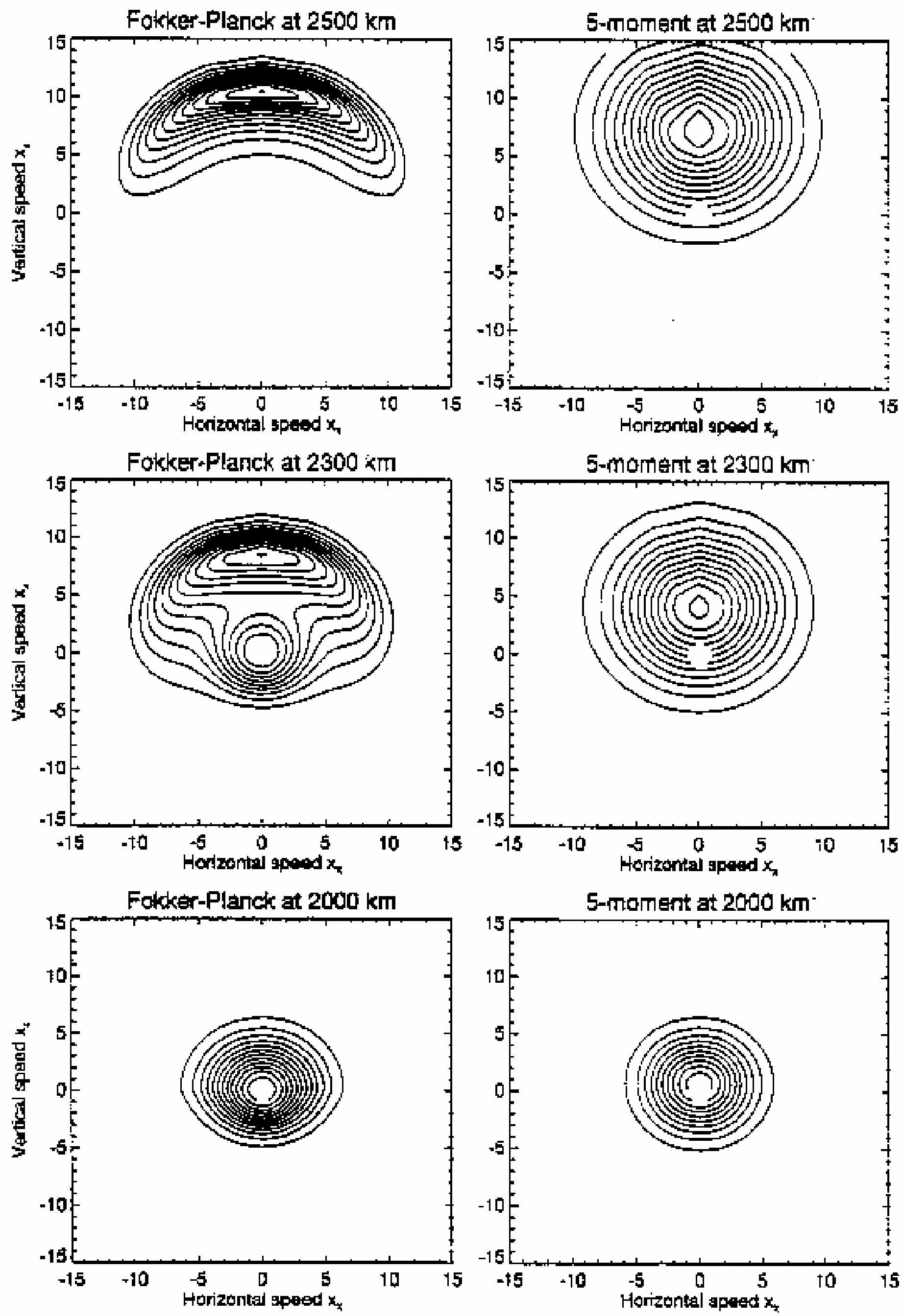
Velocity distribution function 1500 km

Polar wind H^+:

Left: kinetic collisional model (finite elements)

