Hydrodynamic and kinetic models of the solar and polar winds (Part 1)

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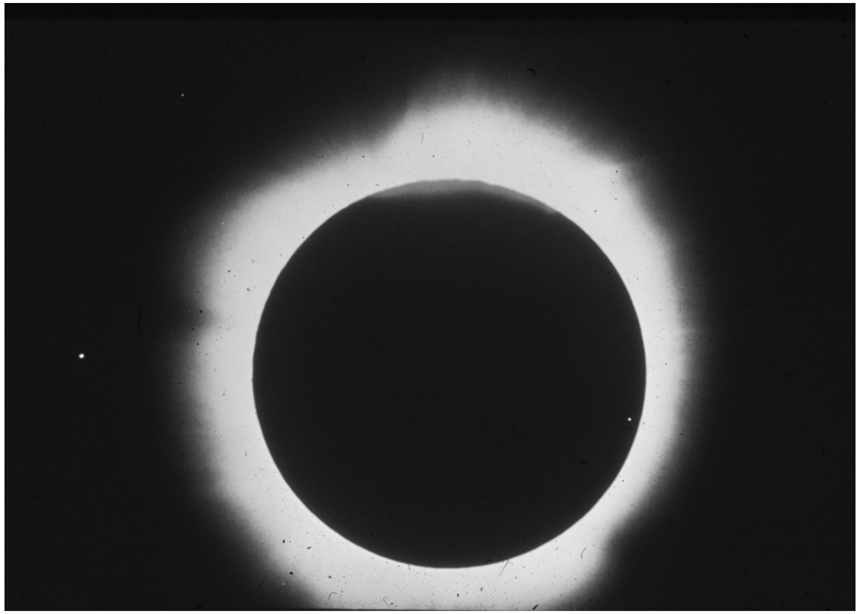
Eclipse / solar maximum

Thompson scattering of sunlight by free electrons



Early hydrostatic models of the solar corona

- Isothermal models
  Alfvén (1941) Edlèn (1942)
  Spitzer (1947)
  van de Hulst (1950, 1953)
  …
- Conductive models
  Chapman (1957)
  De Jager (1959)

$$d(n_i k T_i)/dr = -n_i m_i g + Z_i n_i e E \qquad (: g > 0)$$

$T_i = T_p = T_e = T = C^{st}$ (isothermal equilibrium)

$$k T \, dn_i / dr = -n_i m_i g + Z_i n_i e E$$

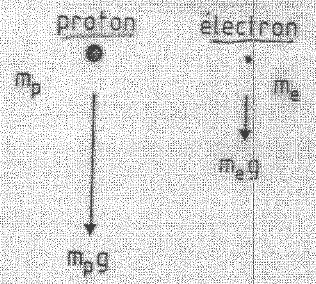
$$\Sigma_i Z_i [\ldots = \ldots]$$

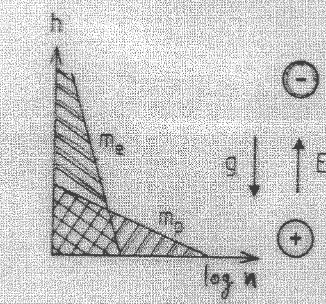
$\Sigma_i Z_i n_i = n_p - n_e = 0$ (quasi-neutrality)

$$e E = -(\Sigma_i Z_i n_i m_i g)/(\Sigma_i Z_i^2 n_i) = -\tfrac{1}{2}(m_p - m_e) g \approx -\tfrac{1}{2} m_p g$$

Pannekoek (1922) -Rosseland (1924)

This E-field is induced by gravitational force inside any plasma when it is in isothermal hydrostatic equilibrium

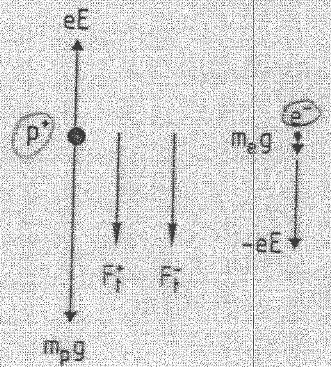


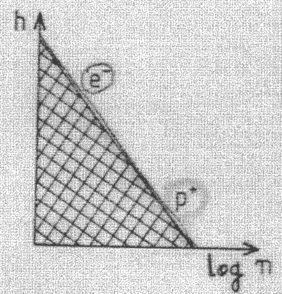


Le champ gravifique (g) induit
dans une atmosphère ionisée
une polarisation électrique (E)

$$e\underline{E} = -\frac{(m_p - m_e)}{2}\,\underline{g}$$

Pannekoeck - Rosseland (1924)





$H_{e^-} = H_{p^+} = 2\, H_{\text{neutral Hydrogen}}$

86.2

$$e\,E_{PR} = -\tfrac{1}{2}(m_p - m_e)\,g \approx -\tfrac{1}{2} m_p\, g$$

$$\Delta\Phi_E = -\int E_{PR}\, d r \approx \tfrac{1}{2}(m_p/e) \int g(r)\, d r = -\tfrac{1}{2}(m_p/e)$$

$$\Delta\Phi_g$$

$\Delta\Phi_E = -150$ Volts between $r = 1.5\, R_S$ and $r = \infty$

When temperatures are increasing with altitude (e.g. in DL),
or when the plasma is expanding away from Sun,
the polarization E-field is larger than E_{PR} …

$$e\,\mathbf{E}_{PR} = -\nabla p_e / e\, n_e$$

♠ $\mathbf{E}_{PR} \cdot \mathbf{B} \neq 0$

Multi-fluid approximation for hydrostatic equilibrium

$$d\,p_i / d\,r \;=\; - n_i\, m_i\, g + Z_i\, n_i\, e\, E$$

$$\Sigma_i\ [\ldots \;=\; \ldots\ ]$$

$p = \Sigma_i\ p_i$

$\rho = \Sigma_i\, m_i\, n_i = \langle m \rangle\, n_e$; $\langle m \rangle = \Sigma_i\, m_i\, n_i / \Sigma_i\, n_i \approx m_p / 2$

$\Sigma_i\, Z_i\, n_i = n_p - n_e \approx 0$ (quasi-neutrality)

$$d\,p / d\,r \;=\; - \rho\, g + 0 \qquad (: g > 0)$$

In this one fluid version of hydrostatic equation,
the (embarassing) E-field term has been veiled:
it has been hidden within the total pressure gradient!!!

