Difficulties of Adding Coulomb Collisions in Monte Carlo Simulations

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Outline

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2. **0-D (homogeneous) simulations:**
   - Binary collisions
   - Diffusion in velocity space
   - Macro-collisions
3. **1-D simulations:**
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   - Effects of mass ratio

- Collisionless “tail” particles

4. **Coupling/comparison with transport equations**
5. **Summary**

1. Introduction

- Characteristics of the M.C. techniques:
  1. Simple algorithm
  2. Stable (advantage?)
  3. Error ~ $N^{-1/2}$
  4. Computationally intensive

- Coulomb Collisions:

1. Widespread
2. $\upsilon \sim \dfrac{1}{g^3}$
    - Hot plasma becomes collisionless.
    - Tail particles are collisionless.
3. Binary small-angle scattering
4. Responsible for Interesting Shapes of the VDF (double hump, Kappa, etc.)

2. 0-D (Homogeneous) simulations

- Binary Collisions:

$\chi \equiv$ scattering angle

$$\approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\mu g^2 \lambda_D}$$

$\sim 10^{-4}$ degrees

- For 1 large-angle scattering, we need N binary scattering where

$N \sim (90/\chi)^2$

$\sim 10^{12}$

Diffusion in the Velocity Space:

- The cumulative effect of C.C. in Δt is represented by drag and diffusion in the velocity space.

$$\text{collision term} = -\frac{\partial}{\partial \mathbf{v}} \bullet \left[\mathbf{A}f - \frac{1}{2}\frac{\partial}{\partial \mathbf{v}} \bullet (\mathbf{D}f) \right]$$

where $\mathbf{D} = D_{\|}\mathbf{e}_{\mathbf{v}}\mathbf{e}_{\mathbf{v}} + D_{\perp}\left(\mathbf{I} - \mathbf{e}_{\mathbf{v}}\mathbf{e}_{\mathbf{v}}\right)$

- It is simulated by randomly changing the velocity s.t.

$$\left\langle \left(\Delta v_{\|}\right)^2 \right\rangle = D_{\|}\ \Delta t$$

$$\left\langle \left(\Delta v_{\perp}\right)^2 \right\rangle = 2D_{\perp}\ \Delta t$$

- Δt is chosen in order to achieve accuracy and efficiency.

- f_b assumed Maxwellian.
- Solution may diverge for self collisions.
- Remedies

  1. Many particles + readjust $(\mathbf{v})$ in order to conserve energy and momentum
  2. Use “Macro Collisions.”.

Macro Collisions:

- The particles are paired randomly.
- The scattering angle χ is randomly picked s.t.

$$\left\langle \chi^2 \right\rangle = \frac{2\pi e^4 n \lambda}{m^2 g^3} \Delta t$$