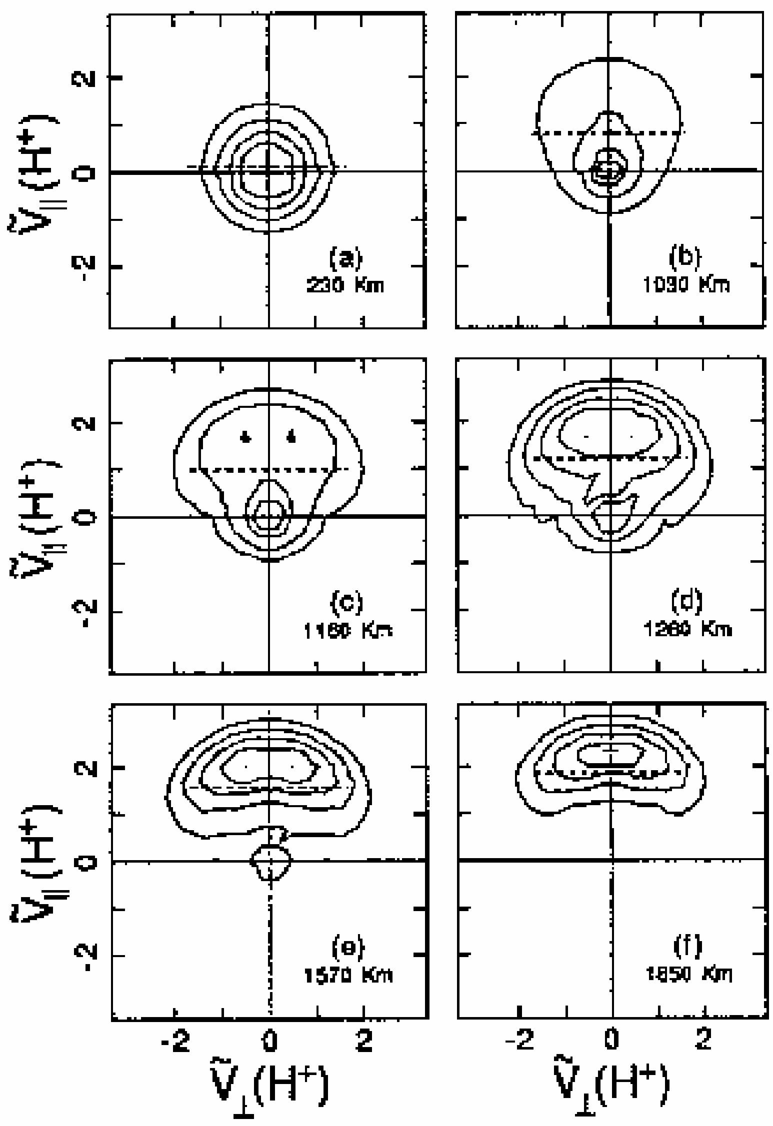
Right: Hydrodynamic model

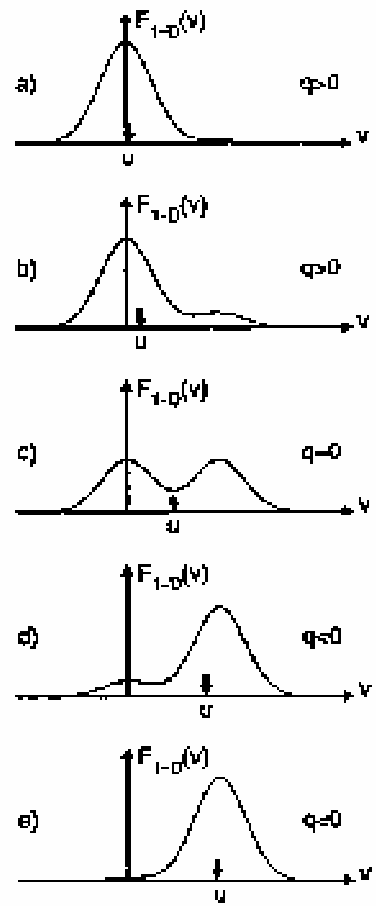
Lie-Svendsen and Rees, JGR, 101, 2415, 1996