The concept of pressure scale height

$$H = - d r / d \ln p = p / \rho g = k T / \langle m \rangle g$$

$H_p = 260$ km (photosphere) ($T = 4400$ K)

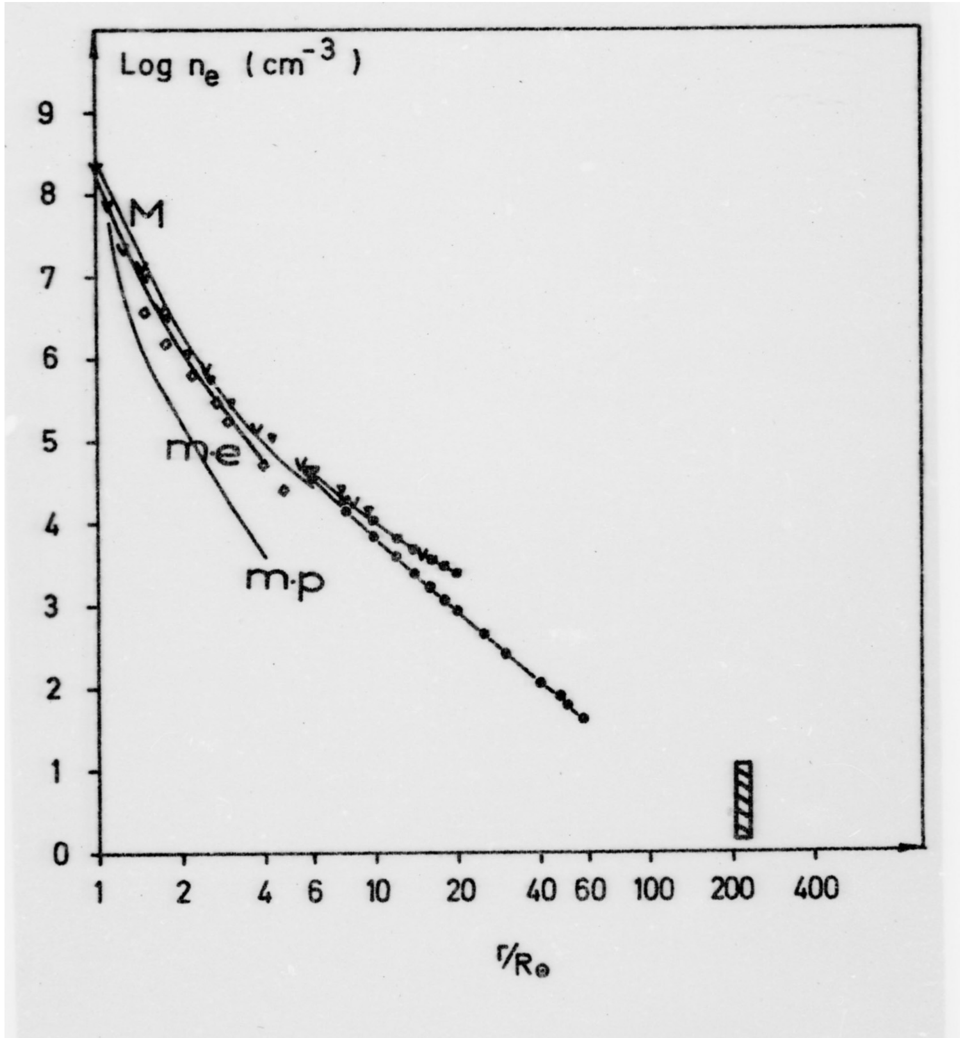
$H_p = 150\,000$ km (solar corona) ($T = 1.4$ MK)

$H = 8$ km (Earth troposphere)

When $dT/dr \neq 0$ density scale height is not the same as pressure scale height :

$$H_e = - d r / d \ln n_e = H_{ions} = - d r / d \ln n_{ions} = 2 H_{neutral\ atm}$$

Radial distributions of coronal electron densities
from eclipses observations



Is the corona really isothermal? (Chapman, 1957)

Conductive heat transport in solar corona

$q = - \kappa_o T^{5/2} \, dT / dr$ (conductive heat flow ; Fourier law)

$4 \pi r^2 q(r) = C^{st}$ (conservation of energy flux)

$R^2 T^{5/2} \, dT / dR = C^{st}$

$T(R) = T_o \, [\xi + (1 - \xi) R_o / R]^{2/7}$

$R = R_o$ $\quad$ $T = T_o$

$R = \infty$ $\quad$ $T = T_o \, \xi^{2/7} = T_\infty \geq 0;$

Chapman's conductive model (1957)

$$T_\infty = 0\ ; \quad \xi = 0$$

$$T(R) = T_o\,[\,R_o / R\,]^{2/7}$$

$$d\,(n_e\,k\,T) = -\,\lambda_o / R^2\,d\,R$$ (hydrostatic equilibrium)

$$n_e(R) = n_{o,e}\,R^{2/7}\exp\,[-\,7\,\lambda_o\,(1 - 1 / R^{5/7}) / 5]$$

$$\lambda_o = r_o / H_o$$

$d(n_e k T_e)/dr = -n_e m_e g - n_e e E$ (hydrostatic equilibrium of electron gas)

$T_p = T_e = T = C^{st}$ (isothermal)

$e E_{PR} = -\frac{1}{2}(m_p - m_e) g$

$g = G M_S / r^2 = g_o / R^2$; $R = r / R_o$

$\lambda_o = G M_S \langle m \rangle / r_o k T = \lambda_{o,e} = \lambda_{o,p}$

$d \ln n_e / dR = -[(m_p + m_e) g / k T] R_S = -\lambda_o / R^2$

$n_e(R) = n_e(R_o) \exp[\lambda_o (1/R - 1/R_o)] = n_p(R)$

$R_o = 1.5$; $T = 1.4$ MK ; $\langle m \rangle = m_p / 2$; $\lambda_o \approx 6.6$;

$n_e(1.5) = 10^8 / cm^3$; $n_e(\infty) = n_e(R_o) \exp[-\lambda_o / R_o] = 1.8\ 10^6 / cm^3 >> n_{obs}(\infty)$

The coronal density distribution has a minimum value at

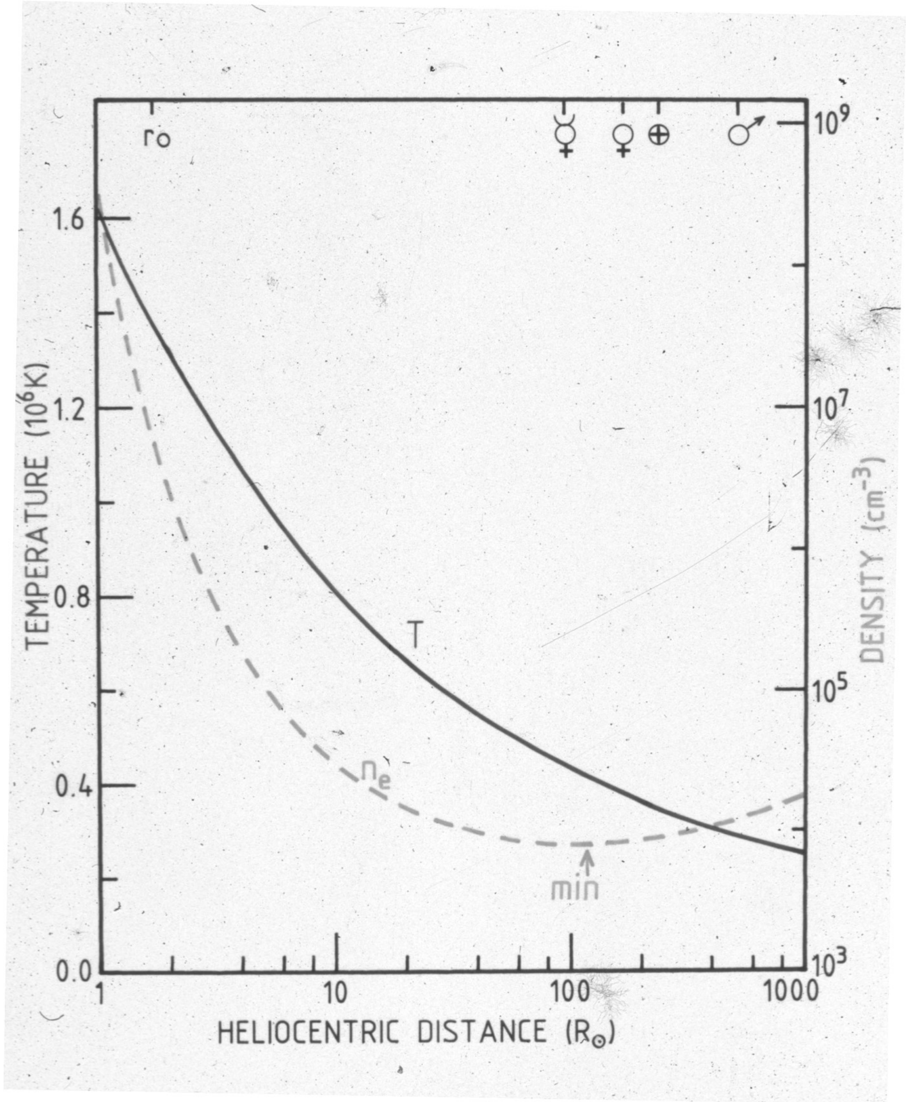
$r_{min} = r_o\ (7\ \lambda_o\ /\ 2)^{7/5} = 125\ R_S$