g	relative speed
λ	Coulomb logarithm
Δt	chosen appropriately

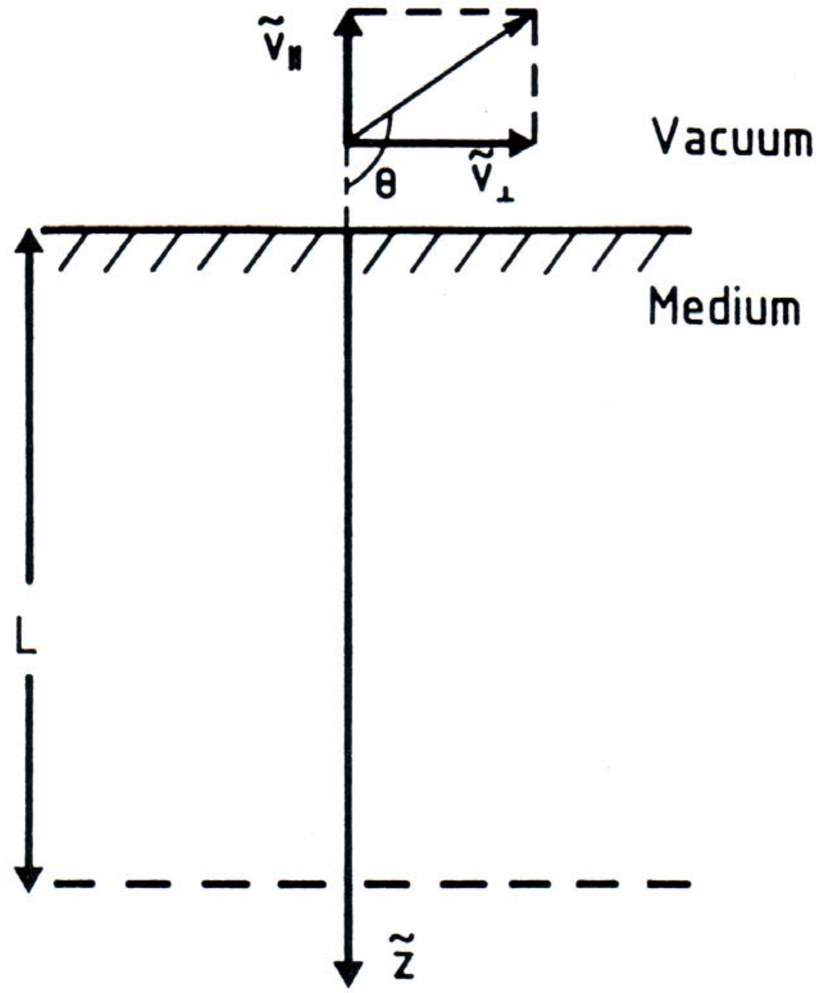
- Conserves energy and momentum

3. 1-D Simulations

Example: Milne Problem

$$\tilde{z} = z / \text{m.f.p.}$$

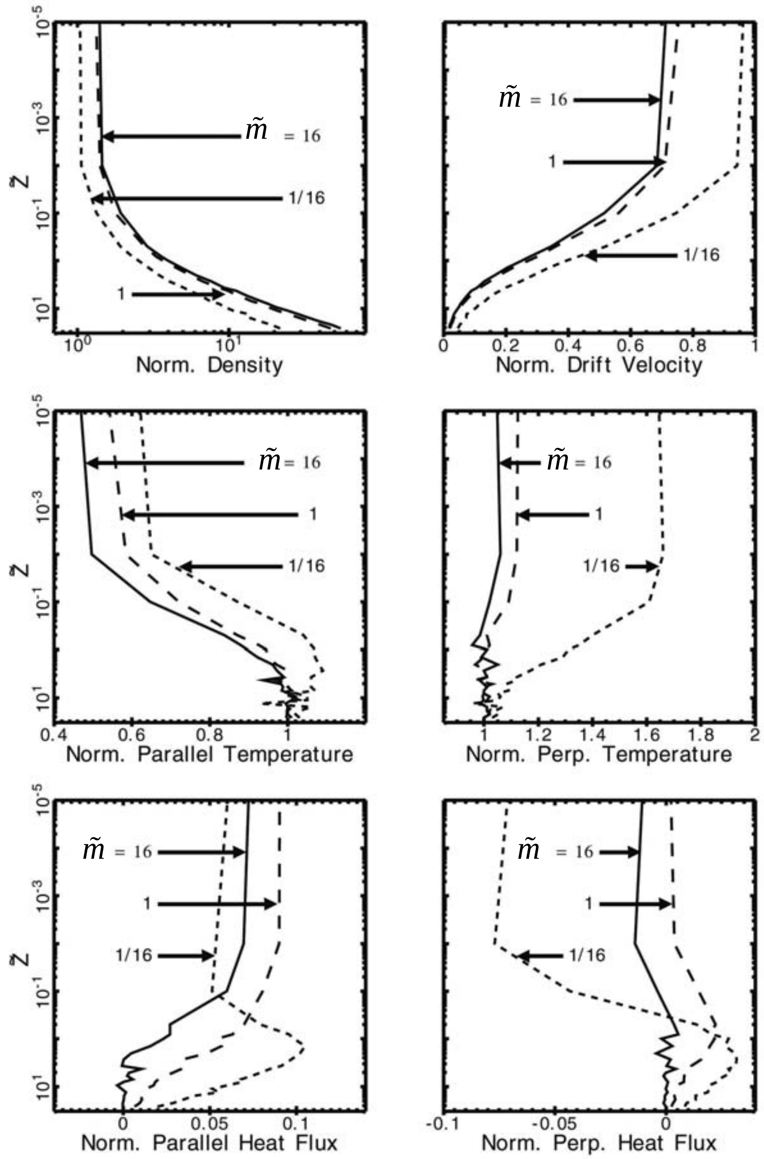
$$n_b = \begin{cases} \text{const.} & \text{for } \tilde{z} > 0 \\ 0 & \text{for } \tilde{z} < 0 \end{cases}$$



Effects of boundaries

- $\tilde{u},\ \tilde{q} \neq 0$ at the boundary
- The effects of the boundary condition vanishes within few mfp's

* Large *L* is needed.



Effect of m/m_b:

- For m/m_b << 1 we notice
  - $\tilde{u}(\tilde{z} \gg)$ increases
  - $\tilde{q}(\tilde{z} \gg)$ increases
  - T/T_b deviates from 1 deeper in the collisional region.

  * Larger L is needed for $m << m_b$.

- Collisionless "Tail" Particles

  1. ν collision frequency ~ v^{-3}
  2. Tail particles thermalize slower.

  * Large L is needed for proper treatments of tail particles.

