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Self-consistent model of mirror structures

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Abstract

The purpose of this work is to give a self-consistent model of the magnetic mirrors using a perturbative magnetohydrostatic approach. With the help of this model a number of features have been revealed like geometry, stability and behavior for different temperature anisotropies ($A = T_\perp/T_\parallel$). The basic relations we use in order to derive the model for the mirror structures are the magnetohydrostatic equilibrium condition and an expression for the anisotropy in the case of bi-Maxwellian distribution (Lee et al., J. Geophys. Res. 92 (1987) 2343). Based on these equations, we have found analytical expressions for the magnetic field (δB), pressure (δp) and temperature (δT) perturbations. From the investigation of the dependence of the magnetic mirrors on the unperturbed anisotropy (A_0), we have found the well-known behavior (opposite phase variations of the magnetic field intensity and number density) for $A_0 > 1$ (Tsurutani et al., Geophys. Res. 87 (1982) 6060). For $A_0 < 1$, the behavior is different but the mirror structures still exist. However, if the anisotropy is in a range of values depending on the plasma parameter $\beta_{0\perp} = p_{0\perp}/(B_0^2/2\mu_0)$, the magnetic mirrors can no longer exist. From the comparison between the current density deduced from the Ampere law, necessary to sustain the magnetic mirror, and the gradient-curvature drift current density actually being inside the magnetic mirror, we have been able to determine instability regions in the $(A_0, \beta_{0\perp})$-plane.

Keywords: Magnetic mirrors; Anisotropy

1. Introduction

Magnetic mirror structures have been observed in the day side of the terrestrial magnetosphere, between bow-shock and magnetopause in the magnetometric data from EQS spacecraft. These structures have also been observed by previous missions both in the day side of the magnetosheath (ISEE 1 and 2, OGO 5) and in the distant magnetotail (ISEE 3).

The basic relations we use in order to derive the model for mirror structures are

$$\nabla\left(p_\perp + \frac{B^2}{2\mu_0}\right) + \nabla\left[\left(p_\parallel - p_\perp - \frac{B^2}{\mu_0}\right)\frac{\underline{BB}}{B^2}\right] = 0, \quad (1)$$

$$A(\rho,z) = \frac{T_\perp(\rho,z)}{T_\parallel(\rho,z)} = \frac{p_\perp(\rho,z)}{p_\parallel(\rho,z)} = \left[1 - \left(1 - \frac{1}{A_0}\right)\frac{B_0}{B(\rho,z)}\right]^{-1}, \quad (2)$$

where $\underline{BB}$ is the tensor with elements $(\underline{BB})_{ij} = B_i B_j$, $p_\parallel$, $p_\perp$ are the parallel and the perpendicular plasma pressures and A_0 is the unperturbed anisotropy. Tsurutani et al., 1982 Eq. (1) is the magnetohydrostatic equilibrium condition and Eq. (2) expresses the temperature anisotropy in the case of bi-Maxwellian distribution.

In the next section, we will derive Eq. (2) that links the anisotropy before and after the perturbation was applied.

2. The anisotropy for a bi-Maxwellian distribution

Lee has derived Eq. (2) for the bi-dimensional case (Lee et al., 1987). We will show, that this relation is still correct for the 3D case. In order to do this, we will consider the plasma obeying a bi-Maxwellian distribution both before and after applying the perturbation. The unperturbed distribution function for electrons and protons is

$$f_0^{(s)}(v_{\perp 0}, v_{\parallel 0}) = \frac{n_0^{(s)}}{T_{\perp 0}T_{\parallel 0}^{1/2}}\left(\frac{m^{(s)}}{2\pi k_B}\right)^{3/2} \times \exp\left\{-\frac{m^{(s)}v_{\perp 0}^2}{2k_B T_{\perp 0}} - \frac{m^{(s)}v_{\parallel 0}^2}{2k_B T_{\parallel 0}}\right\}. \quad (3)$$

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The superscript $^{(s)}$ distinguishes between electrons and protons. The argument of the exponential factor in the distribution function can be expressed in terms of the kinetic energy and the magnetic moment (witch are invariants):

$$-\frac{W^{(s)}}{k_B T_{\|0}} - \frac{\mu^{(s)} B}{\alpha_0 k_B T_{\|0}}. \tag{4}$$

In the previous relation, we have denoted $\alpha_0 = \frac{1}{(T_{\|0}/T_{\perp 0})-1}$.