The temperature gradient becomes superadiabatic : $(1 / \gamma - 1)$ <m> g / k,

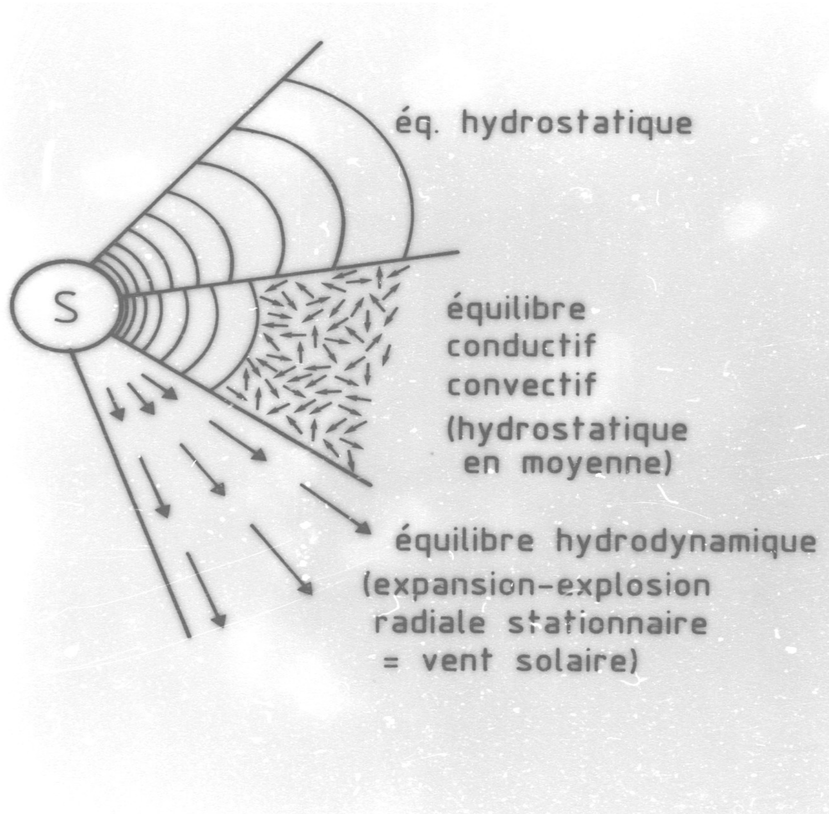
for $r > r_{ad} = 34\ R_S$

$R_{ad} = r_{ad} / r_o = (1 - \xi) \ / \ \{[5\ (1 - \xi) / 7\ \lambda_o]^{7/5} - \xi\}$; $\gamma = 5/3$

$r_o = 1.5\ R_S$; $T_o = 1.4$ MK ; $\lambda_o = 6.6$; $r_{ad} = 34\ R_S$



Minimum density at 125 R_S, indeed Chapman's conductive hydrostatic model becomes already convectively unstable at 34 R_S where the temperature gradient becomes superadiabatic!!!



Solar corona cannot withstand hydrostatic equilibrium.

It becomes convectively unstable above some altitude (34 R_S).
But turbulent convection alla Bhom-Vitense is an
inefficient energy transport process in the outer corona…
Therefore, the solar corona must expand (steady state explosion : solar wind!)
in order to get rid of the energy deposited per unit time at the bottom...
Continous radial expansion (solar wind) is thus the most efficient mechanisms
to evacuate the power deposited at the base of the corona toward interplanetary medium.

Steady state expansion of solar corona : solar wind

Hydrodynamic models / approaches /approximations

Euler approximation of transport equations
(Parker, 1958, 1963, 1965, …)

Assume the densities & bulk velocities of the electrons and protons are equal: $u_e = u_p = u(r)$

$\rho\, u\, (d u / d r) + d p / d r = - \rho\, g$ (one fluid momentum equation)

$\rho\, u\, r^2 = \rho_o\, u_o\, r_o^{\,2} = C^{st}$ (continuity equation)

$p = n\, k\, T$ (perfect gas law)
$T = C^{st}$ (isothermal expansion)
$c^2 = k\, T / \langle m \rangle$ (sound speed)
$\lambda_o = \langle m \rangle\, g_o\, r_o / k\, T = r_o / H_o$

$d \ln(u / c) / d R\, [u^2 / c^2 - 1] = 2 / R - \lambda_o / R^2$

Bernouilli equation
(de Laval Nozzle analogy)

$$\tfrac{1}{2}\,(u^2 - u_o^2)\,/c^2 - \ln\,(u\,/u_o) = \lambda_o\,(1 - 1\,/R) - 2\,\ln R$$