How Expensive is L?

Consider case of $L' = 2L$

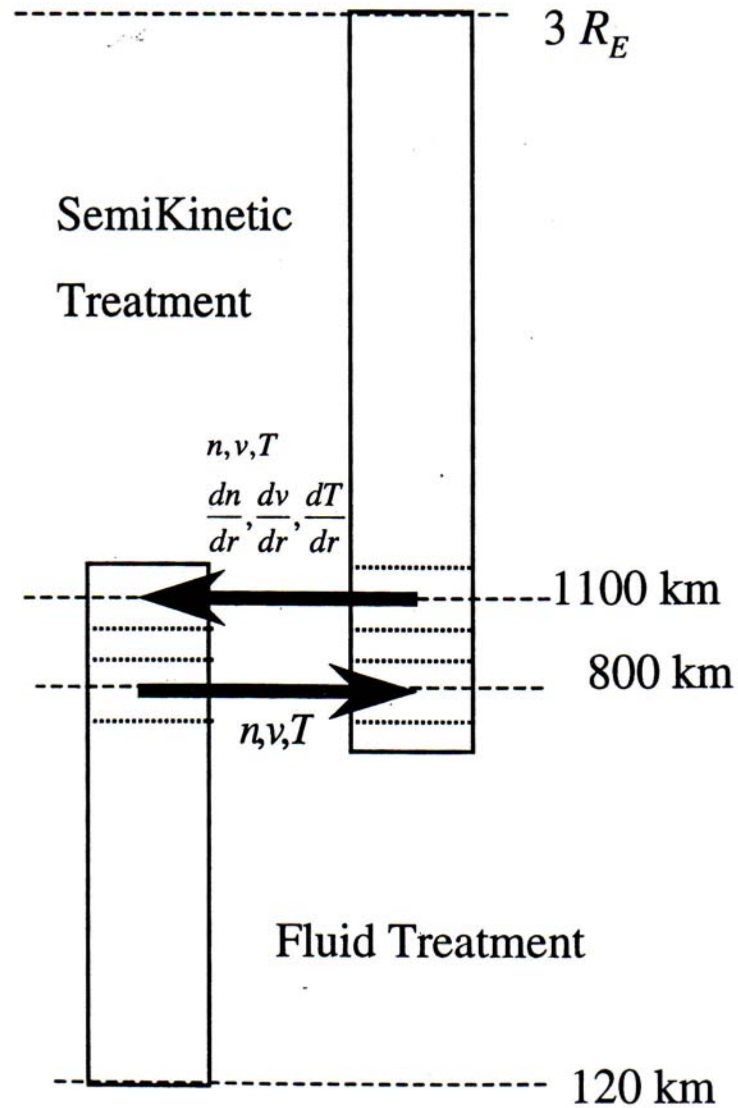
1. Twice as much particles need to be injected.
2. Particles travel twice as far and, hence, make 4 times the number of collisions.

Thus, computer resources increase 8 fold.

Resources ~ L^3

4. Coupling With Transport Equations

- Sophisticated! <u>BUT</u> is it accurate?
- Models should be consistent over the matching region.
- Mismatching may cause artificial discontinuities.



5-moments

$$f_s^{(M)} \sim \exp(-mc^2 / 2kT)$$

13-moments

$$f_s^{(13)} \approx f_s^{(M)} \left[1 + \left(\frac{m}{2ktp} \right) \boldsymbol{\tau}_s : \mathbf{c}_s \mathbf{c}_s + \left(1 - \frac{mc^2}{5kt} \right) \frac{m}{kTp} \mathbf{q}_s \bullet \mathbf{c} \right]$$

Modified VDF

$$f_s^{(\mathrm{mod})} = f^{(BM)} [1 + \alpha_\perp \mathbf{q} \bullet \mathbf{c}_\perp c^2 (1 + \gamma_\perp c^2)$$
$$+ \alpha_{||} \mathbf{q} \bullet \mathbf{c}_{||} c^2 (1 + \gamma_{||} c^2)]$$