Owing to the invariance of $W^{(s)}$ and $\mu^{(s)}$ (the number of particles with certain energy and magnetic moment must remain the same), the distribution function after applying the perturbation is

$$f^{(s)}(v_\perp, v_\|, \rho, z) = \frac{n_0^{(s)}}{T_{\perp 0} T_{\|0}^{1/2}} \left(\frac{m^{(s)}}{2\pi k_B}\right)^{3/2} \exp\left\{-\frac{m^{(s)}}{2\pi k_B T_{\|0}} \left[v_\|^2 + v_\perp^2 \left(1 + \frac{1}{\alpha_0}\frac{B_0}{B(\rho,z)}\right)\right]\right\}. \tag{5}$$

The final number density can be found through integration of the distribution function:

$$n^{(s)}(\rho,z) = n_0^{(s)} \left(1 + \frac{1}{\alpha_0}\right)\left(1 + \frac{1}{\alpha_0}\frac{B_0}{B(\rho,z)}\right)^{-1}. \tag{6}$$

We can express the distribution function in terms of final number density

$$f^{(s)}(v_\perp, v_\|, \rho, z) = \frac{n^{(s)}(\rho,z)}{T_{\|0}^{1/2}\tau}\left(\frac{m^{(s)}}{2\pi k_B}\right)^{3/2} \times \exp\left\{-\frac{m^{(s)} v_\|^2}{2 k_B T_{\|0}} - \frac{m^{(s)} v_\perp^2}{2 k_B \tau}\right\}. \tag{7}$$

Above, we have denoted by τ the following quantity:

$$\tau = T_{\|0}\left(1 + \frac{1}{\alpha_0}\frac{B_0}{B(\rho,z)}\right)^{-1}. \tag{8}$$

Since we find the distribution function to remain bi-Maxwellian after the perturbation was applied, from Eq. (7) we find the final orthogonal and parallel temperatures:

$$T_\|(\rho,z) = T_{\|0}, \tag{9}$$

$$T_\perp(\rho,z) = \tau(\rho,z). \tag{10}$$

These relations give us the anisotropy Eq. (2) in the presence of the perturbation.

3. The magnetic field

The magnetic field in the first order of the perturbation theory is

$$\mathbf{B}(\rho,z) = B_0\mathbf{e}_z + \delta\mathbf{B}(\rho,z), \tag{11}$$

$$B_\varphi = 0 \tag{12}$$

and the pressure is

$$\underline{p} = p_\perp \underline{I} + (p_\| - p_\perp)\underline{e_B e_B}, \tag{13}$$

where $\underline{I}$ is the unit tensor.

From Eq. (2) the pressure perturbation on the parallel direction is

$$\delta p_\|(\rho,z) = (p_{0\perp} - p_{0\|})\frac{\delta B_z(\rho,z)}{B_0} + p_{0\|}\frac{\delta p_\perp(\rho,z)}{p_{0\perp}}. \tag{14}$$

Substituting the first-order expressions of the magnetic field Eq. (1) and of the pressure Eq. (14) in the equation of the magnetohydrostatic equilibrium Eq. (1) and taking into account the divergence-less condition for the magnetic field ($\nabla\mathbf{B} = 0$). We obtain the following system of coupled differential equations:

$$\frac{\partial}{\partial\rho}\delta p_\perp + \frac{B_0}{\mu_0}\frac{\partial}{\partial\rho}\delta B_z + \frac{1}{B_0}\left(p_{0\|} - p_{0\perp} - \frac{B_0^2}{\mu_0}\right)\frac{\partial}{\partial z}\delta B_\rho = 0, \tag{15}$$

$$\frac{p_{0\|}}{p_{0\perp}}\frac{\partial}{\partial z}\delta p_\perp + \frac{2}{B_0}(p_{0\perp} - p_{0\|})\frac{\partial}{\partial z}\delta B_z = 0, \tag{16}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\delta B_\rho) + \frac{\partial}{\partial z}\delta B_z = 0. \tag{17}$$