At $R = 1$ $\Rightarrow u = u_o$

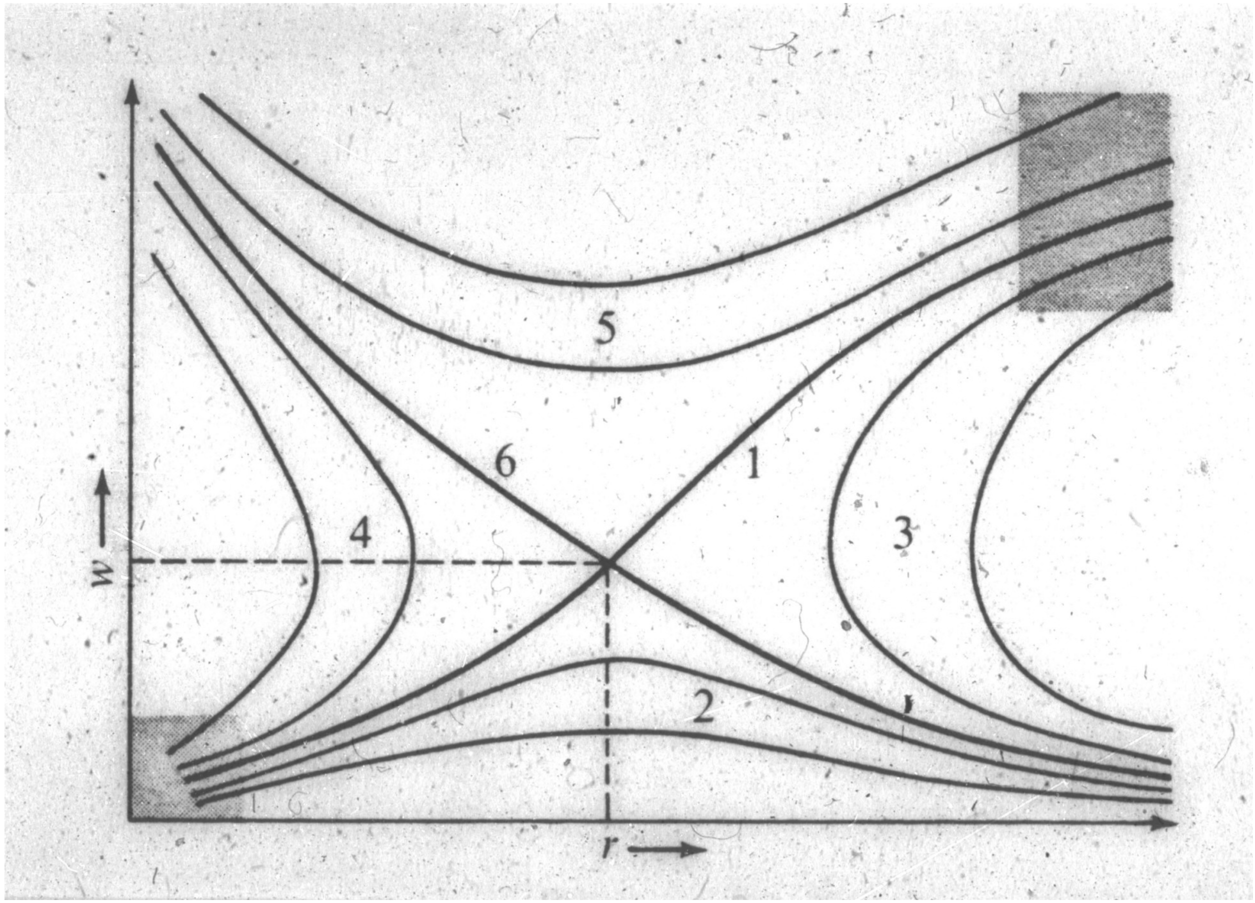
At $R = \infty$ $\Rightarrow u = 0$; $n = n_\infty >> n_{obs}$ for $u < c$ (infrasonic)

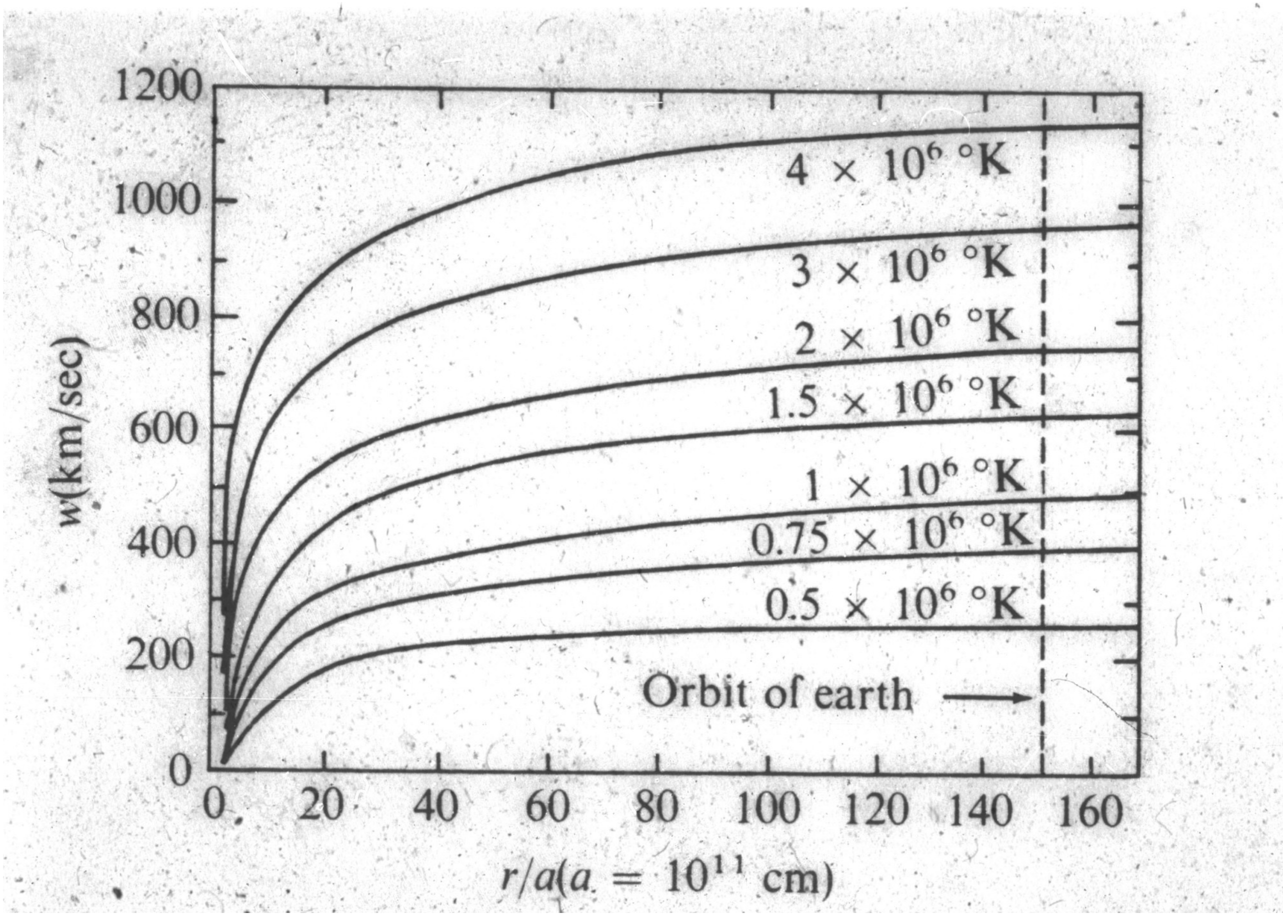
$\Rightarrow u = \infty$; $n = 0$; $p = 0$ for $u > c$ (supersonic)

PARKERS'S CRITICAL SOLAR WIND SOLUTION
IS DETERMINED BY SADDLE POINT WHERE
u = sound speed
(a unique critical solution)

Parker's steady state solar wind expansion (1958)

Euler approximation of hydrodynamic transport equations
or Grad 5-moment approximation





Larger coronal temperatures give larger SW bulk velocities at 1 AU.