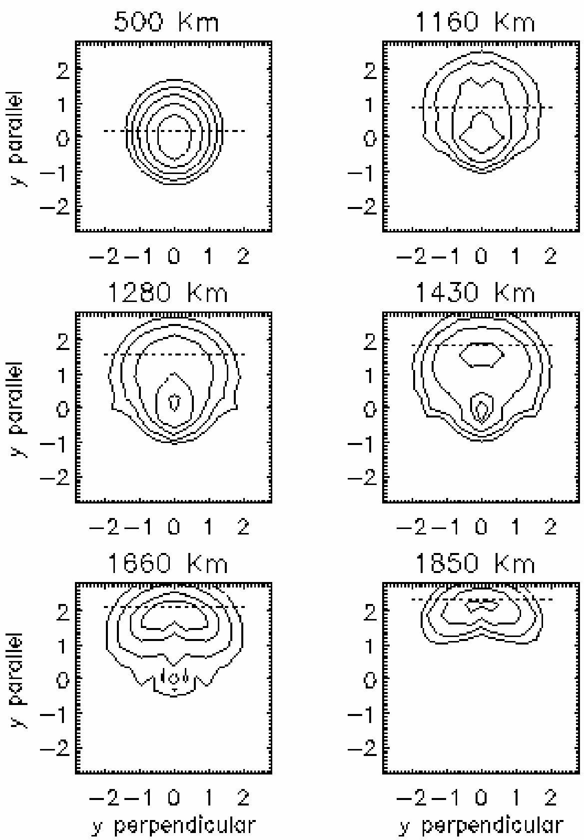


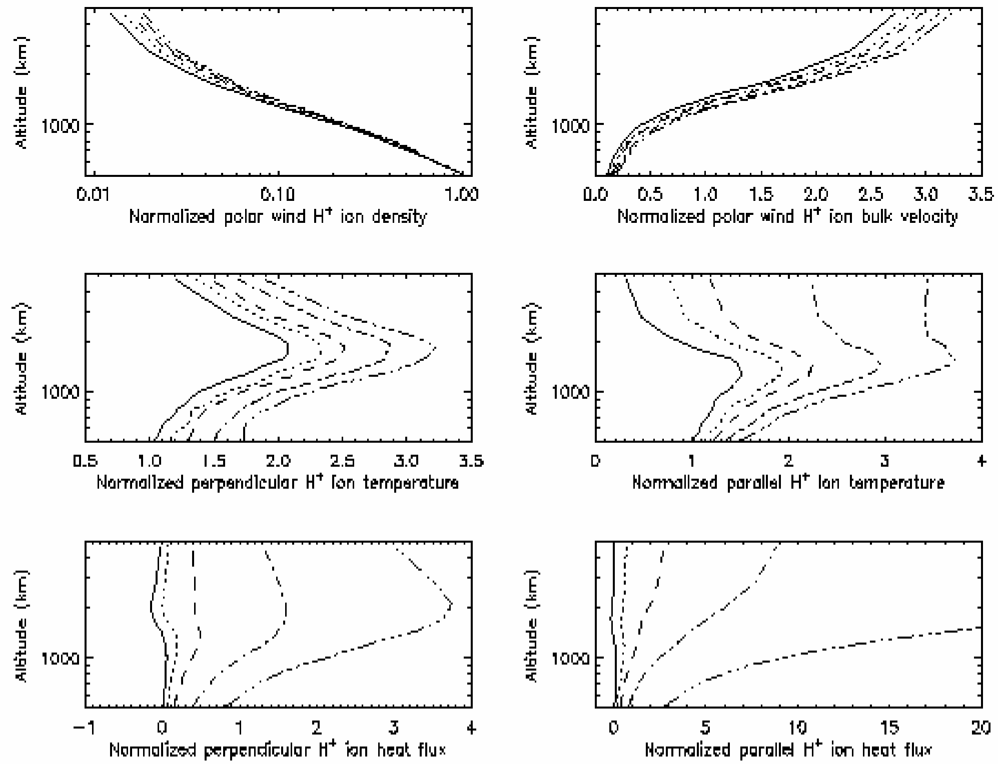
Monte Carlo simulations

Barakat, et al., Geophys. Res. Let., 22, 1857, 1995









____ Maxw
...... kappa=10
------ kappa=6
-.-.-.- kappa=3
-...-...kappa=2

Kappa=2

Monte Carlo simulation assuming kappa VDF

Barghouthi, Pierrard, Barakat and Lemaire, Astrophys. Space Sci., 277, 427, 2001