The magnetic field and the plasma pressure are periodical with respect to z coordinate ($\mathbf{B}(\rho, z + 2L) = \mathbf{B}(\rho, z)$ and $\underline{p}(\rho, z + 2L) = \underline{p}(\rho, z)$); therefore, we can expand them in Fourier series

$$\delta B_\rho(\rho,z) = \sum_{n=-\infty}^{\infty} \delta B_\rho^n(\rho) e^{-in(\pi z/L)}, \tag{18}$$

$$\delta B_z(\rho,z) = \sum_{n=-\infty}^{\infty} \delta B_z^n(\rho) e^{-in(\pi z/L)}, \tag{19}$$

$$\delta p_\perp(\rho,z) = \sum_{n=-\infty}^{\infty} \delta p_\perp^n(\rho) e^{-in(\pi z/L)}. \tag{20}$$

Substituting these Fourier expansions into the system Eqs. (15)–(17) we get a set of Bessel equations, one for each Fourier order of the ρ-component of the magnetic field perturbation:

$$\rho^2\frac{d^2}{d\rho^2}\delta B_\rho^n + \rho\frac{d}{d\rho}\delta B_\rho^n + \left[\left(\frac{n\alpha\rho}{L}\right)^2 - 1\right]\delta B_\rho^n = 0. \tag{21}$$

By α we denoted the subsequent adimensional quantity:

$$\alpha = \pi\sqrt{\frac{\frac{1}{2}(1 - 1/A_0) + 1/\beta_{0\perp}}{A_0 - 1 - 1/\beta_{0\perp}}} \tag{22}$$

$\beta_{0\perp}$ being the plasma parameter, i.e. the ratio between the orthogonal plasma pressure, $p_{0\perp}$ and the magnetic pressure, $B_0^2/2\mu_0$.

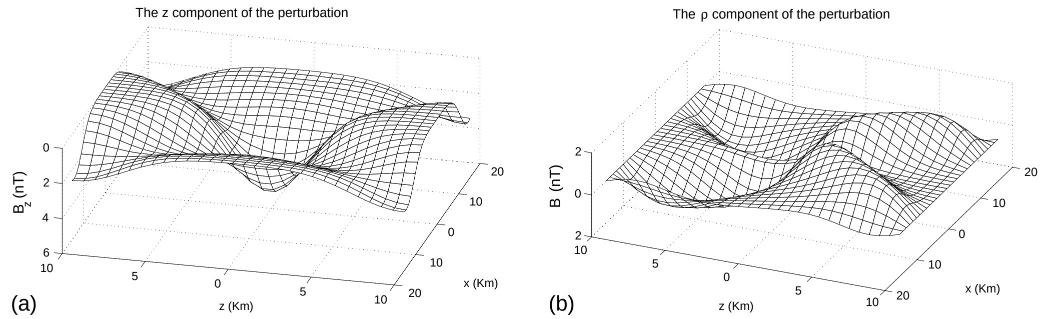

Fig. 1. The magnetic field perturbation for the main structure. The perturbation on the axis has been chosen as Gaussian: $\delta B_z(0,z) = \delta B_z(0,0)\exp(-z^2/a^2), a = L/3$, the initial anisotropy is $A_0 = 0.4$, the unperturbed magnetic field is $B_0 = 20$ nT, the perturbation in the middle of the bottle is $\delta B_z(0,0) = -5$ nT, the unperturbed number density is $n_0 = 50\ \mathrm{cm}^{-3}$, the length of the bottle is $2L = 20$ Km and the initial orthogonal temperature is $T_{0\perp} = 10^6$ K. With these parameters the radius of the main structure is $R = 15.42$ km, $\beta_{0\perp} = 4.33$.

The non-divergent solution of Eq. (21) is

$$\delta B^n_\rho(\rho) = \frac{i\pi}{\alpha} C_n J_1\left(\frac{n\alpha\rho}{L}\right). \tag{23}$$

This solution is non-divergent only if the argument of the Bessel function J_1 is a real number. Hence, $\alpha^2 > 0$ is a condition for the existence of the mirror structures. This condition is equivalent to

$$A_0 > 1 + \frac{1}{\beta_{0\perp}}, \tag{24}$$

or

$$A_0 < \frac{\beta_{0\perp}}{\beta_{0\perp} + 2}. \tag{25}$$

The first inequality, Eq. (24), is the same as the mirror instability condition derived from the kinetic theory (Baumjohann and Treumann, 1997). On the other hand, Eq. (25) shows us that the mirror structures could exist even for anisotropies less than one.