…unfortunately faster solar wind originate in colder coronal hole !?

Steady state hydrodynamic expansions of solar corona

Hydrodynamic models / approaches / approximations

Euler approximation of transport equations
(Parker, 1958, 1963, 1965, …)

Chapman-Enskog approx. (Navier-Stokes, Burnett, super-Burnett…)
(Noble & Scarf, 1963; Parker, 1964; Whang & Chang, 1965; Durney, 1971 …)

Grad 5- 8- 13- 20-moments approximations (Grad, 1949)
The 16-moments approx. (Oraevski, et al, 1968; Demars & Schunk, 1979)
(Whang, 1971, 1972; Cuperman et al., 1980, 1981,1984;
Demars & Schunk, 1990, 1991; Leblanc et al. 2000;
Ofman, 2000, 2004; Li et al, 2006, …)

$\frac{3}{2} kT_s = \frac{1}{2} m_s\langle c_s^2\rangle$, species temperature;

$\mathbf{q}_s = \frac{1}{2} n_s m_s \langle c_s^2 \mathbf{c}_s\rangle$, heat flow vector;

$\mathbf{P}_s = n_s m_s \langle \mathbf{c}_s \mathbf{c}_s\rangle$, pressure tensor;

$\boldsymbol{\tau}_s = \mathbf{P}_s - p_s\mathbf{I}$ stress tensor;

$\boldsymbol{\mu}_s = \frac{1}{2} n_s m_s \langle c_s^2 \mathbf{c}_s \mathbf{c}_s\rangle$, higher-order pressure tensor;

$\mathbf{Q}_s = n_s m_s \langle \mathbf{c}_s \mathbf{c}_s \mathbf{c}_s\rangle$, heat flow tensor;

where n_s is the density of species s, $p_s = n_s kT_s$ is the partial pressure, k is Boltzmann s constant, $\mathbf{I}$ is the unit dyadic, and the bracket symbol denotes the average

$$\langle \mathrm{A}\rangle = \frac{1}{n_s}\int d\mathbf{c}_s f_s \mathrm{A}. \tag{2.6}$$

If we multiply Equation (2·3) by 1, $m_s\mathbf{c}_s$, $\frac{1}{2} m_s c_s^2$, $m_s\mathbf{c}_s\mathbf{c}_s$ and $\frac{1}{2} m_s c_s^2 \mathbf{c}_s$ and integrate over velocity space, we obtain, respectively, the continuity, momentum, energy, pressure tensor and heat flow equations for species s:

Continuity

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{u}_s) = \frac{\delta n_s}{\delta t} \tag{2.7a}$$

Momentum

$$n_s m_s \frac{D_s \mathbf{u}_s}{Dt} + \nabla \cdot \mathbf{P}_s - n_s m_s \mathbf{G} - n_s e_s\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_s \times \mathbf{B}\right) = \frac{\delta \mathbf{M}_s}{\delta t} \tag{2.7b}$$

Energy

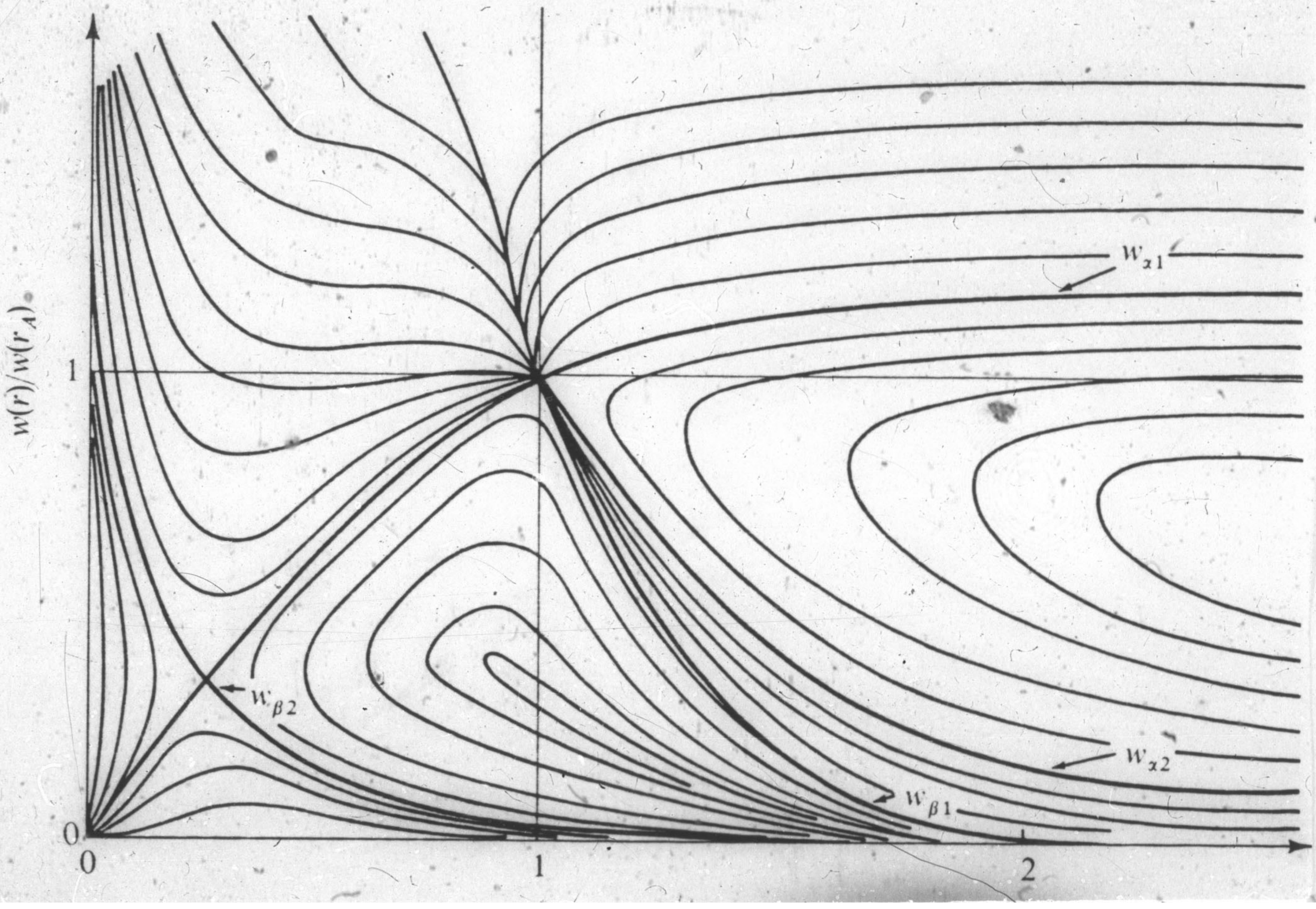
$$\frac{D_s}{Dt}(\tfrac{3}{2}p_s) + \tfrac{3}{2}p_s(\nabla \cdot \mathbf{u}_s) + \nabla \cdot \mathbf{q}_s + \mathbf{P}_s : \nabla \mathbf{u}_s = \frac{\delta E_s}{\delta t} \tag{2.7c}$$