Monte Carlo simulations with wave-particle interactions

Pierrard and Barghouthi, Astrophys. Space Sci., 302, 35, 2006

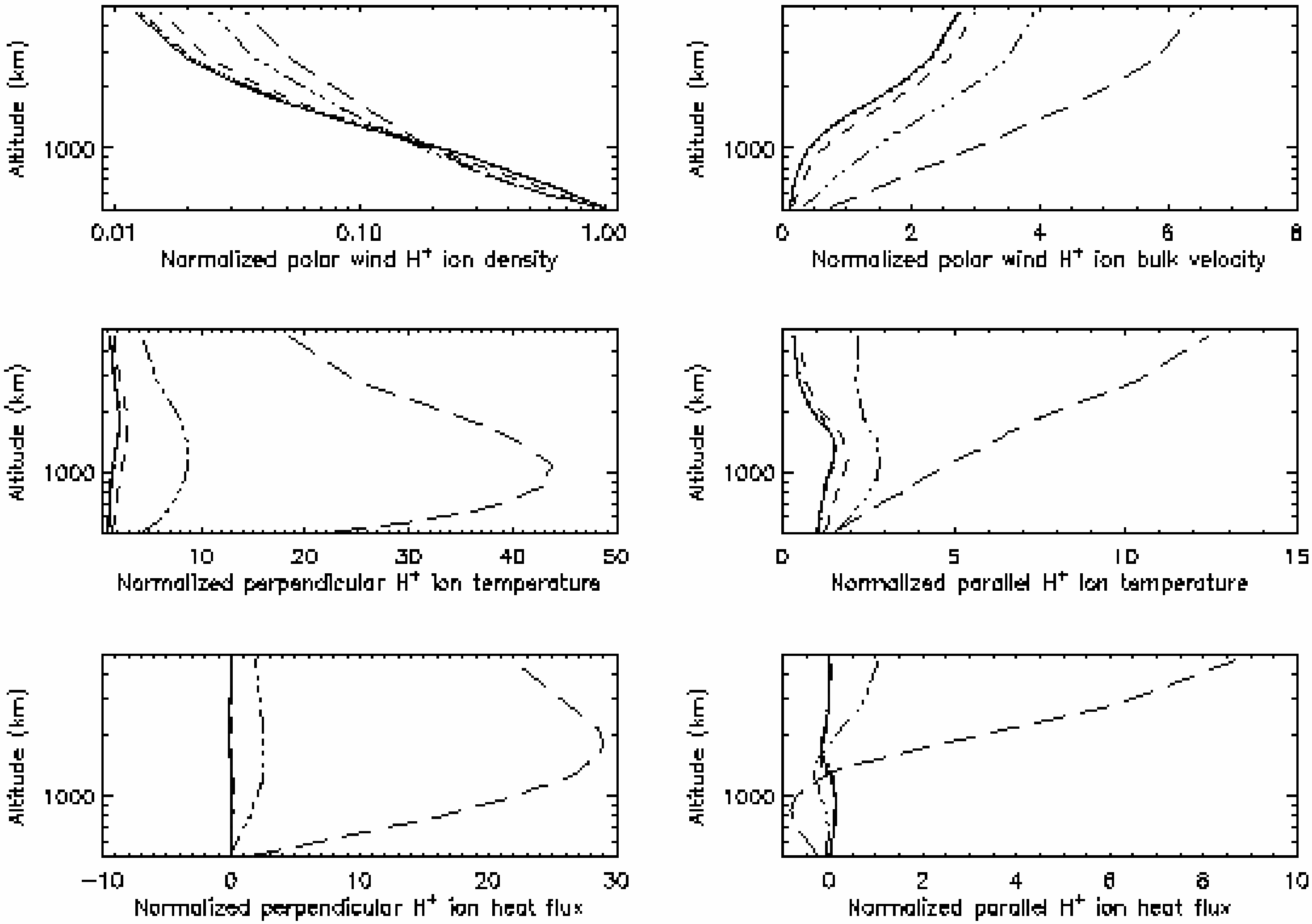
Diffusion rate

___ Dperp=0

---- Dperp=0.01

-...- Dperp=0.1

_ _ Dperp=1



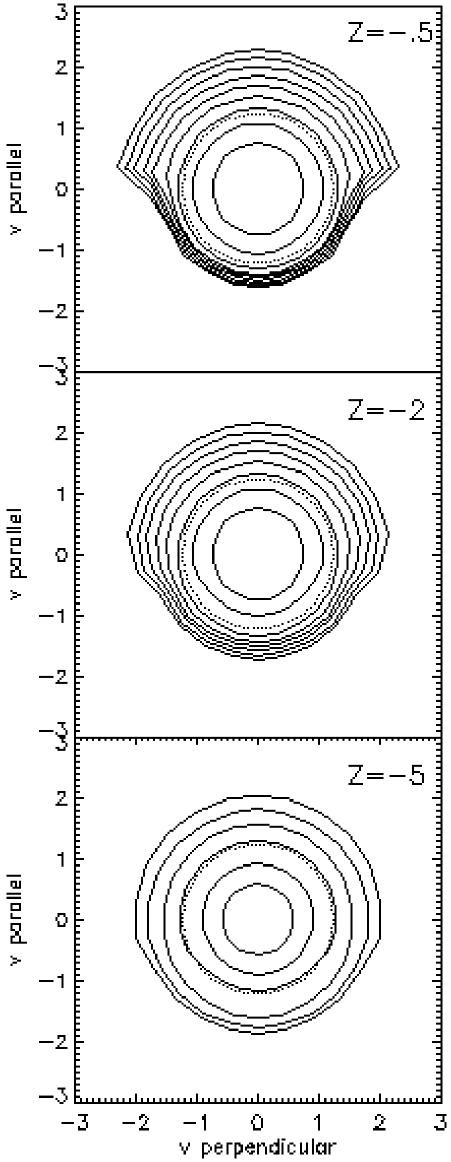
Neutral planetary atmospheres

- **Boltzmann equation: Same spectral method for the escape of neutral particles from the planetary atmospheres**