The Fourier components of the z-component of the magnetic field perturbation are

$$\delta B^n_z(\rho) = C_n J_0\left(\frac{n\alpha\rho}{L}\right). \tag{26}$$

The magnetic field has to be real. In order to ensure this, the coefficients C_n must satisfy the following relation

$$C_{-n} = C^*_n. \tag{27}$$

Here z^* means the complex conjugate of the number z. The boundary condition used to establish the C_n coefficients is the perturbation of the magnetic field on the z-axis.

The components of the magnetic field perturbation become

$$\delta B_\rho(\rho,z) = \frac{2\pi}{\alpha}\sum_{n=1}^{\infty} J_1\left(\frac{n\alpha\rho}{L}\right) \left[a_n \sin\left(\frac{n\pi z}{L}\right) - b_n \cos\left(\frac{n\pi z}{L}\right)\right], \tag{28}$$

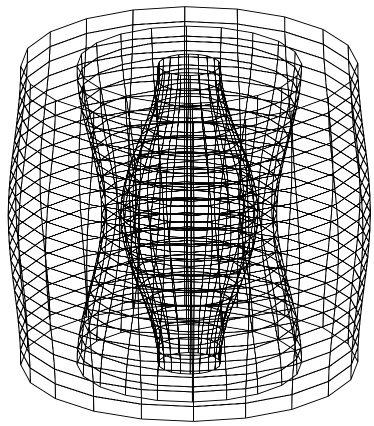

Fig. 2. The surfaces defined by the field lines for the first Fourier component of the magnetic field perturbation. The main structure has the well-known bottle shape. As we move away from the axis, we encounter other structures with similar symmetry, wrapping up each other. (arbitrary units).

$$\delta B_z(\rho,z) = 2\sum_{n=1}^{\infty} J_0\left(\frac{n\alpha\rho}{L}\right) \left[a_n \cos\left(\frac{n\pi z}{L}\right) + b_n \sin\left(\frac{n\pi z}{L}\right)\right]. \tag{29}$$

The quantities a_n and b_n are the real and the imaginary parts of the coefficients C_n. The components of the magnetic field perturbation for a Gaussian boundary condition are shown in Figs. 1(a) and (b).

A closer look at the Fourier components in Eqs. (28) and (29) reveals for each of them a complex geometry that is depicted in Fig. 2.

For each Fourier order, the radius of the main magnetic bottle is established by the first zero of the Bessel function J_1. At this radius, the field lines are straight and parallel to

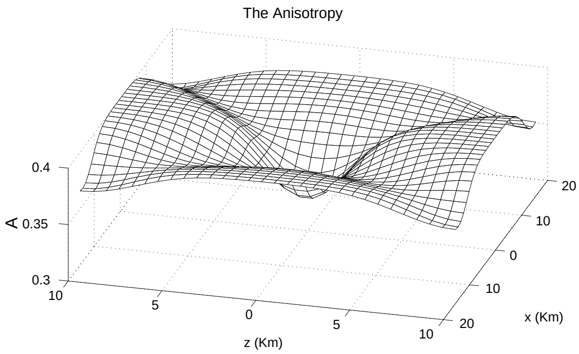

Fig. 3. The anisotropy of the main structure. The parameters are the same as in Fig. 1. For this case ($A_0 < 1$) the variation of the anisotropy is in phase with the variation of the magnetic field magnitude—see Fig. 1(a). For $A_0 > 1$ the variations are in opposite phase.

the z-axis.

$$\frac{\alpha R}{L} = 3.832. \tag{30}$$

The first-order perturbations of the anisotropy and plasma pressure are (Fig. 3)

$$\delta p_{\perp}(\rho, z, N) = -2 p_{0\perp}(A_0 - 1)\, \frac{\delta B_z(\rho, z, N)}{B_0}, \tag{31}$$

$$\delta p_{\parallel}(\rho, z, N) = -p_{0\parallel}(A_0 - 1)\, \frac{\delta B_z(\rho, z, N)}{B_0}, \tag{32}$$

$$\delta A(\rho, z, N) = -A_0(A_0 - 1)\, \frac{\delta B_z(\rho, z, N)}{B_0}. \tag{33}$$

These last three relations have also been obtained from the kinetic theory (Hasegawa, 1969).