Pressure tensor

$$\frac{D_s \mathbf{P}_s}{Dt} + \nabla \cdot \mathbf{Q}_s + \mathbf{P}_s(\nabla \cdot \mathbf{u}_s) + \frac{e_s}{m_s c}[\mathbf{B} \times \mathbf{P}_s - \mathbf{P}_s \times \mathbf{B}] + \mathbf{P}_s \cdot \nabla \mathbf{u}_s + (\mathbf{P}_s \cdot \nabla \mathbf{u}_s)^T = \frac{\delta \mathbf{P}_s}{\delta t} \tag{2.7d}$$

Heat flow

$$\frac{D_s \mathbf{q}_s}{Dt} + \mathbf{q}_s \cdot \nabla \mathbf{u}_s + \mathbf{q}_s(\nabla \cdot \mathbf{u}_s) + \mathbf{Q}_s : \nabla \mathbf{u}_s + \nabla \cdot \boldsymbol{\mu}_s$$
$$+ \left[\frac{D_s \mathbf{u}_s}{Dt} - \mathbf{G} - \frac{e_s}{m_s}\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_s \times \mathbf{B}\right)\right] \cdot (\boldsymbol{\tau}_s + \tfrac{5}{2}p_s\mathbf{I}) - \frac{e_s}{m_s c}\mathbf{q}_s \times \mathbf{B} = \frac{\delta \mathbf{q}_s}{\delta t} \tag{2.7e}$$



w(r)/w(r_A)
1
0
0
1
2
w_α1
w_α2
w_β1
w_β2

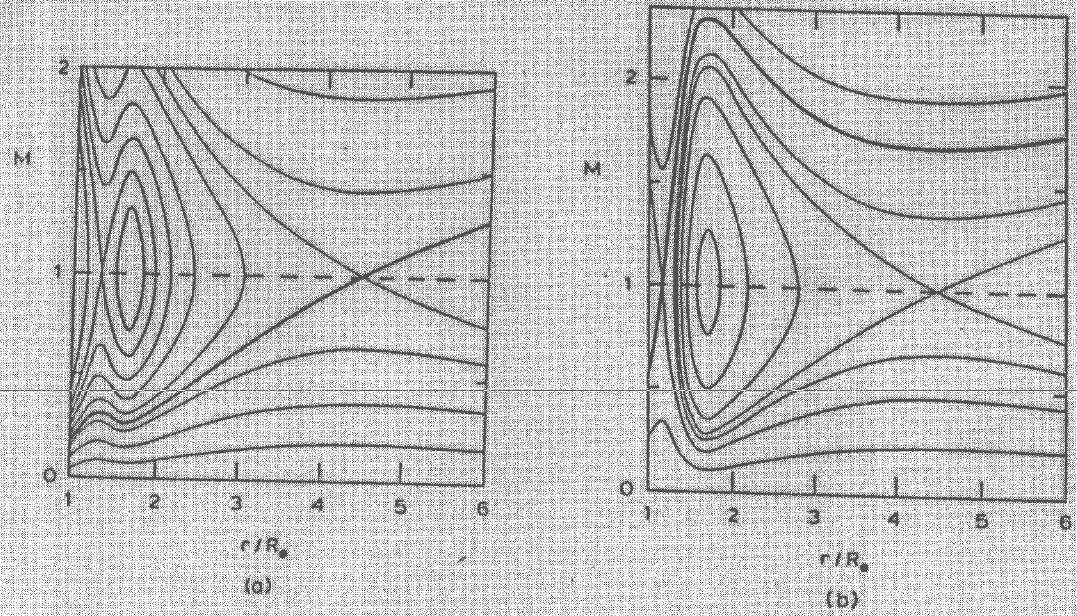


Fig. 12. Solution topologies for super-radial expansion. There are three critical points: two X-type points with an O-type point between them. The energy per gram of the fluid is the same in both figures, but the expansion parameter is much larger in b) than in a). The physically realizable solution is shown with a bold line (Kopp and Holzer, 1976). (Copyright by D. Reidel Publishing Company. Reprinted by permission of Kluwer Academic Publishers).

Knudsen number : Kn

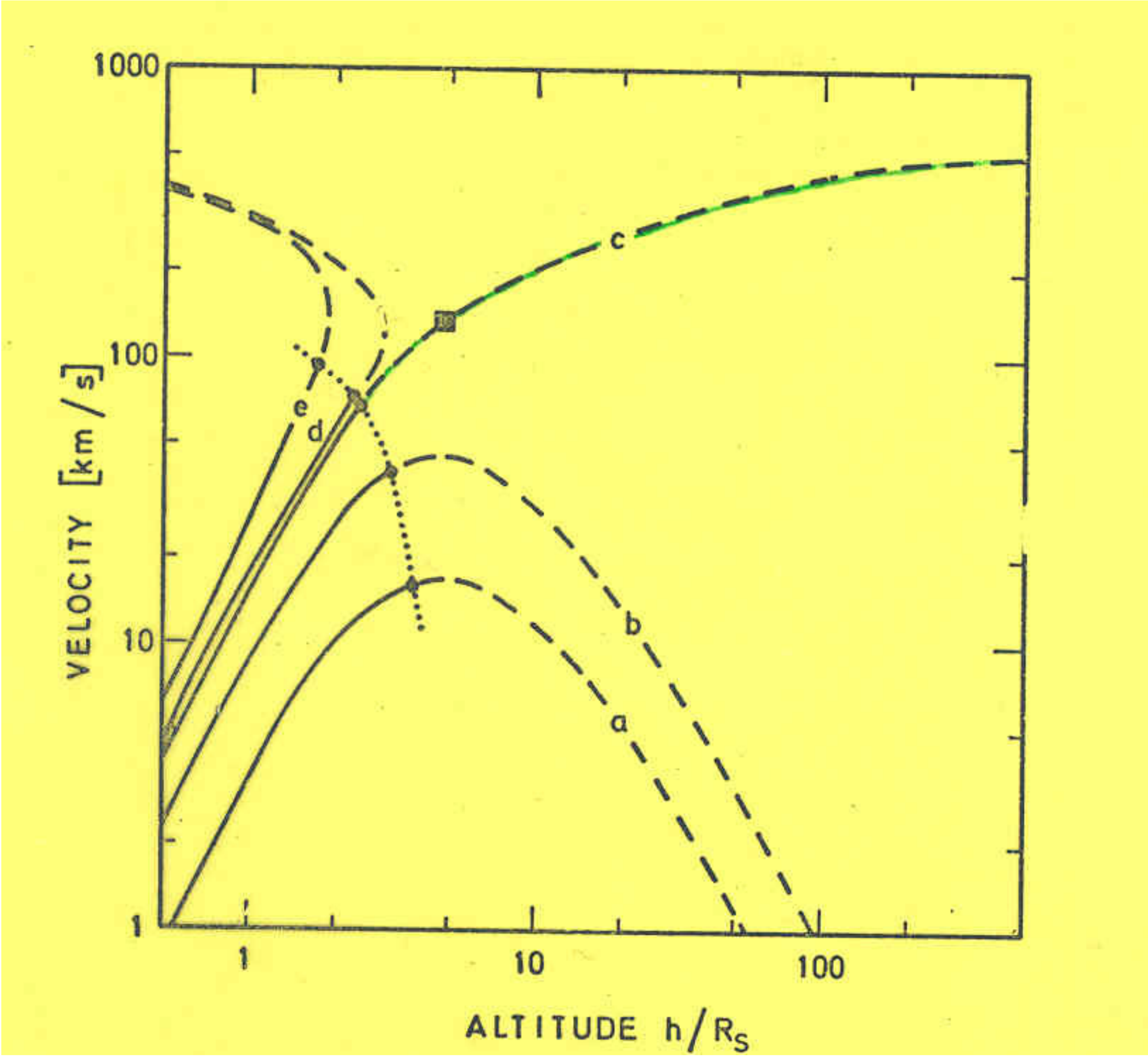
Mean free path : mfp

Electrons : $\ell_{D,e} = 0.75\ T_e^2 / n_e \ln \Lambda$ [km]