Altitude of the exobase:
Earth 500 km
Mars 250 km

Escape velocity:
$V_e = (2GM/r)^{1/2}$

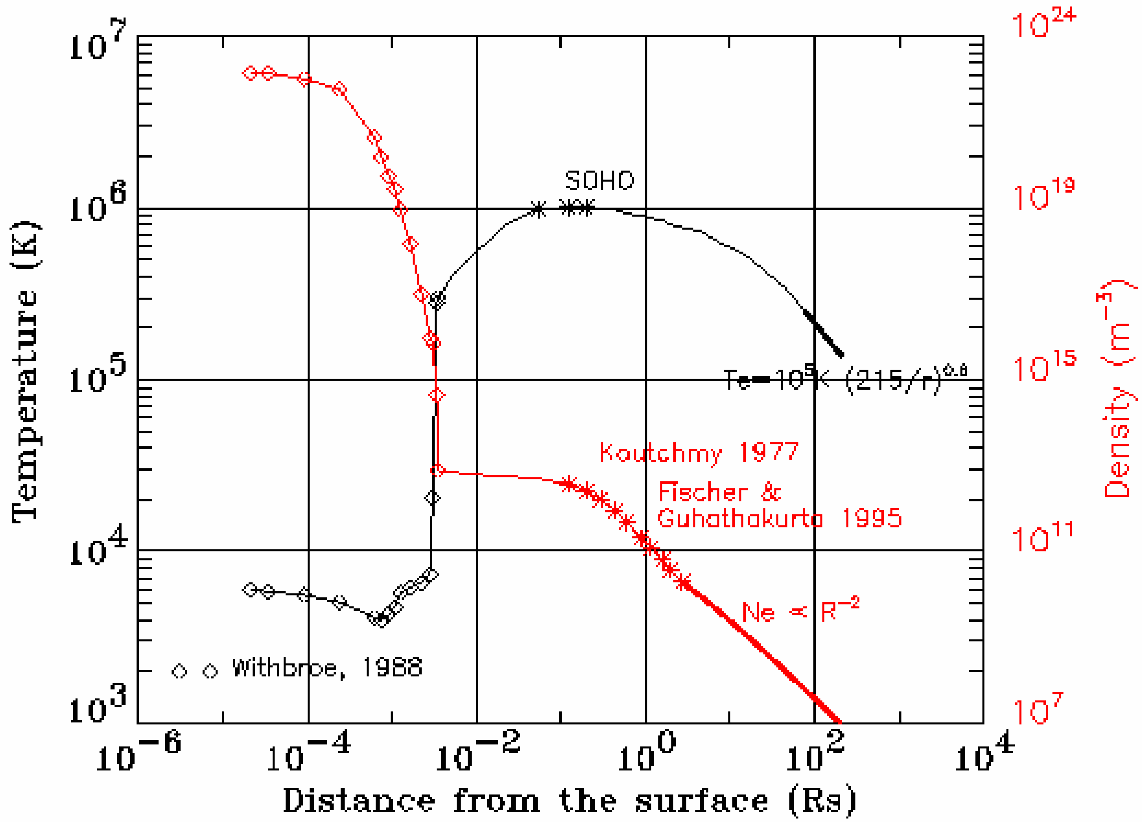
Earth: 10.8 km/s
Mars: 4.8 km/s



- Pierrard, Planet. Space Sci., 51, 319, 2003

Conclusions

- Models of polar and solar winds based on the resolution of the Fokker-Planck equations are necessary for the transition region but not only
- The velocity distribution functions found with kinetic collisional models are more realistic than the truncated VDF of the exospheric models
- The solutions of the hydrodynamic models depend on the approximations made to solve the system of equations
- Suprathermal tails generally observed in space plasmas
- Suprathermal tails can be maintained even when collisions are considered, when they are imposed in the boundary conditions.
- Suprathermal tails are not created if maxwellians are assumed as boundary conditions.
- The kinetic collisional models can be improved by solving simultaneously the equations for all particle species



Temperature (K)
Density (m^{-3})
Distance from the surface (Rs)
SOHO
Te=10^5K (215/r)$^{0.6}$
Koutchmy 1977
Fischer &
Guhathakurta 1995
Ne ∝ R^{-2}
◇ ◇ Withbroe, 1988