4. The instability mechanism

Using Ampere's law, we find the electric current density that exists inside the mirror structure:

$$\mathbf{j}_B(\rho, z) = \frac{2\pi^2}{\mu_0 \alpha L}\left[1 + \left(\frac{\alpha}{\pi}\right)^2\right] \sum_{n=1}^{\infty} n J_1\left(\frac{n\alpha\rho}{L}\right) \left[a_n \cos\left(\frac{n\pi z}{L}\right) + b_n \sin\left(\frac{n\pi z}{L}\right)\right] \mathbf{e}_{\varphi}. \tag{34}$$

This ring current is illustrated in Fig. 4.

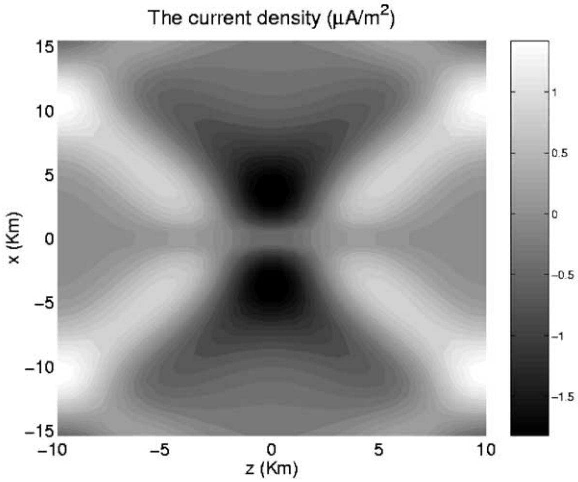

Fig. 4. The ring current density $\mathbf{j}_B$. A central ring current bordered upon opposite sense ring currents can be seen. The parameters are the same as in Fig. 1.

The actual current density $\mathbf{j}_d$ existing in the mirror structure can be calculated from the gradient-curvature drift

$$\mathbf{j}_d = \frac{1}{4}\, \frac{3\beta_{0\perp} + 2}{2A_0 + 1}\, \mathbf{j}_B. \tag{35}$$

At equilibrium $\mathbf{j}_d = \mathbf{j}_B$. In this case, the magnetic field perturbation produces a drift current which in turn sustains the perturbation. Depending on A_0 and $\beta_{0\perp}$, the drift current might be greater or smaller than the current $\mathbf{j}_B$ required to sustain the magnetic field perturbation. If $\mathbf{j}_d > \mathbf{j}_B$, then the perturbation induced by the drift current will be greater than the original perturbation, consequently, the drift current intensity will increase. In this situation, the mirror structure is unstable. Similarly, if $\mathbf{j}_d < \mathbf{j}_B$, then the magnetic field perturbation will decrease. Therefore, the instability condition for the magnetic mirrors is

$$A_0 < \tfrac{1}{4}(\tfrac{3}{2}\beta_{0\perp} - 1). \tag{36}$$

Making use of Eqs. (36), (24) and (25) we have plotted the existence and instability domains for the magnetic mirrors in Fig. 5.

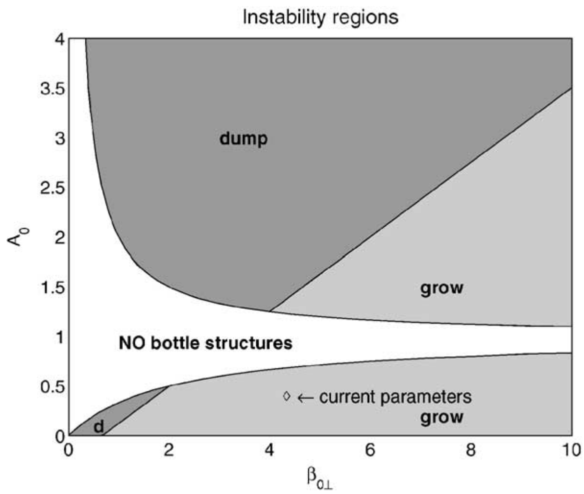

Fig. 5. The existence and instability domains in the $(A_0, \beta_{0\perp})$-plane. In the white region, the existence condition is not fulfilled, in the light gray regions the mirror structures are unstable and in the dark gray regions the perturbation will be dumped.

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