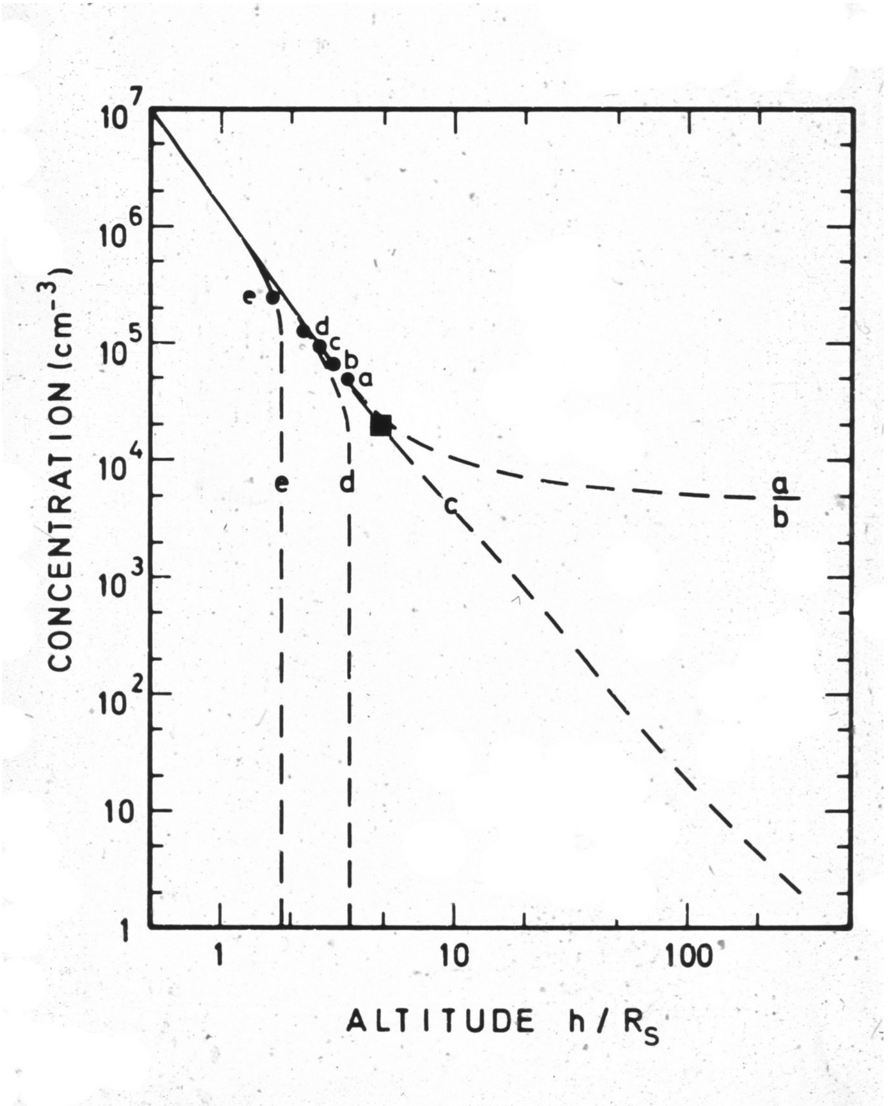
Protons : $\ell_{D,p} = 1.80\ T_p^2 / n_e \ln \Lambda$ [km]