$$f_s^{(BM)} \sim \exp(-m(c_{||}^2 / 2kT_{||} + c_\perp^2 / 2kT_\perp)]$$

1. Deep into the collision region ($\tilde{z} \gg 1$), we can expand the moments as follows:

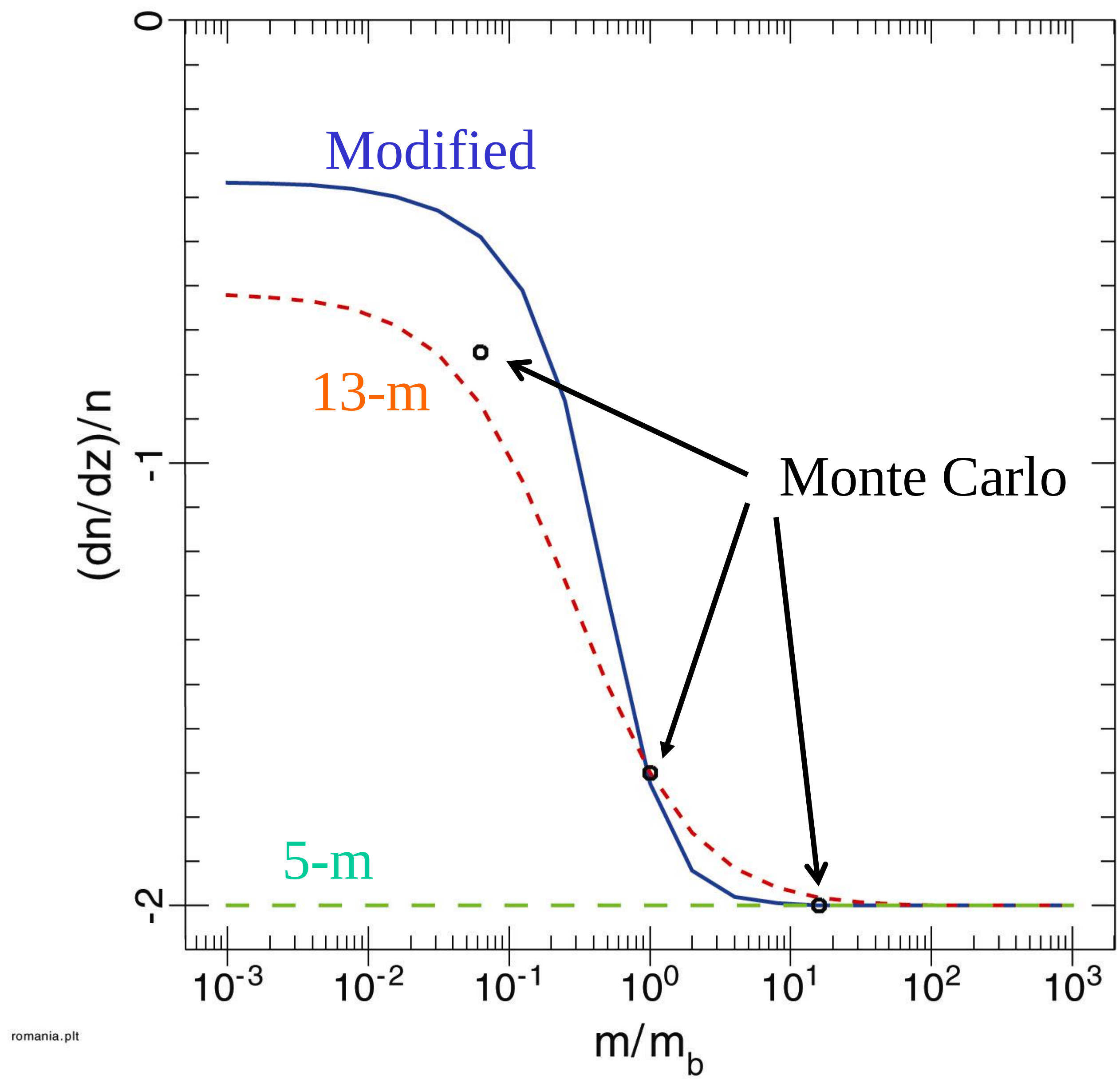
$$n = n_{-1}\tilde{z} + n_0 + n_1\tilde{z}^{-1} + \ldots$$

$$\tilde{u} = \tilde{u}_1\tilde{z}^{-1} + \tilde{u}_2\tilde{z}^{-2} + \ldots$$

....

Where $\tilde{x}$ is dimensionless normalization of x

2. Keeping the highest-order terms, we can find the value of $\left(dn/d\,\tilde{z}\right)/n$ for the three approximations.



Modified
13-m
5-m
Monte Carlo
(dn/dz)/n
0
-1
-2
10-3
10-2
10-1
100
101
102
103
m/mb
romania.plt

Conclusions

1. Including Coulomb Collisions is (often) essential for proper modeling.
2. Monte Carlo simulation of Coulomb collisions can get very expensive (computationally intensive).
3. Potential pitfalls could be due to:
   a) Boundary conditions effects
   b) Small m/m_b ratio
   c) Collisionless tail particles

Conclusions (Continued)

4. Possible remedies:

   a) Large L?

   b) Using appropriate algorithms (e.g. diffusion in the velocity space, macro-collisions,…etc.)

   c) Combining with other techniques (e.g., transport equations).

5. When matching the M.C. with another technique, we should pay special attention in order to avoid artificial discontinuities.