Kn = mfp / scale height $= \ell_D / H$

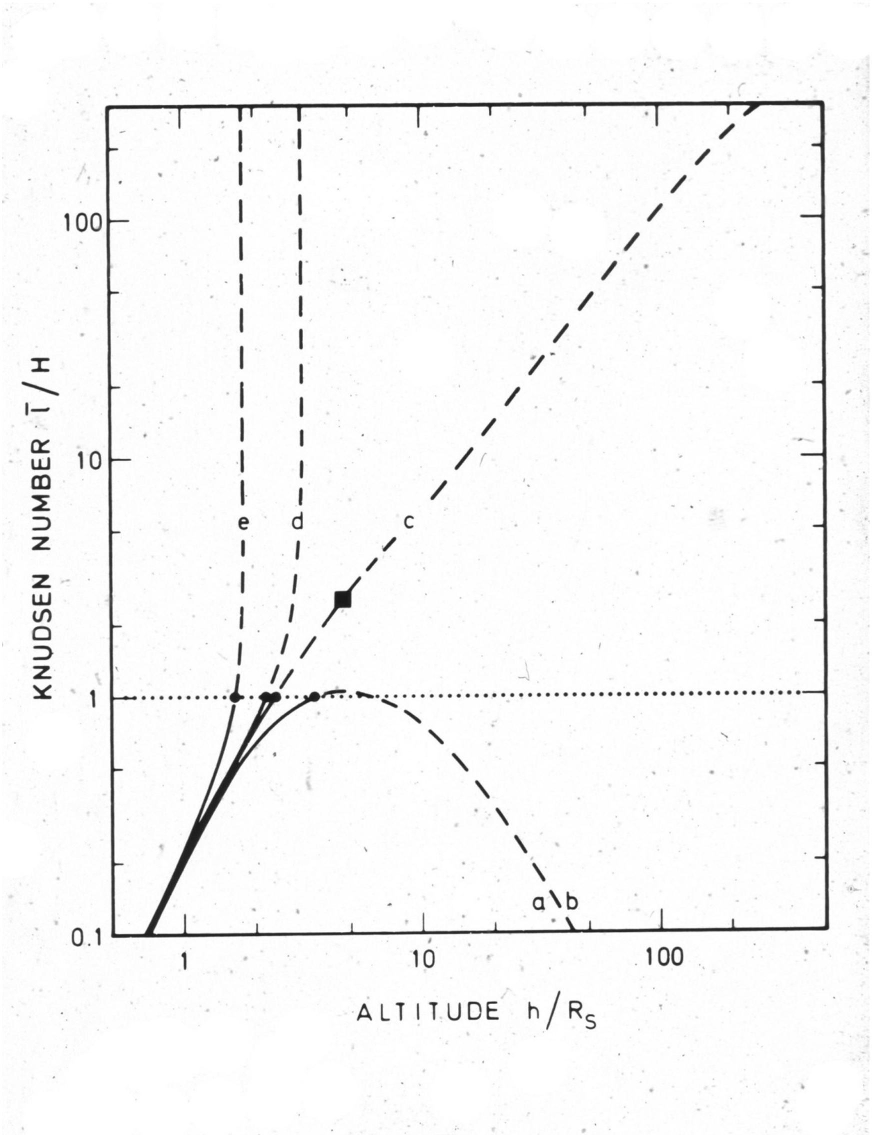
Exobase level where Kn = 1


1000
100
10
1
VELOCITY [km/s]
1
10
100
ALTITUDE h/R_S
a
b
c
d
e

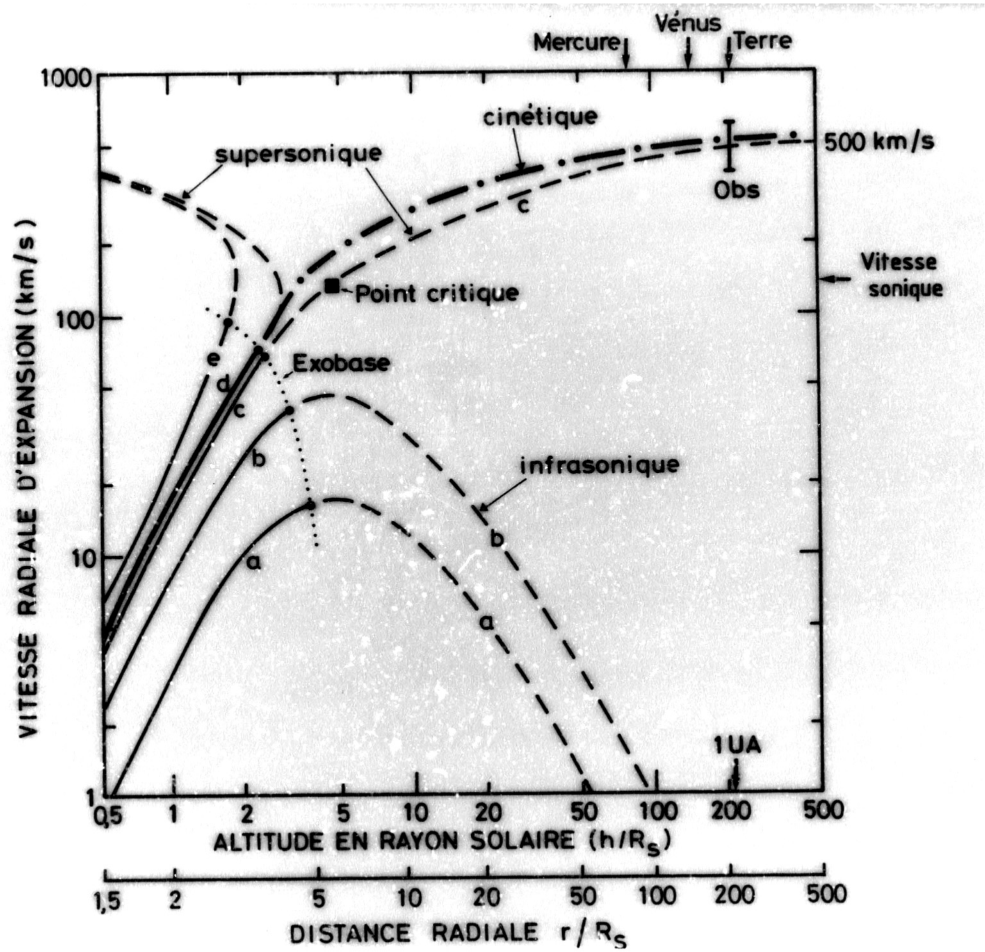
Density distributions in Parker's hydrodynamical solar wind solutions



Knudsen number becomes larger than unity beyond 2-6 R_S



Beyond the exobase Coulomb collisions become rare.
Hydrodynamic approximations of transport equations are questionable;
Use kinetic approaches (exospheric, Fokker-Planck solutions, Monte-Carlo ...)



Conclusions Part 1 :

- There is a polarization electric field in the corona as well as in any gravitationally bound plasma
- Generally $\mathbf{E}_{PR}$ has a non vanishing parallel component!
- $\mathbf{E}_{PR}$ keeps the electron density scale height equal to ion density scale height; it accelerates also the solar and polar wind protons to supersonic velocities
- This E-field is hidden in one fluid approximations of transport equations!

- Hydrostatic coronal models are convectively unstable
- Hydrodynamic expansion/explosion is needed to carry energy away to space (conduction alone is not efficient enough)
- Hydrodynamic solutions become collisionless at an altitude smaller than sonic critical point (saddle point)

Additional Slides

Cover page of CUP book “THE BASICS OF THE SOLAR WIND “ by N. Meyer-Vernet

Review paper : kinetics models of the solara and polar winds,

Reviews of Geophysics and Space Physics, Vol 11, (2) pp 427-468, 1973.

Kinetic Models of the Solar and Polar Winds

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In this paper the application of the kinetic theory to the collisionless regions of the polar and solar winds is discussed. A brief historical review is given to illustrate the evolution of the theoretical models proposed to explain the main phenomenon and observations. The parallelism between the development of the solar wind models and the evolution of the polar wind theory is stressed especially. The kinetic approaches were in both cases preceded by the hydrodynamic models, and their publication gave rise to animated controversies; later on, semikinetic and hydromagnetic approximations were introduced. A kinetic method, based on the quasi neutrality and the zero current condition in a stationary plasma with open magnetic field lines, is described. The applicability of this approach on the solar and polar winds is illustrated by comparison of the predicted results with the observations. The kinetic models are also compared with hydrodynamic ones. The validity of the criticism and remarks uttered during the Chamberlain-Parker controversy (for the solar wind), and the dispute between Banks and Holzer on the one hand, and Dessler and Cloutier on the other (for the polar wind), are carefully analyzed. The main result of this study is that both approaches are in fact not contradictory but complementary. The classical hydrodynamic descriptions are only appropriate in the collision-dominated region, whereas the kinetic theory can be applied only in the collision-free domain.

CONTENTS

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Kinetic Physics of the Solar Corona and Solar Wind

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Abstract

Kinetic plasma physics of the solar corona and solar wind are reviewed with emphasis on the theoretical understanding of the *in situ* measurements of solar wind particles and waves, as well as on the remote-sensing observations of the solar corona made by means of ultraviolet spectroscopy and imaging. In order to explain coronal and interplanetary heating, the micro-physics of the dissipation of various forms of mechanical, electric and magnetic energy at small scales (e.g., contained in plasma waves, turbulences or non-uniform flows) must be addressed. We therefore scrutinise the basic assumptions underlying the classical transport theory and the related collisional heating rates, and also describe alternatives associated with wave-particle interactions. We elucidate the kinetic aspects of heating the solar corona and interplanetary plasma through Landau- and cyclotron-resonant damping of plasma waves, and analyse in detail wave absorption and micro instabilities. Important aspects (virtues and limitations) of fluid models, either single- and multi-species or magnetohydrodynamic and multi-moment models, for coronal heating and solar wind acceleration are critically discussed. Also, kinetic model results which were recently obtained by numerically solving the Vlasov–Boltzmann equation in a coronal funnel and hole are presented. Promising areas and perspectives for future research are outlined finally.